

Algebra II (2008)

Exercise 4/week 45

1. Find the degree, and the leading and the constant coefficients of $x + x^3 + 2x^4 \in \mathbb{F}_2[x]$
2. Let $f(x) = 3 + x^2 + 2x^3, g(x) = 2 + x + 17x^5 \in \mathbb{F}_3[x]$. Give explicitly the polynomials $f(x) + g(x)$ and $f(x)g(x)$. What is the additive inverse of $f(x)$?
3. Find $\gcd(x^3 + 6x + 4, x^5 + x^3 + 1)$ in $\mathbb{F}_7[x]$ by using Euclidean algorithm.
4. Form the calculation tables of the ring $\mathbb{F}_2[\alpha] := \mathbb{F}_2[x]/(x^3 + x^2 + x + 1)$. Find the units of $\mathbb{F}_2[\alpha]$. Is $\mathbb{F}_2[\alpha]$ a field?
5. Form the calculation tables of the ring $\mathbb{F}_2[\alpha] := \mathbb{F}_2[x]/(x^3 + x + 1)$. Find the units of $\mathbb{F}_2[\alpha]$. Is $\mathbb{F}_2[\alpha]$ a field?
6. (a) Let $f(x) = f_0 + f_1x + \dots + f_nx^n \in \mathbb{Z}[x]$ with $f_0f_n \neq 0$. Prove: If $f(a) = 0$ for some $a \in \mathbb{Q}$, then $a = b/c$ for some integers b and c satisfying $b|f_0$ and $c|f_n$.
(b) Find the canonical factorization of $3x^3 + 4x^2 - 7x + 2$ in $\mathbb{Q}[x]$ and in $\mathbb{R}[x]$.
7. Find all irreducible polynomials over \mathbb{F}_2 of degree 4.