Algebra II (2008)

Exercise 2/week 43

1. Calculate $\text{ord}(\bar{2})$ in $\mathbb{Z}_{101}^*$. Is $\mathbb{Z}_{101}^*$ cyclic?
2. Find the cosets of $< \bar{3} >$ in $\mathbb{Z}_{16}^*$, and give a complete set of representatives for the cosets.
3. The group of *quaternions* $(Q, \cdot)$ is defined as follows:

$$Q = \{I, A, A^2, A^3, B, BA, BA^2, BA^3\}$$

   where

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ B = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \qquad (i^2 = -1).$$

   Find the cosets of $\langle A \rangle$ in Q.
4. Let G be a group having only the trivial subgroups. Show that G is cyclic.
5. Let G be a cyclic group. Prove: $G \simeq \mathbb{Z}$ or $G \simeq \mathbb{Z}_n$ for some $n \geq 1$
6. Let G be a group having only finite number of subgroups. Show that G is finite.
7. Let $f : G \to G'$ be a homomorphism. Show that $\ker f = \{e\}$ if and only if f is injective.