**Algebra II (2008)**

Exercise 1/week 42

1. Solve the congruence $104x \equiv 56 \ (132)$ by using Euclidean algorithm.
2. (a) Show that $(\mathbb{Q} \setminus \{1\}, \circ)$ is an Abelian group if the group operation $\circ$ is defined as follows:
$$a \circ b = ab - a - b + 2.$$
   (b) Solve: $x^2 = 0$ and $x^2 = 2$ where $x^2 = x \circ x$.
3. (a) Give the group table of $\mathbb{Z}_9^*$.
   (b) Use the group table to find $\bar{7}^{-1}$ in $\mathbb{Z}_9^*$.
4. Prove the subgroup criterion: A non-empty set $H$ of a group $G$ is a subgroup of $G$ if and only if $ab^{-1} \in H$ for all $a, b \in H$.
5. Let $m$ be an integer. Show that the solutions of $x^m = 1$ in $\mathbb{C}$ form a group.
6. Find an element of order 3 in $\mathbb{Z}_{21}$.
7. Find all finite subgroups of $(\mathbb{R}^*, \cdot)$.
8. Let $(G, \star)$ be a group. Prove the following assertions:
   (a) $(a \star b)^{-1} = b^{-1} \star a^{-1} \quad \forall\, a, b \in G$.
   (b) if $x \star x = e$ for all $x \in G$ then $G$ is Abelian.