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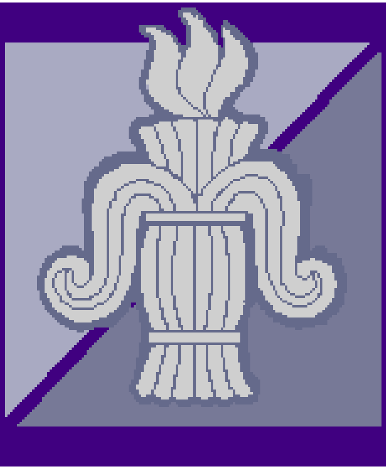
Timo Salmi – Ilkka Virtanen

Deriving the Internal Rate of Return from the Accountant's Rate of Return

A Simulation Testbench



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Deriving the Internal Rate of Return from the Accountant's Rate of Return

A Simulation Testbench

Timo Salmi
Professor of Accounting and Business Finance
Email: ts@uwasa.fi
http://www.uwasa.fi/~ts/

Ilkka Virtanen
Professor of Operations Research and Management Science
Email: itv@uwasa.fi
http://www.uwasa.fi/~itv

University of Vaasa, Finland
PO Box 700
FIN-65101 Vaasa
Finland

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ABSTRACT

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This paper presents a realistic simulation testbench for evaluating the various methods used to estimate the long-term profitability of firms in terms of the internal rate of return (IRR) on their capital investments. The simulation model extends the earlier, rigid approaches by incorporating business cycles and capital investment shocks. Kay's IRR estimation method is used to demonstrate the improved simulation approach. When the growth rate and profitability are near each other, Kay's method yields accurate estimates as is expected by theory. The more growth and profitability differ, the less accurate the estimates will be. The magnitude (and even the direction) of the error depends on the depreciation method applied and the capital investments' contribution distribution. It is also seen that Kay's method is insensitive to full business cycles, whereas it is disrupted by excessive capital investment shocks.

Timo Salmi and Ilkka Virtanen, Faculty of Accounting and Industrial Management, University of Vaasa, P.O. Box 700, FIN–65101 Vaasa, Finland.

Key words: *Long-term profitability, accountant's rate of return, internal rate of return, Kay's IRR estimation model, simulation.*

1. INTRODUCTION

1.1 Background

It is hardly an exaggeration to say that the questions of income determination and the valuation of the firm's assets are the most important questions in accounting research. The measurement of profitability is intimately linked to these fundam-ental areas. The question of a theoretically sound and pragmatic profitability measurement is of crucial importance for an economy's welfare. The allocation of resources in an economy is directly affected by the validity and reliability of the decision makers' measures of the firms' performance (profitability) and financial position.

The accountant traditionally measures profitability as the ratio between the firm's annual income and the book value of its assets. This ratio is often called the accountant's rate of return (ARR) in literature. Other common terms for it are the return on the capital invested (ROI) and the book yield. This measure looks at profitability in retrospect. The economist has a different definition of income. It is based on the changes in the market value of the firm defined as its discounted future cash flows. The economist's definition is based on expectations about the future. The internal rate of return (IRR) is consistent with the economist's concept of income. The internal rate of return also is prominent in the capital investment theory.

One traditional way of looking at the firm is to regard it as a series of capital investments. It is fairly well-accepted that theoretically the IRR of the capital investments making up the firm is the valid measure of the firm's profitability. The problem with this theoretical notion is, however, that the IRR of the firms is not readily measurable in actual business and financial analysis practice, while the ARR is calculated routinely for business firms. There is a considerable body of literature that discusses the possibility of analytically deriving or empirically estimating the firm's IRR from its ARR, estimating the IRR from the firm's cash recovery rate (CRR) which is easier to estimate than IRR, or estimating the IRR directly from the published financial statements. For a review of the literature on the profitability measurement of the firm as IRR estimation see the review article by Salmi and Martikainen (1994, Ch. 3) and the comprehensive references in it.

The results and the various methods to estimate the IRR have been controversial. There is no clear consensus as to the validity and the reliability of the different models to estimate the long-term profitability from the published financial statements. The difficulty is that even if empirical estimates of the IRR given by the various methods have been compared, their relative validity and reliability cannot be established unless the true IRR of the firms are known, and this is not the case when using actual financial statement data.

1.2 Research Problem and Methodology

Since the results and views on the validity of IRR estimation in literature are controversial, an objective and operational methodology is needed to assess the validity and reliability of the models to measure the firm's IRR. The general aim of this paper is to develop a realistic simulation approach for the evaluation.

The simulation approach to evaluate IRR estimation methods was introduced by Salmi and Luoma (1981). We extend and generalize their simulation approach to form a basis for a later comparison of the different IRR estimation methods. This paper extends the simulation model by improving its realism in the capital investment behavior of business firms and in the accounting practices in the depreciation methods. In this paper we demonstrate the usage of the simulation approach on the IRR estimation method presented by Kay (1976).

Before any IRR estimation method can be applied on the simulated (or actual financial) statements, the IRR estimation method must be made operational for the financial data available. This process for Kay's (1976) IRR estimation method has been presented in Salmi and Luoma (1981).

Kay's model is analyzed using simulated financial data where the true IRR will thus be known in advance. First, knowing the IRR in advance enables assessing whether a model estimates the true IRR correctly. Second, the sensitivity of a method to the firm's parameters can be studied. In this paper these parameters in evaluating Kay's method include the investment policy (the growth rate and pattern), the pattern of the contributions from the capital investments (the contribution distribution), and the depreciation method (straight-line and double declining balance methods). Furthermore, we also evaluate the results with data that deviate from the usual steady state assumptions.

2. SIMULATION MODEL

This chapter presents the theoretical layout and the derivation of our simulation model. First, we present the part of the model describing the firm as a capital investment process. Second, we discuss some of the essential assumptions in measuring long-term profitability. Third, we consider the process for simulating the annual profits and asset valuation. Fourth, we discuss the alternative depreciation methods for the simulated, annual profit assessment.

2.1 The Firm as a Capital Investment Process

The simulated firm is basically made up by the annual cash outflows to the capital investments and the cash inflows generated by the capital investments. This is because in the theory of accounting a firm can be deemed a series of cash outflows to investments and cash inflows from them. After a depreciation method is chosen, the annual operating income of the simulated firm becomes defined, and the book value of the firm is determined. In the numerical simulations the data to be analyzed is taken from the period after the process is past the transient, initial stage.

Denote

g_t = capital expenditures in year t
f_t = cash inflow in year t
b_i = relative contribution from capital investment i years back
f_{ti} = absolute contribution in year t from capital investment i years back
d_t = depreciation in year t
p_t = accountant's profit (operating income) in year t
v_t = book value of the firm's assets at the end of year t
w_t = market value of the firm's assets at the end of year t
T = length of the simulation period
n = length of the observation period (number of years under observation
for the profitability estimation)
N = life-span of every capital investment project
r = true internal rate of return
IRR = estimated internal rate of return
k = growth rate
A = amplitude of the business cycle
C = length of the business cycle
φ = phase adjustment for the business cycle
S = capital investment shock coefficient
τ = the year of the capital investment shock ($\tau = \infty$ for no shock in the simulation)

An economic time series is made up by several constituents. These are the growth trend, the business cycle, the seasonal variation and the noise.

Furthermore, there can be regular or irregular shocks. We use the following model for the capital investments in our simulation model:

(1) $$g_t = g_0 (1+k)^t \{1+A \sin[(2\pi t / C)+\varphi]\}[1+\delta_{t\tau} S],$$

where δ is Kronecker's delta, i.e.

(2) $$\delta_{t\tau} = 1 \text{ when } t = \tau, \text{ and } 0 \text{ otherwise.}$$

Technically, t runs from 1 to T in the simulation runs. The observation period is from T-n+1 to T. For simplicity, this fact is not repeated for the later formulas.

In the above the constant g_0 is the initial level of the capital investment expenditures. The trend is an exponential growth trend with growth rate k. In the simulation model of Salmi and Luoma (1981) only this steady state growth was used. We generalize the model by introducing business cycles and shocks into the simulation model. The cycle is given by the sinusoidal component in Formula (1) with an amplitude of A and a length of the cycle C. The term φ is a technical phase adjustment. It slightly shifts the continuous sine curve so that its maximum and minimum values agree with the discrete observations. For the average length of six years of real-life business cycles φ becomes $\pi/6$.

Our model also incorporates the possibility of introducing shocks into the system. The term $[1+\delta_{t\tau} S]$ defines the shock as a coefficient relative to the regular level of capital investments.

Seasonal variations do naturally not arise. This is because the simulation model is a discrete model with one-year intervals.

It is natural that in building a computer model for numerical simulation simplifications have to be made while trying to retain essential realism. The time-series of capital investments defined by Formula (1) does not involve random fluctuations even if it includes the possibility of an investment shock. Random fluctuations are excluded from our simulation model because they might mask the underlying regularities. Statistical estimation problems would require complicating considerations of their own.

The capital investments g_t produce later cash inflows which can be defined in terms of a contribution distribution. It is denoted by coefficients b_i where the contributions cover the life-span of each capital investment. As is familiar from capital investment literature, the capital investment model involves a discretization of what basically are partly continuous events. An initial outlay made at time $t = 0$ is assumed to produce its corresponding contributions at times $t = 1,...,N$. Likewise, the depreciations for a capital expenditure made at time $t = 0$ will take place at $t = 1,...,N$. The same pattern is repeated for all capital investments for the simulation period. Our simulation model considers all the events as discrete. Consequently, the contribution in year t from a capital investment made in year t-i is defined as

$$(3) \qquad f_{ti} = b_i \, g_{t-i}; \;\; i = 1,...,\min(N,t).$$

The total contribution f_t in year t is cumulated from the contributions from the capital investments made in the earlier years:

$$(4) \qquad f_t = \sum_{i=1}^{\min(N,t)} f_{ti} = \sum_{i=1}^{\min(N,t)} b_i \, g_{t-i}.$$

2.2 Discussion on IRR Uniformity, and on the Role of Financing

A familiar, but a very strict simplifying assumption has to be made in considering the capital investment process and the profitability. This simplification is fully in line with the literature on long-run profitability estimation. It is a basic fact that the internal rate of return of a single capital investment project is independent of its scale. We assume that contribution distribution b_i is the same for all the capital investments g_t which the simulated firm makes. Consequently, the internal rate of return (economist's long-run profitability) for the entire simulated firm can be solved from knowing any g_t and the corresponding f_{ti} values from Formula (3) independently of the size and pattern of capital investments defined by Formula (1). For an illustration of the contribution distribution see Ruuhela, Salmi, Luoma and Laakkonen (1982: 331-332). The true internal rate of return for (3) and (4) can be solved from

N

(5) $$\sum_{i=1} b_i (1+r)^{-i} = 1.$$

Profitability defined as the IRR in our simulation is assessed from the contributions of the capital investments only. The financing issue does not come to the fore. This separation of capital investments from financing is in line with the classic results of Modigliani and Miller. For a discussion on this issue, see for example Yli-Olli (1980). This separation also is in line with the standard usage of IRR in connection with the capital investment decision. In making the decision, the decision maker compares the IRR of the capital investment project prior interest to the cost of capital. Including the interest (i.e. the cost of financing) in the cash estimates for the project's flows would be double accounting as pointed out by any good textbook on capital investments.

The question of financing and its costs do not arise in our simulations as long as it can be safely assumed that the firm remains sufficiently profitable to be able to obtain new capital as the need arises. Hence chronically declining activities (divestments) or infeasible combinations of growth and profitability will not be considered in our research, since in actual business practice this would in the long-run cause restrictions or even a cessation of the availability of capital to the firm. For a discussion of feasible growth/profitability combinations see Suvas (1994).

2.3 Profits and Valuation

The accountant's profit is defined by the cash flow less depreciation

(6) $$p_t = f_t - d_t.$$

The book value of the firm at the end of period t is defined by

(7) $$v_t = v_{t-1} + g_t - d_t + e_t.$$

In a business enterprise also the retained earnings increase the book value. The retained earnings e_t are given by the operating income p_t less the interest expenses and the direct taxes and the dividends. We do not consider financing

financing in evaluating the profitability of the firm. As explained earlier, this is in line with the separation of capital investments from financing. Hence, we assume that no earnings are retained in the firm. In other words we set $e_t = 0$. If all the profits are not distributed as dividends in a real-life estimation, they become part of financing the next period's activities. This does not pose a problem, since, because of the separation, we do not need to consider in estimating the IRR whether the financing of the capital investments is by debt, retained earnings or by issuing new stock.

Depreciation and the choice of the depreciation method is a central question in the theory of income measurement. (Depreciation is discussed more fully in the next section.) The accountant's rate of return is directly dependent on it. It is given by

$$(8) \qquad ARR_t = p_t \;/\; v_t = (f_t - d_t) /\; v_t.$$

The well-known economist's valuation of the firm is defined by

$$(9) \qquad w_t = \sum_{i=t+1}^{\infty} (f_t - g_t)\,(1+r)^{t-i}.$$

Formulas (8) and (9) are not part of our current simulation model, but they are needed here for pointing out the following important theoretical result about the different depreciation and income concepts. The discussion on ARR vs IRR is basically a question about the compatibility and a connection between Formulas (5) and (8). In accordance to the classic results, IRR and ARR (appropriately weighted if not constant) agree if the annuity method of depreciation is used for depreciating the book value of the firm's assets. This result is tantamount to proving that if the economist's valuation w_t and accountant's valuation v_t of the firm's assets agree, then IRR and ARR agree. A second, relevant classic result is that if the steady state growth of the firm is equal to its internal rate of return, then ARR and IRR agree. For a discussion and a presentation of the proofs see for example Salmi and Luoma (1981). Furthermore, being able to simulate w_t is needed in our intended further research on IRR estimation models which include market values of the firms' stock. This aspect does not come up in this paper, which uses Kay's estimation method as its case.

The economist's and the accountant's valuations will agree if the annuity method of depreciation is used. Annuity method is a theoretical concept. The result referred to in the above about the equivalence between IRR and ARR under agreeing economist's and accountant's valuations would not be readily applicable for actual business practice. Contrary to the accountant's valuation economist's valuation assumes a knowledge of the future cash flows. The related annuity method of depreciation requires knowing in advance the internal rate of return of the firm's capital investments. This involves a circular deduction as pointed out for example by Salmi and Luoma (1981).

2.4 Depreciation Methods

We build the choice of three alternative depreciation methods into our simulation model. The alternatives are the annuity depreciation method, the straight-line depreciation method and the double declining balance method. The annuity depreciation method is included for theoretical reasons to verify whether the simulation and the profitability estimation algorithm give the expected results. In other words to see that the estimated profitability (IRR) is equivalent to the underlying true profitability (r) of the simulated capital investments. The derivation of any IRR estimation method becomes suspect if it fails this feasibility test.

An important part of research is to be able to evaluate how the different IRR estimation methods perform under realistic conditions. The other two included depreciation methods are prevalent in business practice. The idea of straight-line depreciation method is that it allocates the costs evenly based on the passage of time over the expected life-span of the asset. Decreasing charge depreciation methods are based on the idea of equipment being more efficient in their early life. We choose double declining balance method as a representative of the decreasing charge methods because it is by definition (the doubled rate) related to the corresponding straight-line method.

The well-accepted definition for the annuity depreciation is that the profit (before interest and taxes) p_t is assessed as the interest on the initial capital stock v_{t-1} in year t. Thus

(10) $$p_t = r\, v_{t-1}$$

and hence from Formula (6) we get

(11) $$d_t = f_t - r\, v_{t-1}.$$

As discussed above, this is a theoretical concept, since it is necessary to know the value of r (the internal rate of return) in order to be able to apply the annuity depreciation method. In a simulation model, however, this is possible since the true internal rate r is defined in advance.

For the straight-line depreciation method in our simulation model we have

(12) $$d_t = \sum_{i=1}^{\min(N,t)} (1/N)\, g_{t-i}.$$

For the double declining balance method we have

(13) $$d_t = \sum_{i=1}^{\min(N,t)} q\,(1-q)^{i-1}\, g_{t-i},$$

where $q = 2/N$. Since a double declining balance forms an infinite geometric series, the remaining book value at the end of the life-span N of each capital investment is depreciated in full in our simulation. When this is taken into account, Formula (13) can be rewritten (for years $t \geq N$) as

(14) $$d_t = \sum_{i=1}^{N} q\,(1-q)^{i-1}\, g_{t-i} + (1-q)^N\, g_{t-N}.$$

2.5 Further Considerations on the Simulation Model

The indexing warrants a technical comment. Consider a single capital investment depicted below

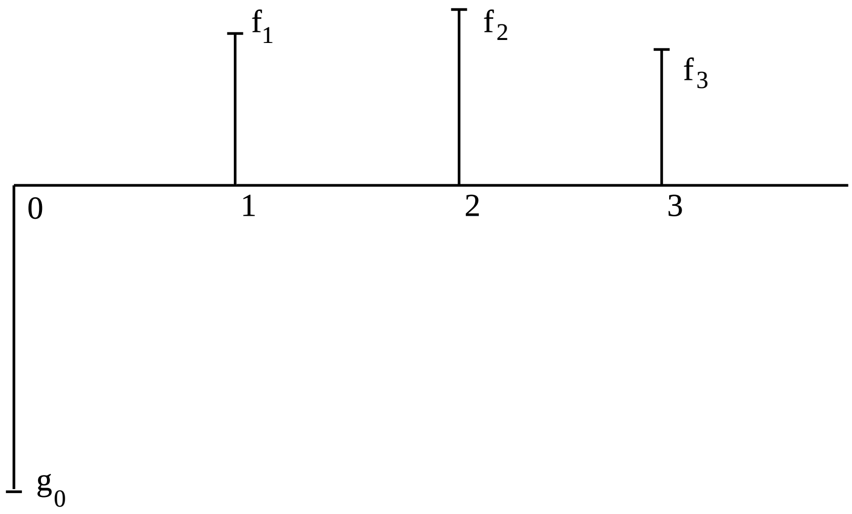

Figure 1. Structure of a Single Capital Investment Project

In actual practice the events can take place continuously during each year, but in our simulation events only occur at discrete points of time. A choice has to be made in the model about the timing of the first contribution from a capital investment. We use the same convention as the traditional capital investment model. The initial outlay is effected at instance 0 and the first contribution comes in at time 1. Depreciation must be treated consistently with this traditional approach. Thus the first depreciation for the depicted capital investment will take place at time 1, not at time 0. This will mean that the first depreciation will effectively take place a year later than the corresponding capital investment. This is an unavoidable problem in all discrete financial modelling. It is not a characteristic of our model, only.

The presented data-generating simulation model is programmed as three Turbo Pascal 7.0 source code programs on a standard MS-DOS PC. Each depreciation method gives rise to a separate program. These three programs generate the simulated data based on the input and parameter data to be discussed in Chapter 3. The listings of the programs are not included in this paper. They are, however, available upon emailed requests to ts@uwasa.fi through Internet by FTP or mail server from the garbo.uwasa.fi electronic repository at the University of Vaasa.

2.6 Kay's IRR Estimation Model

The simulation model developed in the above is applied in this paper to analyze and evaluate Kay's IRR estimation model as an example. Kay's model has been presented by Kay (1976) and further operationalized as a discrete-time version by Salmi and Luoma (1981). We repeat the discrete-time version estimation formula below derived from Salmi and Luoma

$$(15) \qquad IRR = \left[\sum_{t=2}^{n} p_t (1+IRR)^{-t} \right] / \left[\sum_{t=2}^{n} v_{t-1} (1+IRR)^{-t} \right].$$

The indexing of the years in the data-generating models runs from 0 to T and the observation period is from T-n-1 to T. For notational simplicity the indexing of the years in the IRR estimation phase has been adjusted accordingly to run from 1 to n.

The annual accountant's profit (operating income) p_t and the book values of the firm's assets v_t at the end of each year are now observed for years 1 to n. Therefore the first v_{t-1} available is for year t = 2. The estimation Formula (15) has been presented accordingly.

Kay's method is coded as a Turbo Pascal 7.0 program to produce the IRR estimates from the simulated data. The recursive estimation of IRR from Formula (15) is done using the secant method of numerical analysis. Likewise, the data-generating programs utilize the secant method to solve the true internal rate of return r from Formula (5).

3. SIMULATION DESIGN

3.1 The Outline of Data Generation

The logic of the simulation procedure is delineated in Figure 2.

Our data-generating programs produce the following layout of simulated time series data (see Table 1). As an example we present the simulated output for a uniform contribution distribution with a life-span of 20 years, double declining balance depreciation, growth rate of 8%, true profitability of 8%, amplitude coefficient 0.50 for business cycles, no shock in the form of an exceptionally large one-time capital investment. These are the factors that will be varied in our simulations. The observation period will be 13 years from the simulated year 22 to 34 (the lines not denoted by the *).

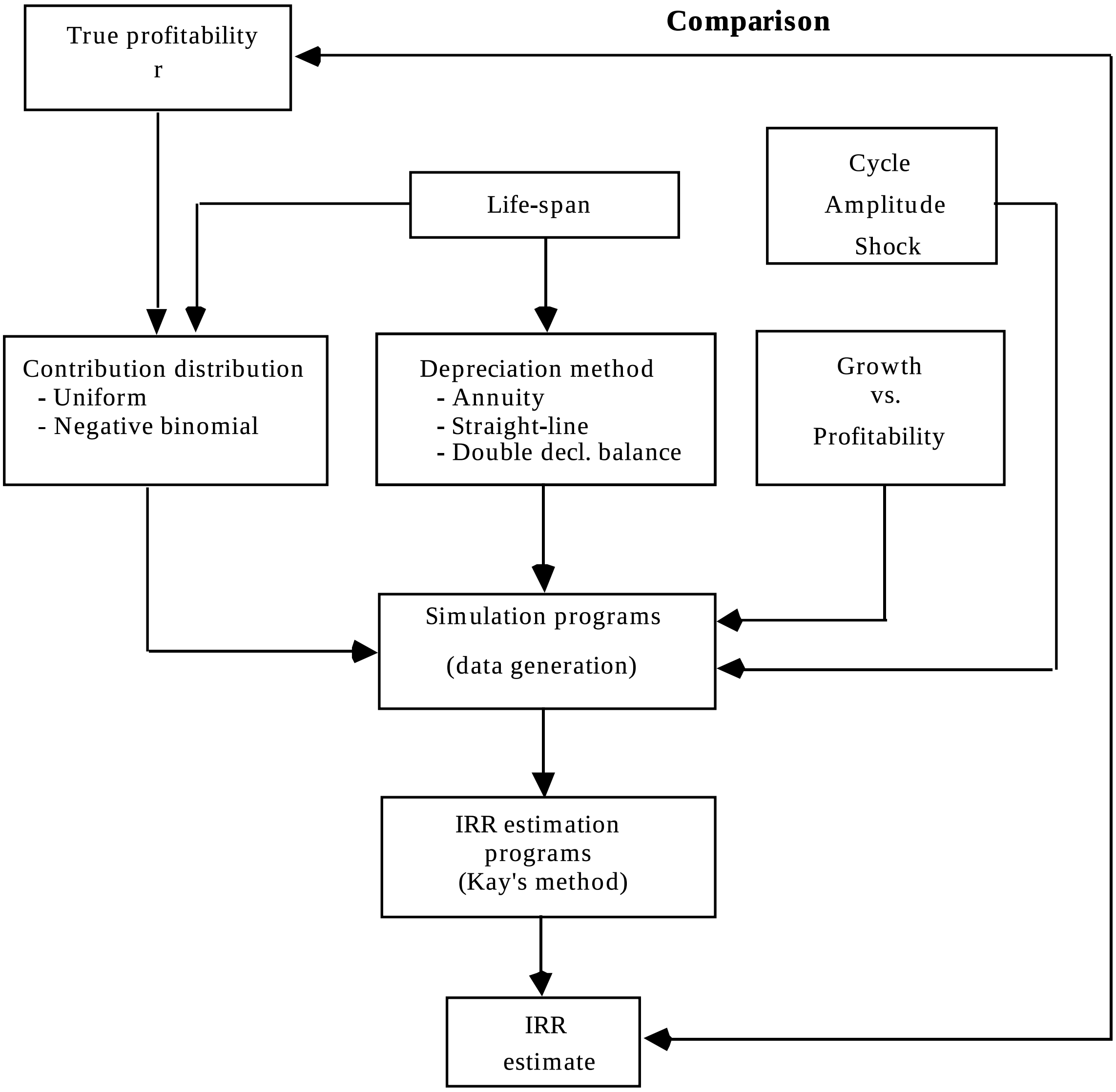

Figure 2. The Structure of the Simulation Design

The visualization of this data is given in Figure 3. Because of their different scale, book values are excluded from the visualization.

Figure 3 can be visually compared to the corresponding time series of actual business firms. Contrary to the rigid, steadily growing series of earlier simulation research, the series produced by our simulation model and parameters are realistic in terms of empirical observations. This contention is

Table 1. Example of Simulated Observations

Year t	Capital expenditure g_t	Funds from operations f_t	Declining depreciation d_t	Operating income p_t	Book value v_t
* 0	100.00	0.00	0.00	0.00	100.00
* 1	162.00	10.18	10.00	0.18	252.00
* :	:	:	:	:	:
* 21	377.53	530.25	317.10	213.15	3034.50
22	271.82	552.20	321.17	231.02	2985.15
23	440.35	565.04	310.00	255.04	3115.50
24	792.64	600.27	319.82	280.44	3588.33
25	1027.27	674.07	372.23	301.84	4243.37
26	924.54	767.48	448.45	319.02	4719.46
27	599.10	841.44	503.20	338.24	4815.37
28	431.35	876.28	509.66	366.61	4737.06
29	698.79	896.65	491.93	404.71	4943.92
30	1257.83	952.55	507.51	445.03	5694.23
31	1630.15	1069.67	590.68	478.98	6733.70
32	1467.13	1217.89	711.63	506.25	7489.20
33	950.70	1335.26	798.51	536.75	7641.38
34	684.50	1390.54	808.77	581.77	7517.11

readily corroborated by the empirical time series data gathered in the course of several research projects at University of Vaasa, such as Ruuhela, Salmi, Luoma and Laakkonen (1982). The only deviation, in principle, from actual business data is that, as explained, we have not included annual random variation in our simulated series. Such an inclusion would divert the focus to statistical estimation issues and remains a subject of potential, further research.

3.2 Contribution Distribution

The true internal rate of return is a function of the contribution distribution characterized by b_i from Formulas (3) and (5). The true form of the contribution distribution is not generally known for real-life business firms. Hence, we will use two alternative contribution distributions: a uniform contribution distribut-ion and a negative binomial contribution distribution. From using two different contribution distributions we see what kind of a bearing the form of the contribution distribution might have on the results.

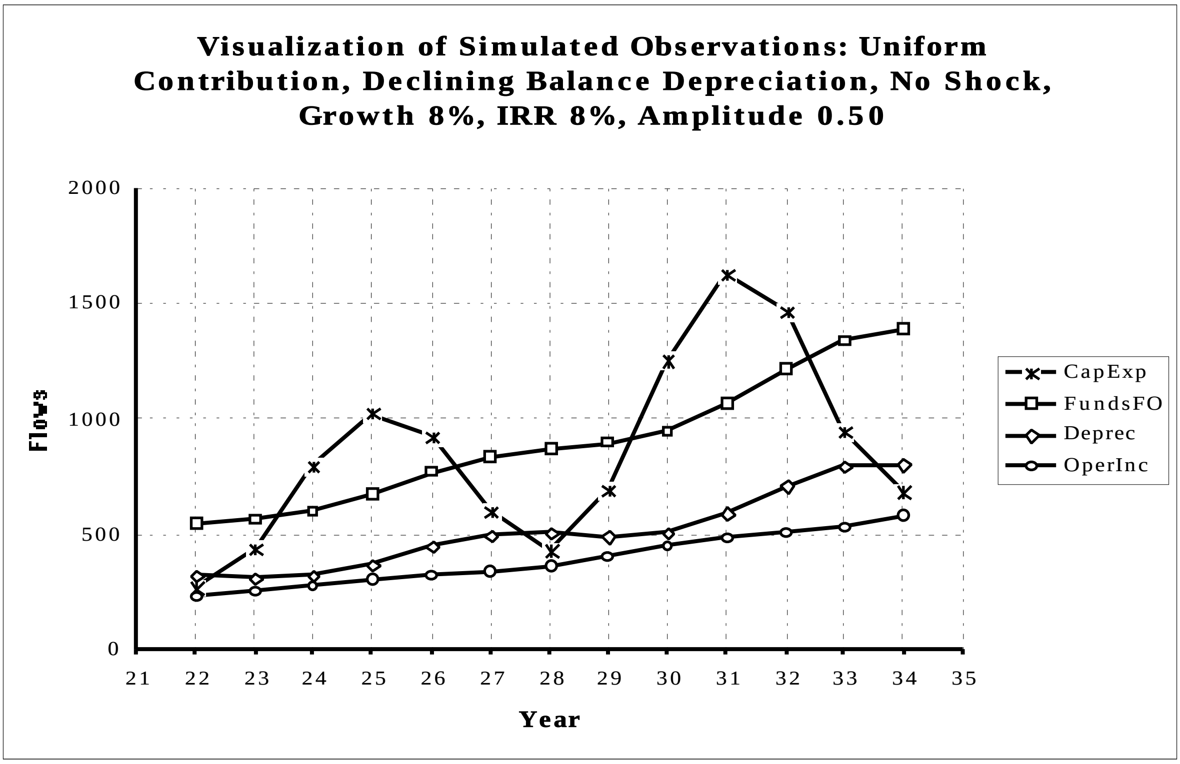

Figure 3. Visualization of Simulated Observations

A uniform contribution distribution for the life-span of the investments is an obviously neutral choice. After this choice it is easy to establish the contribution coefficients which lead to preselected true profitability figures to be discussed in the next sections. They are b_i = 0.0736 for a profitability of 4%, 0.1019 for 8%, 0.1339 for 12%, and 0.1687 for 16% when a typical life-span of 20 years is selected.

The typical life-cycle of a product includes an early growth phase, maturity, and decline. A negative binomial distribution corresponds to this cycle. For our simulation it has the further advantage of being different from the uniform contribution distribution in two important respects. It is not constant and it is not symmetrical.

The general definition for the negative binomial distribution is given by Formula (16) where the distribution parameters p and r must not be confused with our earlier definitions. We have

$$(16) \qquad P_m = \binom{m-1}{r-1} p^r \ (1-p)^{m-r} \quad \text{for } m = r, r+1, \ldots$$

where p is a shape parameter and r is a location parameter. For our simulation we choose $p = 0.85$ and $r = 2$ which leads to a typical life-cycle profile.

For our purposes, two technical adjustments to the generic negative binomial distribution are needed. First, the distribution is cut from the right at the life-span instead of letting it continue to infinity. Second, the distribution is shifted to the left to coincide with the capital investments' life-span. Hence we have as our negative binomial contribution coefficients

$$b_i = s\,(i+1)\,p^2\,(1-p)^i \quad \text{for } i = 1,2,\ldots,N, \tag{17}$$

where s is a scaling factor inducing the desired level of true profitability.

3.3 Depreciation

The life-span (N) of the capital investments is taken to be 20 years, which is a reasonable average for the simulated firm's fixed assets. This selection is based on the practical experience of one of the authors as a former writer of financial analyses in a Finnish business daily for several years.

As discussed earlier the time series are produced for three depreciation methods:

- annuity depreciation
- straight-line depreciation
- double declining balance method depreciation.

The assumed 20-years life span of the simulated capital investments means that the annual rate of depreciation in generating the simulated data is 5% in the straight-line method and 10% in the the double declining balance method.

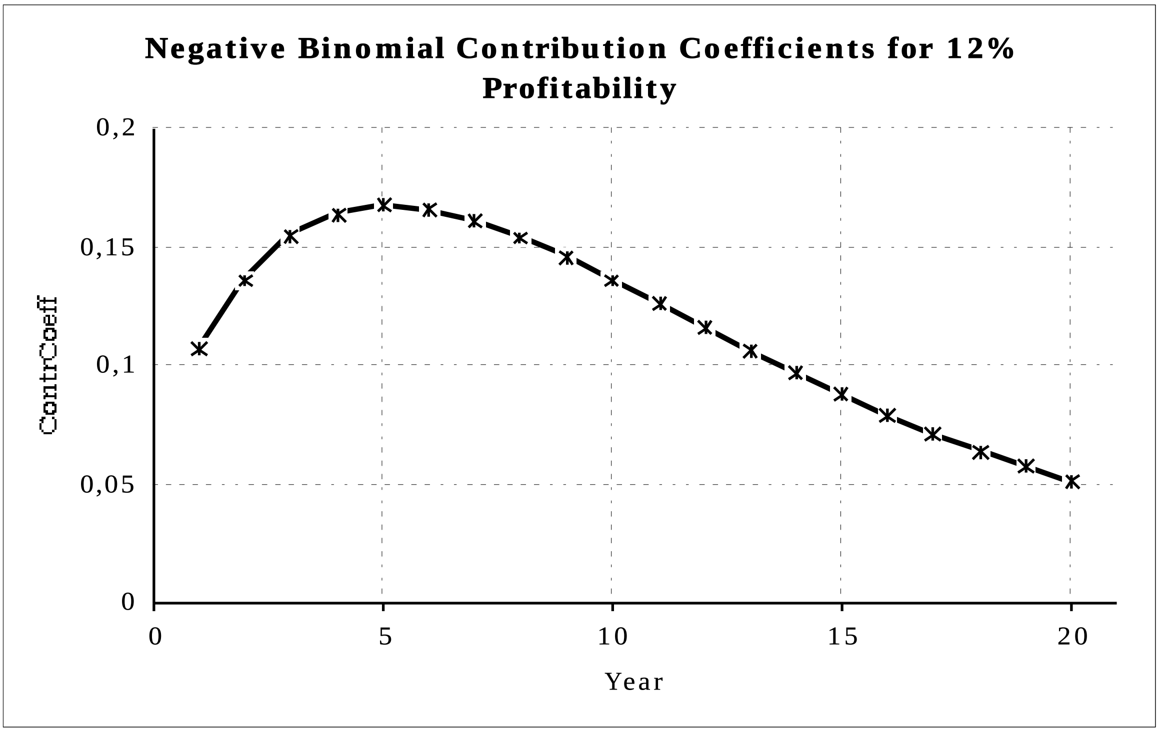

Figure 4. Negative Binomial Contributiom Distribution for 12% Profitability

3.4 Capital Investment Variation

Reconsider Formula (1) defining the level of capital investments. We assume a growth rate (k) of 8%. Only a positive growth rate is considered for the reasons explained earlier in this paper. Simulated data is generated to produce true profitability figures (r) on both sides of the growth rate. Although the growth rate has been made fairly realistic, the actual point is the relation between the profitability and growth. Either could be fixed and the other varied to achieve cases of low profitably (4%) compared to growth, equal rates (8%) and high profitabilities (12% and 16%) in relation to growth (8%). We have decided to fix the growth rate in the simulation and vary the profitability, but it could have easily been done the other way round.

The second component in the capital investment pattern in Formula (1) is the business cycle component within the braces {...}. The inclusion of the business cycle is an extension to the simulation model in Salmi and Luoma (1981). It is realistic to assume that the long-run average length of a business cycle is six years (C = 6). Three alternative amplitudes are simulated. With an amplitude A = 0 there are no cyclical fluctuations in capital investments, only the trend. With A = 1 the capital expenditures double from the trend and fall to zero in six year cycles. The amplitude A = 0.5 is between the two.

3.5 Capital Investment Shocks

The robustness of a profitability estimation method can be tested by including capital investment shocks in the model. In business terms such a shock is usually related to a major deviation from the level of capital investment pattern. It often also means a structural change to firm's activities. Therefore, it can be debated whether long-run profitability measurement stays valid under such circum-stances. We wish, however, to see what the technical effect of such instances have on the estimation when the level (but not the contribution pattern) of the capital investment deviates.

We alternatively simulate an early or a late shock during the observation period. An early shock takes place in the third year of our thirteen year simulation period. A late shock takes place in the ninth year. Both the potential shocks take place towards the end of the boom in the cycle. Two different levels are considered: a realistic, big shock and a totally unrealistic shock to test a potential estimation model break-down. In Formula (1) the former corresponds to a shock coefficient S = 5.309 and the latter to S = 17.924. The numerical values of the shock coefficients were chosen to give suitable absolute capital investment levels. Figure 5 delineates an early, realistic capital investment shock.

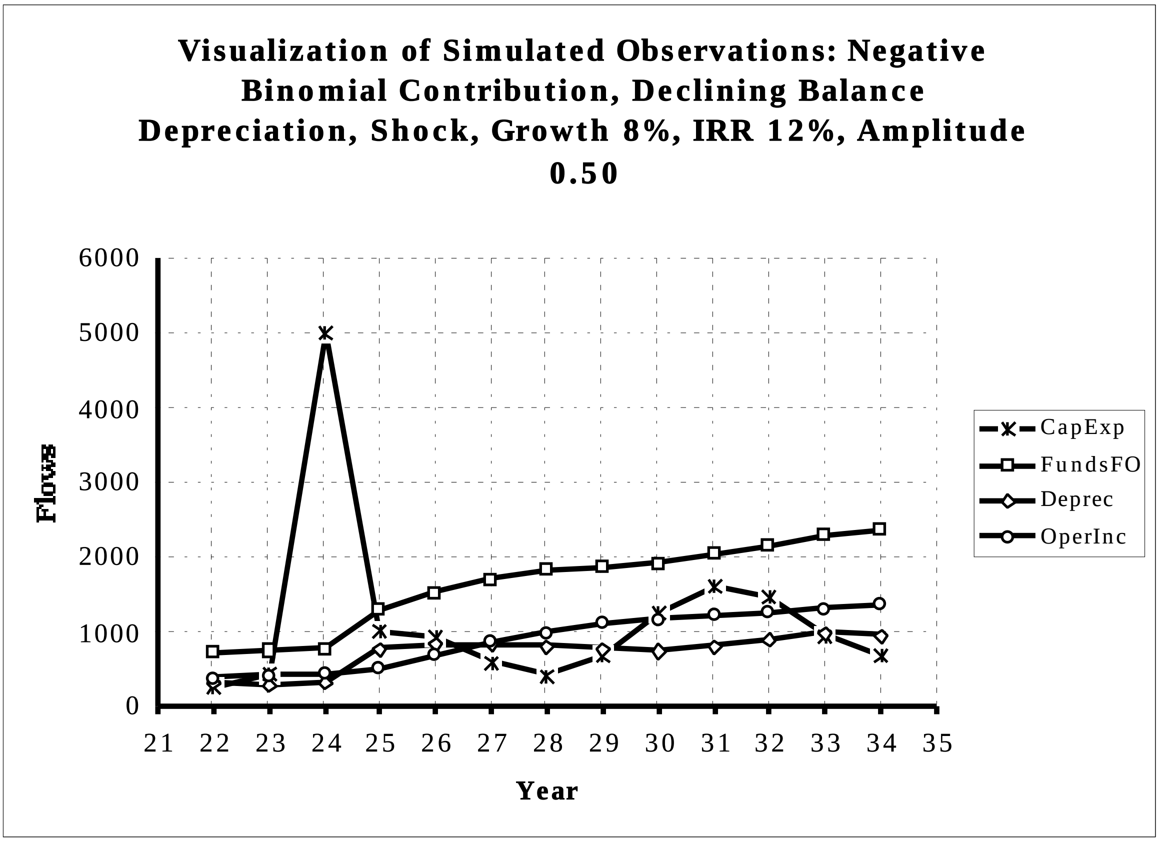

Figure 5. Visualization of Simulated Observations with Shock

4. EMPIRICAL RESULTS TO EVALUATE KAY'S IRR ESTIMATION MODEL

4.1 Research Questions for the Empirical Estimation

We apply our simulation approach on Kay's IRR estimation model summarized by Formula (15). Evaluating and comparing other IRR estimation models remains a subject of our further, intended research. The following questions and issues are of particular interest in the rest of the current paper. The first two questions concern general assertions about IRR profitability estimation which have been proven in earlier literature. Questions from 3 to 5 relate to the introduction of realistic business cycles in a simulation testbench for evaluating IRR profitability estimation methods. Question 6 concerns the robustness of estimation methods subject to unexpected capital investment shocks.

1) As discussed earlier, it can be proven mathematically that the ARR and IRR are equal when the annuity method of depreciation is used (see e.g. Salmi and Luoma, 1981:28). Hence, if the discrete format interpretation of Kay's model by Salmi and Luoma (1981) is correct, IRR estimation should provide the correct r for all the simulations.

2) It has been shown that for constant growth the accountant's rate of return and the internal rate of return equal when growth equals profitability as proven by Solomon (1966). Thus the application of Kay's model on the simulated data with constant growth (no cycles nor shocks) should provide the correct r when growth and profitability are set equal.

3) It is intuitive and mathematically sound to expect that with the introduction of regular business cycles the result in item 2 still holds if the length of the estimation period is a multiple of the business cycle, as we have in our simulated data.

4) If growth and profitability deviate from each other, it is of interest to see how sensitive Kay's method is with the introduction of the business cycle fluctuations in the capital investments. If the results show low sensitivity this will corroborate the general validity of Kay's method under realistic business conditions.

5) The next issue is what kind of effect irregularities in the capital investment pattern will have on profitability estimation. A weaker instance of irregularity obviously arises if the estimation period is not a multiple of the business cycle or if the business cycle is not symmetrical.

6) It is of interest to see how much a profitability estimation method like Kay's is affected with the introduction of a strong irregularity in the form of a capital investment shock. It is to be expected that, in particular, a shock has a disruptive influence on the estimation since the period of observation realistically is shorter than the life-cycle of the capital investment. The disruptive influence is expected to be aggravated the bigger or later the shock.

Our simulation model contains a number of further parameters depicted by Figure 2. They include the contribution distribution (uniform and negative binomial distributions), the practical depreciation method (straight-line and double declining balance methods), and the relationship between growth and profitability. The status of the following issues are of interest in varying these parameters. If an IRR profitability estimation method, like Kay's method, is robust, it is to be expected that it is not sensitive to variations in these parameters. Under steady-state growth and steadily declining or increasing contribution distribution it is possible to predict the direction and magnitude of the estimation errors. The same need not necessarily hold with the introduction of the cyclical fluctuations in the capital investments and/or non-symmetric contribution distribution.

4.2 Results with Regular Business Cycles and Uniform Contributions

The results for uniform contribution distribution are given below in Table 2.

Table 2. Estimation of IRR with Kay's model, uniform contribution distribution, growth rate k = 8%, no shock.

Cycle amplitude		A = 0.00			A= 0.50			A= 1.00		
Depreciation		Ann	Str.l	Decl	Ann	Str.l	Decl	Ann	Str.l	Decl
True r	4%	4.0	3.6	2.9	4.0	3.6	2.8	4.0	3.6	2.8
	8%	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0
	12%	12.0	12.9	13.8	12.0	12.9	13.9	12.0	13.0	13.9
	16%	16.0	18.3	20.1	16.0	18.3	20.3	16.0	18.4	20.4

As was to be expected from theory, applying annuity depreciation (Ann) always equates the estimated internal rate of return (IRR) with the true internal rate of return (r). As predicted, for the case r = k (= 8%) the estimated IRR is 8% for the amplitude A = 0 (the case with no cycles). Furthermore, the result holds with the introduction of the cycles (amplitudes 0.50 and 1.00).

It is readily seen that when $r < k$ Kay's method under-estimates the true profitability. When $r > k$ IRR is an over-estimate of r. The error grows monotonically. The estimation error is bigger if double declining balance depreciation is applied than if straight-line depreciation is applied.

The biggest deviation in Table 2 takes place when the true internal rate of return deviates most from growth and the declining balance method is used.

When the true profitability is 16% the estimate is off by over 4% (by 25 per cent in relative terms). This is a marked deviation. However, it is not easy to evaluate how serious this error is from the point of view of decision making. It depends on whether any alternative methods would give better estimates. Most importantly the seriousness of the deviation would depend on what would be the consequences of the management of the firm having erroneous profitability information. Predicting such consequences in quantitative terms is a very involved question and is outside the scope of our research.

The introduction of business cycles increases the estimation error only negligibly when the length of the business cycle has been estimated correctly. Our further simulations indicated that if the length of the business cycle is misidentified, it affects the estimates. While not negligible the effect is moderate. The direction of the effect is not easily established. We can conclude, however, that the method is robust to regular business cycles.

4.3 Results with Regular Business Cycles and Non-Symmetric Contributions

As pointed out earlier, the shape of the contribution distribution of the capital investments is not readily known for real-life firms. Therefore it is of interest to test whether the IRR estimation results are sensitive to this factor. Table 3 lists the estimation results when negative binomial distribution has been used in the simulation.

Table 3. Estimation with Kay's model, negative binomial contribution distribution, growth rate k = 8%, no shock.

Cycle amplitude		A = 0.00			A = 0.50			A = 1.00		
Depreciation		Ann	Str.l	Decl	Ann	Str.l	Decl	Ann	Str.l	Decl
True r	4%	4.0	4.1	3.5	4.0	4.1	3.4	4.0	4.1	3.4
	8%	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0
	12%	12.0	12.3	13.1	12.0	12.3	13.2	12.0	12.4	13.2
	16%	16.0	17.0	18.6	16.0	17.1	18.8	16.0	17.1	18.9

The results in Table 3 have much in common with the results in Table 2. However, some differences can be observed. When r > k, it is seen that the error in the IRR estimate is systematically smaller with the negative binomial distribution than with the uniform contribution distribution. In this example the error is about halved.

When r < k, IRR is no more systematically underestimated. This indicates that when the true contribution distribution is not known it is not possible to be certain of the direction of the estimation error.

4.4 Results with Inclusion of Shocks

This section looks at the effect of capital investments shocks to the robustness of the estimation. The results are presented in Tables 4 and 5.

As was to be expected from theory, applying annuity depreciation (Ann) still equates the estimated internal rate of return (IRR) with the true internal rate of return (r). However, the effect of the shock is so disruptive that for r = k (= 8%) the estimated IRR is no more 8% (with the natural exception of the annuity method). Furthermore, a late, great shock is the most disruptive. This behavior is easy to explain. The one-time investment shock becomes dominating, and its effects are much outside the period under observation.

Table 4. Estimation with Kay's model, binomial contribution distribution, growth rate 8%, early shock ($\tau = 24$), amplitude A = 0.50.

Shock factor		S = 0.00			S = 5.309			S = 17.924		
Depreciation		Ann	Str.l	Decl	Ann	Str.l	Decl	Ann	Str.l	Decl
True r	4%	4.0	4.1	3.4	4.0	4.3	3.3	4.0	4.5	3.2
	8%	8.0	8.0	8.0	8.0	8.1	7.6	8.0	8.1	7.3
	12%	12.0	12.3	13.2	12.0	12.2	12.3	12.0	12.0	11.6
	16%	16.0	17.1	18.8	16.0	16.5	17.3	16.0	16.2	16.1

Table 5. Estimation with Kay's model, binomial contribution distribution, growth rate 8%, late shock (τ = 30), amplitude A = 0.50.

Shock factor		S = 0.00			S = 5.309			S = 17.924		
Depreciation		Ann	Str.l	Decl	Ann	Str.l	Decl	Ann	Str.l	Decl
True r	4%	4.0	4.1	3.4	4.0	4.0	2.4	4.0	3.8	1.3
	8%	8.0	8.0	8.0	8.0	7.6	6.7	8.0	7.2	5.2
	12%	12.0	12.3	13.2	12.0	11.8	11.6	12.0	11.0	9.6
	16%	16.0	17.1	18.8	16.0	16.4	17.1	16.0	15.3	14.6

5. CONCLUSIONS AND DIRECTIONS FOR FURTHER RESEARCH

In this paper we developed a realistic simulation testbench to evaluate the methods for estimating the long-term profitability of business firms in terms of the internal rate of return (IRR) of their capital investments. Our simulation model extends the earlier, rigid approaches by incorporating business cycles and capital investment shocks into the model. Our approach includes the effect of alternative contribution distributions and alternative depreciation methods. This kind of an approach is needed to shed further light on the much debated question whether the accountant's rate of return is a valid approximation of the firm's internal rate of return (and thus a valid profitability measure).

After developing the simulation testbench we applied it, at this stage, to one long-run profitability estimation model, Kay's IRR estimation model. The following, main results concerning Kay's method were observed. When the growth rate and profitability are near each other, Kay's method yields accurate estimates as expected by theory. The more growth and profitability differ the less accurate will the estimates be. The magnitude of the error depends on the depreciation method applied and the capital investments' contribution distribution. It is also seen that Kay's method is insensitive to full business cycles, but disrupted by excessive capital investment shocks.

In this paper we applied our testbench to one profitability estimation model. We are continuing this research project by applying our method on the major IRR estimation methods presented in literature as listed in Salmi and Martikainen (1994). Besides evaluating each method individually it will be of interest to compare the performance (accuracy and sensitivity) of the methods with each other. Technically, this will not always be a trivial task since, as pointed out by Salmi and Luoma (1981), for example Kay's model is not readily applicable to factual observations from real life business firms before developing an operational, discrete version of the model. We will also look into the correlation consistency between the estimates of the alternative IRR estimation methods under the varying parameters. Furthermore, it will also be of particular interest to see if the elaborate IRR estimation methods really fare better than using the straight average of annual accounting rate of returns as the estimate of the long-run profitability. The reason why this last question is of particular interest is that there does not seem to be a consensus in literature whether ARR is an operational proxy of IRR and thus a valid profitability measure.

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