# HOW TO MEASURE THE AVAILABILITY OF AN ELECTRICAL POWER SYSTEM: A GENERALIZATION OF THE TRADITIONAL AVAILABILITY CONCEPT

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The paper deals with the availability characteristics of complex systems, especially in such cases when the systems have several different levels of performance. An example of such systems is a power plant, where a power block can be in operation at a lower output than the maximum (e.g. a failure of a smoke ventilator leads to a reduction in available power, but the block is still operable). The traditional theoretical availability characteristic, assuming the system to be in one of the two states: either failure-free and capable of full performance or failed and inoperable, is now unsuitable. Therefore, a generalized availability concept is introduced for systems with states of reduced efficiency. The generalized availability is in full agreement with indexes used in practice to quantify the availability of power plants.

## INTRODUCTION

The number of quantitative characteristics used for describing the reliability of a complex system is quite large, different indices of reliability playing the determining role when different systems or their different uses are considered. The most important and common reliability characteristics, when repairable production systems are considered, are the indexes of availability. Availability indexes depend, not only on the ability of the system to function without failures, but also on the efficiency with which the repair and maintenance of the system have been arranged, i.e. they describe the general ability of the system to maintain its quality and to preserve its output within established limits.

In practice, the indexes most commonly used for quantifying the availability of a power plant are the <u>time availability</u> (or <u>availability coefficient</u>) and the <u>energy availability</u> (or <u>utilization coefficient</u>). <u>The time availability is defined as the ratio of the hours of use of the plant</u> and the calendar time under consideration, i.e. it expresses the mean portion of time during which the plant has been in a functioning state ('available'). The energy availability is defined as the ratio of the energy actually produced during the observation period and the potential energy that would have been produced if the plant had operated with the maximum power during the whole period. The energy availability thus expresses the mean portion of potential energy output that has been got into use ('available').

When the time availability of a power plant is dealt with, the plant is considered as a two-stage system: either as operable (with the maximum or less than the maximum power) or as inoperable. This means, for example, that the failures, maintenance and repair works, changing of fuel etc. which only reduce the output of the plant are not taken into account as factors decreasing the availability of the plant. In the energy availability, on the other hand, the effect of both stoppages and partial reductions of output are revealed. The plant is then considered as a multi-stage system with many different levels of performance.

In the mathematical theory of reliability, where the characteristics of reliability are based on probability theory, the systems have been traditionally considered as simple two-stage operable or inoperable systems. As far as availability is especially concerned, this means that among the empirical indexes of availability the time availability only has an analogous theoretical characteristic. Theoretical foundations or analogous theoretical characteristics for the energy availability have not been presented. In this paper a generalized characteristic of availability is introduced. The characteristic possesses completely analogous empirical properties in the meaning of energy availability as the traditional concept of availability possesses in the meaning of time availability. The generalized characteristic also contains the traditional one as a particular case, and in ordinary operable or inoperable systems the two characteristics coincide. Further, both of the characteristics can be derived with the same mathematical formalism.

## AVAILABILITY OF A TWO-STAGE SYSTEM

### Availability as a probability concept

In the following we consider a repairable system which is assumed at every moment to be in one of the two states: failure-free and thus capable of full performance or failed and thus under repair. Besides actual repairs also preventive maintenance works, changing of fuel etc., which presume the system to be stopped, are inclued in repairs. Both the operation and repair times are assumed to be random variables.

A basic characteristic of availability of the system is the (pointwise) availability A(t), which is defined (see e.g. [1], p. 7 and [3], p. 239) as the probability that the system will be functioning at the instant t, or

$$A(t) = \Pr\left\{\text{the system is operable at time } t\right\}. \tag{1}$$

As the limiting value of this probability we get

$$\bar{A} = \lim_{t\to\infty} A(t), \tag{2}$$

called the availability coefficient ([2], p. 358). It is easy to show (cf. [2], p. 358) that $\bar{A}$ is the mean portion of time in the long run that the system is in a functioning state. But this interpretation of $\bar{A}$ coinsides with the definition of time availability which is used in empirical availability calculations. The theory and practice are found to be in a good agreement when two-stage systems are considered.

### Availability in general mathematical terms

In this section we present the general mathematical definition of reliability, as a special case of which we obtain the availability A(t). The presentation follows the formulation of GNEDENKO et al. [2]. Generally, let S be the set of all the different states of the system, called the phase space. Let X(t) be a random variable denoting the state of the system at time t. Let x(t) be a single value of X(t). Then $x(t) \in S$ for all $t \geq 0$. The set $X = (X(t) \mid t \geq 0)$, ordered to the time variable t, is then a stochastic process describing the course of the states in time. A time series $\bar{x} = (x(t) \mid t \geq 0)$ is a trajectory of this process. The set of the trajectories we denote by $\bar{X}$. Further, let $\Phi$ be a functional defined on the set $\bar{X}$, whereupon to every trajectory $\bar{x}$ there corresponds a unique real number $\Phi(\bar{x})$. The reliability $\varphi$ of the system is now defined as the expected value of the functional $\Phi$:

$$\varphi = E\left\{\Phi(\bar{X})\right\}. \tag{3}$$

In order to get the availability A(t) we must specify the functional $\Phi$. Let $S^0$ be a subset of the phase space S such that the system is inoperable, when the state x(t) belongs to $S^0$ and operable otherwise. Let the functional $\Phi$ be defined as follows

$$\Phi(\bar{x}) = \begin{cases} 0, & \text{if } \bar{x}(t) \in S^0 \\ 1, & \text{if } \bar{x}(t) \in S - S^0. \end{cases} \tag{4}$$

Then we obtain A(t) as a special case of the definition (3), for we have

$$\begin{aligned}\varphi = E\left\{\Phi(\overline{X})\right\} = Pr\left\{\Phi(\overline{X}) = 1\right\} = Pr\left\{X(t) \in S - S^0\right\} \\ = Pr\left\{\text{the system is operable at time } t\right\} = A(t).\end{aligned} \tag{5}$$

It can be shown (see [4], pp. 28 - 30) that, by a suitable definition of the functional $\Phi$, all the conventional characteristics of reliability can be obtained using definition (3).

## THE GENERALIZED AVAILABILITY FOR A MULTI-STAGE SYSTEM

### The problem

Above we have considered availability from the point of view of the traditional approach: the system is at every moment regarded either as operable or inoperable. Among production systems there exist, however, several examples which show that the ordinary concept of availability is too narrow to cover all systems. One example is given by an electrical power system, where a power block can be in operation at a lower level of performance than the maximum: a failure of a smoke ventilator leads to a reduction in available power, but the block is still operating. Generally, the problem comes from the fact that situations may exist when the system is neither fully operable nor totally inoperable, but the theory assumes, however, that one or the other of these states exists.

In practical calculations of availability the states of reduced efficiency are taken into account: for electrical power plants we have, for example, the index energy availability where the effect of operation with a less power than the maximum is revealed (whereas in time availability only total stoppages are registrated). In the following we present a generalized concept of availability, which makes it possible to consider also the availability of systems with states of reduced efficiency. One of the empirical properties of the generalized availability concept will be that its steady state value equals to the energy availability of the system. The presentation is based on the doctoral thesis of the author [4].

### The generalized availability

In order to make the generalization in a theoretically wellfounded and empirically and logically adequate way, we set for the new concept the following requirements:

(a) Failures having a limiting effect on the efficiency of the system are referred to factors, which decrease the availability of the system, and their influence is dependent on the seriousness of the consequences of the failure.
(b) When the generalized availability is applied to general multi-stage systems, we get empirical interpretations analogical to those which result, when the traditional availability is applied to ordinary two-stage operable or inoperable systems.
(c) When a two-stage system is under consideration, the generalized availability coincides with the traditional availability A(t).
(d) The definition of the generalized availability can be presented in the form of the general mathematical definition of reliability, i.e. in the form (3).
(e) The numerical value of availability can be determined directly from the behavioural properties of the system, e.g. from the state probabilities.

Notation. Let the possible levels of performance of the system be $c_0, c_1, \ldots, c_K$, where $c_0 = 0$ stands for total inoperability, $c_K = C$ indicates full operability ($C$ is the capacity of the system: the maximum power of a power plant, for example), and $c_1$ to $c_{K-1}$ are levels of reduced performance. The respective proportional levels of performance we denote by $w_0, w_1, \ldots, w_K$ ($w_i = c_i/C$). Let the possible states of the system be numbered as follows: the set of the states corresponding to the performance level $c_i$ is

$$S_i = \left\{s_i^1, s_i^2, \ldots, s_i^{n_i}\right\}, \quad i=0,1,\ldots,K. \tag{6}$$

The state probabilities of the system we denote by $P_{ij}(t)$, where

$$P_{ij}(t) = \Pr\left\{\text{the state of the system at time t is } s_i^j\right\}. \tag{7}$$

Further, let the probability distribution of the proportional level of performance $W(t)$ be denoted by $P_i(t)$, $i=0,1,\ldots,K$. Then we have

$$P_i(t) = \Pr\left\{W(t) = w_i\right\} = \sum_{j=1}^{n_i} P_{ij}(t), \quad i=0,1,\ldots,K. \tag{8}$$

The characteristic 'mean availability of the capacity'. We define the generalized availability, called the mean availability of the capacity, as the mathematical expectation of the proportional level of performance of the system at time t:

$$A_C(t) = E\left\{W(t)\right\} = \sum_{i=0}^{K} w_i P_i(t) = \sum_{i=0}^{K} w_i \sum_{j=1}^{n_i} P_{ij}(t). \tag{9}$$

In the following we show, that the characteristic $A_C(t)$ fulfills the requirements (a) to (e) set above (for a more detailed analysis see [4], pp. 40 - 43).

First, all kinds of failures have a decreasing effect on the availability of the system, and the decrease is dependent on the degree of the failure: the more serious the consequences of the failure, the more will the availability decrease.

Secondly, $A_C(t)$ has under the steady state e.g. the following properties and interpretations. The steady state value of $A_C(t)$, denoted by $\bar{A}_C$, gives (1) the mean proportional level of performance of the system during the observation period, (2) the ratio of the actual and potential outputs of the system during that period, i.e. $\bar{A}_C$ coincides with the empirical index energy availability, and (3) the portion of the observation period during which the full capacity should be available in order that an equivalent system output would be achieved. All these properties are analogical to those which result, when the traditional availability of ordinary two-stage systems is analysed.

In the third place, for an operable or inoperable system we have: $K=1$, $w_0=0$ and $w_1=1$. Thus we can write

$$A_C(t) = 0 \times \Pr\left\{W(t)=0\right\} + 1 \times \Pr\left\{W(t)=1\right\} = \Pr\left\{W(t)=1\right\} = A(t), \tag{10}$$

so that in two-stage systems $A_C(t)$ coincides with $A(t)$.

Fourth, by defining the functional $\Phi$ as follows: $\Phi(\bar{x}) = w_i$, when $\bar{x}(t) \in S_i$, we have

$$E\left\{\Phi(\bar{X})\right\} = \sum_{i=0}^{K} w_i \Pr\left\{\Phi(\bar{X})=w_i\right\} = \sum_{i=0}^{K} w_i \Pr\left\{X(t) \in S_i\right\} = A_C(t), \tag{11}$$

and we have obtained $A_C$ as a particular case of definition (3).

Fifth and last, in (9) we have shown already, that $A_C(t)$ can be calculated directly from the state probabilities of the system.

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