

# Non-linear Advertising Capital Model with Time Delayed Feedback between Advertising and Stock of Goodwill

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*Motto:*

*Doing business without advertising is like winking at a girl in the dark.*

*You know what you are doing, but nobody else does.*

*(Steuart Henderson Britt, New York Herald Tribune 30.10.1956,*

*as cited by Colin Dexter in The Way Through the Woods).*

## **Abstract**

According to the classical Nerlove-Arrow model, advertising expenditure can be considered as a capital investment to create present and future demand for the firm's products and, hence, to create present and future revenues for the firm. Advertising is assumed to influence via stock of goodwill which cumulatively counts for the effects of the firm's current and past advertising outlays. The paper presents a time delayed feedback model describing the relations between advertising and goodwill. Three different types of effects of advertising upon the dynamics of goodwill are modelled. The advertising policy of the management is incorporated into the model via a non-linear advertising function. The advertising function controls the advertising outlay e.g. by budget constraint and by the actual and target values of goodwill. The behavior of the model is analysed both analytically and numerically. Special attention is given for deriving the stability conditions for the limiting solution. The cases of repelling or chaotic limiting solutions are analysed by bifurcation and state space diagrams. Several numerical examples are given.

*Keywords: dynamic systems, advertising models, chaos theory, bifurcation diagrams*

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