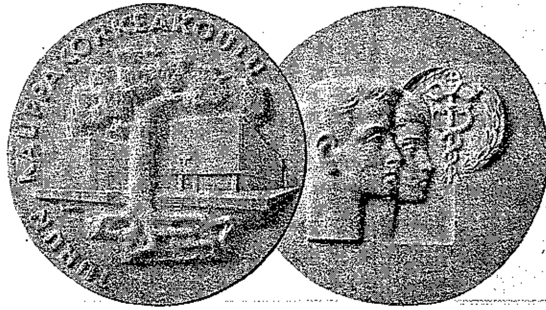

TURUN KAUPPAKORKEAKOULUN JULKAISUJA

SARJA A - 3:1977

PUBLICATIONS OF THE TURKU SCHOOL OF ECONOMICS

SERIES

Ilkka Virtanen

ON THE CONCEPTS AND DERIVATION OF RELIABILITY IN STOCHASTIC SYSTEMS WITH STATES OF REDUCED EFFICIENCY

Eripainos julkaisusta

"Tutkielmia ja tutkimusraportteja 1977"

THE TURKU SCHOOL OF ECONOMICS
Economic Mathematics and Statistics
Ilkka Virtanen

ON THE CONCEPTS AND DERIVATION OF RELIABILITY IN STOCHASTIC SYSTEMS WITH STATES OF REDUCED EFFICIENCY†

Abstract

The paper deals with the concepts and characteristics of system reliability, especially in such a case when the system has several possible different levels of performance. This kind of a situation e.g. arises when the failure of a component or subsystem only reduces the efficiency of the system instead of making the system completely inoperable. The traditional concepts of reliability are shown to be too narrow in extent in order to cover the systems with many different levels of performance.

In order to remove this deficiency, a new definition with an extension in content is given for the concepts of reliability. This conceptual extension is done in such a way that

- the failures having a partly reducing effect on the efficiency of the system are referred to factors which decrease the reliability of the system, but do that by an amount that is less than the decrease in reliability caused by a failure with total system inoperability
- the new more comprehensive concepts of reliability get among the general systems with many levels of performance analogical empirical interpretations as the conventional concepts of reliability have among the two-state operable-inoperable systems
- when a two-state operable-inoperable system is under consideration the new concepts become consistent with the traditional concepts of reliability
- the mathematical definition of the new concepts stays within the limits of the general mathematical definition of reliabili

The extension of the concepts of reliability is in detail carried out for the traditional characteristic 'availability'. Two new availability characteristics are presented:

Availability of the levels of performance $A_0(c,t)$ is the probability

$A_0(c,t)$ = P{the level of performance of the system at time t is $\geq$ c}.

Mean availability of the capacity $A_C(t)$ is

$A_C(t)$ = the mean value for the relative level of system performance at time t.

Further, expressions for calculating the new characteristics of reliability straight on the basis of the state probabilities of the system are derived in the paper. Also several remarks on empirical interpretation and statistical estimation of these characteristics are given.

† Paper to be presented at The Sixth Nordic Congress on Operations Research NOAK 77 in Turku, Finland 24th - 25th October 1977

THE TURKU SCHOOL OF ECONOMICS
Economic Mathematics and Statistics
Ilkka Virtanen

ON THE CONCEPTS AND DERIVATION OF RELIABILITY IN STOCHASTIC SYSTEMS WITH STATES OF REDUCED EFFICIENCY

1 Introduction

Numerous problems concerning the improvement of the efficiency of a production system have acquired special importance in the last few decades. In a central place in these efficiency problems there is the ability of the system to function in the way intended by the user in advance. This general property of the system is commonly used as a characterization for the concept 'reliability of the system'.

The importance of reliability has grown in proportion as the equipments have automatized, become more complex in construction and begun to take the responsibility for ever larger entire ties of tasks without continuous and direct control of man. The failures meaning the breakdown of a system or any other inconvenience in its operation, i.e. the unreliability of the system, have now several injurious consequences for the system itself, for the user, and for the environment of the system. Most clearly these consequences are revealt in cost increased, time wasted, the psychological effect of inconvenience, and in certain instances personal and national security.[1] Preventatio of system failures as well as control of the failures arising in spite of everything and reduction of their inconveniences have thus become an important problem when towards an efficien management of the system is tended.

This paper deals with some definition and interpretation problems appearing in the connection with the content of the concept 'reliability', especially with the quantitative characteristics of reliability of certain kind of systems. New definitions for extending the intension and extension of the concepts of reliability turn out to be essential in order to carry out a valid reliability analysis for systems with states of reduced efficiency, i.e. for systems which after failure may be able to operate in a decreased level of performance instead of being totally inoperable.

1 Lloyd and Lipow (1962), p. 1

In section 2 we consider the definitions and characteristics of the traditional reliability, i.e. reliability that has been defined for two-state operable-inoperable systems (the components of the system and so the whole system are assumed to be either failure-free and thus capable of full performance or failed and thus totally inoperable and under repair). The section also contains the general mathematical definition of reliability, which is shown to cover all the quantitative characteristics of reliability and as particular cases of whic these characteristics may be obtained.

In section 3 the traditional concepts of reliability are shown to be for general purposes too narrow both in intension and in extension: the reliability of those systems which as a consequence of failures have several possible levels of performance remains open or gets a value in contradiction to empirica observation. The concepts of reliability are now extended so that these deficiencies become removed. The extensions are carried out supporting them on the empirical interpretation of reliability and so that the new concepts preserve in ordina operable-inoperable systems their previous content and meaning It will further be shown that the new more comprehensive concepts of reliability agree with the general mathematical defin ition of reliability.

2 The traditional concepts of reliability

21 Qualitative definitions for reliability

The definitions of reliability are in general based on the occurence or non-occurence of a failure at a given time or during a given time interval. By their main features the defin itions are divided into two groups, into qualitative and quant ative definitions. In the following we list a few typical representatives for the group of qualitative definitions of reliability:

1 Reliability is the ability of the equipment to preserve its output characteristics (parameters) within established limits in given conditions of operation[1]

2 By unit reliability we mean the ability of the unit to maintain its quality under specified conditions of use[2]

3 Reliability is defined as 'the probability of a succes ful operation of the device in the manner and under the conditions of intended customer use'[3]

4 Reliability is the probability of a device performing its purpose adequately for the period of time intende under the operating conditions encountered[4]

5 Reliability is the probability that a system will perform satisfactorily for at least a given period of time when used under stated conditions[5].

Although the qualitative definitions of reliability form a grou that can be regarded quite homogenous, the measurement and rendering of reliability still remain open to some extent. The final specification of the concepts of reliability is made in the form of quantitative definitions.

22 Quantitative characteristics for reliability

The number of quantitative characteristics of reliability is quite large, different indices play the determining role when different systems or different use of a given system are considered. In the following we present the three most general and important quantitative characteristics of reliability, viz reliability (function), (pointwise) availability and mean time to system failure.

1 Polovko (1968), p. 1
2 Gnedenko et al. (1969), p. 70
3 Lloyd and Lipow (1962), p. 20
4 Barlow and Proschan (1965), p. 6
5 von Alven (1964), p. 6

The reliability of a system at time t, R(t), is defined as the probability of failure-free operation of the system during the time t[1], i.e.

(1) R(t) = P{the system operates without failure from 0 to t

Reliability R is typically a characteristic for systems that are non-repairable or are considered as non-repairable. The object of interest in the behaviour of the system there is onl the first failure of the system and the time of its occurence. The existence of a repair facility, the duration time of repai etc. do not have any influence on the reliability achieved.

The (pointwise) availability of the system at time t, A(t), is the probability[2]

(2) A(t) = P{the system is operable at time t}.

The availability of the system depends, besides on the ability of the system to operate without failures, also on the efficiency with which the repair of the system has been arranged. The availability of an easily failing system may still be high if the repair times of the system are very short. As a charact eristic for reliability, A(t) so is reasonable only in the cas of repairable systems.

Mean time to system failure, T, is, according to its name, the mean or expected value of the time that the system uninterrupt edly operates without failures.[3] When the characteristic T is concerned, we can consider the time before the first system failure as well as the time from the completion of the repair of a former failure to the occurence of the next failure. Thus T is a characteristic of reliability both for a repairable and for a non-repairable system.

1 The term 'reliability' is in reliability literature in comm use in two different sense. The one meaning is the system's general ability to function in the way intended by the user (section 21). The other meaning is one particular character istic of the general concept, also known as 'reliability fun tion', see Gnedenko et al.(1969), p.79, Barlow and Proschan (1965), p.7, and Jorgenson et al.(1967), p.14
2 Cf. Barlow and Proschan (1965), p.7, Rau (1970), p.239
3 Also known as 'mean time of failure-free operation of the system', Gnedenko et al.(1969), p.77, 'system mean time before failure', Rau (1970), etc.

23 The general mathematical definition of reliability

In section 22 we defined the three quantitative characteristics of reliability as certain probabilities or expected values. In spite of definitional dissimilarities between the different characteristics and the fact that each one of the characteristics gives weight to in some degree different points in the operation of the system, they all measure and reveal, however, the capability of the system to manage the tasks and requirements given to it, i.e. just the reliability that has been set as the object of the measurement. The compatibility of these characteristics is formally revealt so that it turns out to be possible to construct an abstract mathematical concept of reliability, which as particular cases contains all the most important and usual characteristics of reliability, especially those three considered in the foregoing section.[1]

Generally, let s denote a state, in which a system at a given time can exist. Then S = {s} is the set of all the possible different states of the system, called the phase space of the system. The state of the system may be either a scalar or a vector quantity. With the passage of time, various changes may take place in the constituent parts of the system, the state o the system changes. Let x(t) denote the state of the system at the instant t. Then $x(t) \in S$ for all values of t ($t \geq 0$). When the stochastic nature of the state transitions is taken into account, the stochastic state (the set of the states) at time t or the time distribution at time t can be described by a rando variable X(t), a single (sample) value of which each observed or in general each at time t possible x(t) is. The ordered set $X = (X(t)|t \geq 0)$ of the random variable X(t), indexed on the tim variable t, is then a stochastic process describing the course of the states in time. Any time serie $(x(t)|t \geq 0)$ is a realization or trajectory of this stochastic process. It is evident that the value of every single trajectory $\hat{x}$ at any instant $t^* \geq$ is one of the elements of the phase space S, i.e. $\hat{x}(t^*) \in S$ for all $\hat{x}$ and for all $t^* \geq 0$. The set of these trajectories we denote by $\hat{X}$.

1 The definition of reliability in this kind of a general form is presented by Barlow and Proschan (1965), pp. 6-7, and by Gnedenko et al. (1969), pp. 74-78; in this study we follow the latter presentation

After definition of the phase space S and the stochastic process X we can formulate the general definition of reliability. Let Φ be a functional defined on the trajectories of the process X (on the set $\hat{X}$), whereupon to every trajectory $\hat{x} \in \hat{X}$ there corresponds a unique real number $\Phi(\hat{x})$. The reliability φ is now defined as the expected value of that functional:

(3) $$\varphi = E\{\Phi(\hat{X})\}.$$

The final specification of the concept of reliability, i.e. the choice of the quantitative characteristic to be used, so remains to depend on the definition of the functional Φ. In the following we will show, that e.g. the characteristics considered in section 22 are obtained as particular cases of this general definition of reliability.

Reliability R(t). Let S^o be a subset of the phase space S such that the system is inoperable, when its state x(t) belong to S^o. Let the functional Φ_1 be defined as follows:

(4) $$\Phi_1(\hat{x}) = \begin{cases} 0, \text{if for at least one value } 0 \leq u \leq t,\ \hat{x}(u) \in S^o \\ 1, \text{otherwise.} \end{cases}$$

Then we have

(5) $$\begin{cases} \varphi_1 = E\{\Phi_1(\hat{X})\} = P\{\Phi_1(\hat{X}) = 1\} \\ \quad = P\{\text{the system does not visit } S^o \text{ before time } t\} \\ \quad = P\{\text{the system operates without failure from 0 to} \\ \quad = R(t). \end{cases}$$

By defining the functional Φ according to equation (4) we thus obtain from equation (3) the characteristic 'reliability'.

Availability A(t). Let S^o be the same subset of S as above, and define the functional Φ_2 as follows:

(6) $$\Phi_2(\hat{x}) = \begin{cases} 0, \text{ if } \hat{x}(t) \in S^o \\ 1, \text{ if } \hat{x}(t) \notin S^o. \end{cases}$$

Now we have

$$(7)\quad \begin{cases} \varphi_2 = E\{\Phi_2(\hat{X})\} = P\{\Phi_2(\hat{X}) = 1\} = P\{X(t) \notin S^o\} \\ = P\{\text{the state of the system at time t is non of the inoperable states}\} \\ = P\{\text{the system is operable at time t}\} \\ = A(t), \end{cases}$$

so that we have come to the characteristic 'availability'.

Mean time to system failure T. Let the functional Φ_3 be defined by the equation

$$(8)\qquad \Phi_3(\hat{x}) = \int_0^\infty I_{S-S^o}(t)\, dt,$$

where

$$(9)\qquad I_{S-S^o}(t) = \begin{cases} 1, \text{ if } \hat{x}(u) \in S-S^0 \text{ for all } 0 \le u \le t \\ 0, \text{ if there exists at least one } 0 < \tau \le t \\ \quad \text{such that } x(\tau) \in S^o, \end{cases}$$

i.e. $\Phi_3(\hat{x})$ gives the first time point (denoted by τ), at which the trajectory $\hat{x}$ falls into the subset S^o. Or in other words, the value of the functional Φ_3 is the lenght of the failure-free operation time of the system. As the expected value for the lenght of this failure-free operation time we obtain the third characteristic of reliability, mean time to system failure T:

$$(10)\qquad \varphi_3 = E\{\Phi_3(\hat{X})\} = T.$$

3 The extended concepts of reliability

31 Problems concerning the concepts of reliability in the case of partial reduction in the level of performance of a system

In section 2 we considered reliability from the point of view of the traditional approach established in literature. The characteristic feature of the traditional reliability consideration is the property of the system that it at every moment is regarded either as operable or as inoperable. At the occurence of any phenomenon differing from the system's normal operation, for instance, this means, that the phenomenon must be classified either as a failure or as a factor having no influence on the operability of the system.

Among production systems there exist, however, several examples which show that the usual concepts of reliability are too narrow in extent to cover all kinds of systems. This is particular clearly revealt when a production system of the type of a processing plant will be considered. The problematic situation comes from the property of the system that it on some occasions may be able after failure to operate with reduced efficiency (in a reduced level of performance), instead of becoming totally inoperable due that failure. The reduction in the level of performance may be a consequence from the respective property of an individual component (the component is able to operate also after failure, but only retarded or otherwise partly) or from the type of the composition of the system's components (there are parallel branches in the system, some of the branches having failed).

It is evident that the production power of a system which as a consequence of a failure has operated in a reduced level of performance can not be as high as the production power of such a system which all the time has operated normally. On the other hand, its production power is higher than that of such a system which for the time in question has been totally inoperable. This kind of circumstances should also be revealed in the reliability of the system: the reliability of the system should decrease due to the failure, not as much, however, as due to a total system failure. But if the traditional concepts of reliability are used, the situation either remains open (the reliability of the system can not be determined at all), or it becomes uncorrectly treated (the reduction in the level of performance is left without any attention or the system is regarded totally inoperable).

The objective of the paper is to extend the concepts of reliability in order to make it possible to determine also the reliability of systems with states of reduced efficiency. In the extension care must be taken that it will be done in a theoretically wellfounded and empirically adequate way. Any violation against the traditional concepts must not be made, either. These conditions will be fullfilled, when we set the following requirements for the new concepts:

1. The failures having a partly reducing effect on the efficiency of the system are referred to factors which decrease the reliability of the system, but do that by an amount that is less than the decrease in reliability caused by a failure with total system inoperability.

2. The new, more comprehensive concepts of reliability get among the general systems with many levels of performance analogical empirical interpretations as the conventional concepts of reliability have among the usual two-state operable-inoperable systems.

3. When a two-state operable-inoperable system is under consideration the new concepts become consistent with the traditional concepts of reliability.

4. The mathematical definition of the new concepts stays within the limits of the general mathematical definition of reliability (by Gnedenko, equation (3) in this paper).

5. The numerical value of reliability can be determined straight on the basis of the behaviour of the system, i.e. from the state probabilities.

Explicitely we carry out the extension of the concepts of reliability for the characteristic 'availability' only. In connection with the derivation of two new, extended "availabilities", 'availability of the levels of performance' and 'mean availability of the capacity', the general principles of the extension procedure will be given, however. Following these principles analogical extensions for the other characteristics of traditional reliability can be accomplished.[1]

1 see Virtanen (1977), where analogical extended concepts for the traditional 'reliability' and 'mean time to system failure' are presented

32 The extended availability characteristics

321 Notation and preliminary remarks

Let us assume that the possible levels of performance of the system are $c_0, c_1, \ldots, c_K$, where c_0 stands for total inoperability, c_K indicates full operability (the level of performance is equal to the capacity C of the system), and $c_1, \ldots, c_{K-1}$ are reduced levels of performance caused by one or several coexistent failures. Let the respective proportional levels of performance (proportional level of performance = ratio of the level of performance to the capacity) be denoted by w_0, $w_1, \ldots, w_K$. Then we have $w_0=0$, $w_K=1$, and $0<w_k<1$, when $k=1,2,\ldots K-1$. Between the states and the levels of performance there is an obvious relation. For each state there is an uniquely determined level of performance, several states, instead, may lead to one and the same performance level. The passage of the states and the levels of performance of the system during a time interval $[0,t]$ are graphigally described in figure 1.

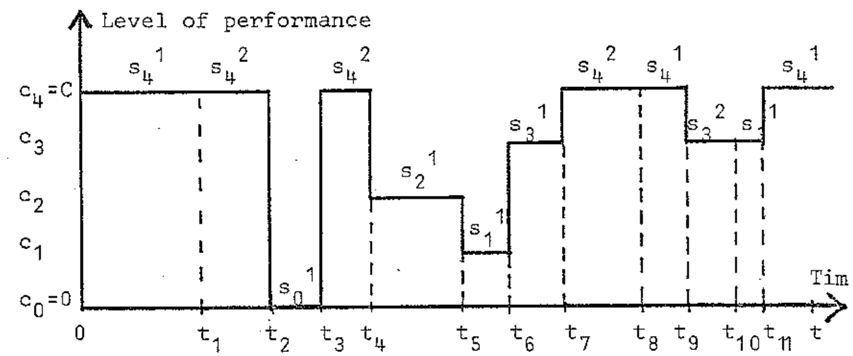

Figure 1. The states and levels of performance of a system with the passage of time

In figure 1 the state of the system changes at time points $t_1, t_2, \ldots, t_{11}$. Out of these eleven state transitions the ones happening at instants t_1, t_8 and t_{10} are such that they do not indicate any change in the level of performance of the system (e.g. failure of one single component out of several still operable redundantly connected components, ending of repair of this kind of a component etc.).

The introduced concepts 'level of performance' and 'proportional level of performance' do not prevent us from considering an ordinary operable-inoperable system within these new frameworks. In such a case we only have K=1, whereupon c_0 means the total inoperability of the system and c_1=C the full (capacity level) operability of the system. The only possible proportional levels of performance of the system are $w_0=0$ and $w_1=1$, respectively.

Let the states of the system to be numbered as follows: the states corresponding to the performance level c_i (i=0,1,...,K) are $s_i^1, s_i^2, \ldots, s_i^{n_i}$. Let the set of these states be denoted by S_i. Then we have $S_i = \{s_i^1, s_i^2, \ldots, s_i^{n_i}\}$. The state probabilities of the system we denote by the symbol $P_{s_i^j}(t)$, where

(11) $$P_{s_i^j}(t) = P\{\text{the state of the system at time } t \text{ is } s_i^j\},$$
$$i=0,1,\ldots,K, j=1,2,\ldots,n_i.$$

From this notation not only the current state of the system, but its level of performance is revealt as well (on the basis of the subscript of the state symbol s_i^j).

322 Availability as a function of the level of performance

In section 22, see equation (2), the availability of an ordinary operable-inoperable system was defined to be the probability that the system is operable at time t. The reduced performance levels, which are consequences of the failures of certain kind of components or subsystems, can be taken into account in the calculation of availability, when a new availability characteristic, 'availability of the levels of performance', denoted by A_0, is defined as follows:

(12) $$A_0(c,t) = P\{\text{the level of performance of the system at time } t \text{ is } \geq c\}.$$

While the conventional availability A(t), calculated at a fixe time t, is a simple number, $A_0(c,t)$ (at time t) is now a funct ion on the level of performance (in the range $0<c\leq C$). We can immediately see, that A_0 is within the frames of the general definion of reliability (3) suited. Defining, for given values of c and t, the functional Φ as follows:

(13) $$\Phi_4(\hat{x}) = \begin{cases} 0, & \text{if } \hat{x}(t) \in \bigcup_{c_i<c} S_i \\ 1, & \text{if } \hat{x}(t) \in \bigcup_{c_i\geq c} S_i, \end{cases}$$

we get $A_0(c,t)$ as the expected value of this functional:

(14) $$\begin{cases} E\{\Phi_4(\hat{X})\} = P\{X(t) \in \bigcup_{c_i\geq c} S_i\} \\ \quad = P\{\text{the level of performance of the system at time } t \text{ is at least } c\} \\ \quad = A_0(c,t). \end{cases}$$

In practice, we can easily calculate $A_0(c,t)$, if the state probabilities of the system are known:

(15) $$A_0(c,t) = \sum_{c_i\geq c} \sum_{j=1}^{n_i} P_{s_i^j}(t).$$

When the value of $A_0(c,t)$ is being calculated, into factors decreasing it (from the maximum value 1) the failures are included, as a consequence of which the level of performance of the system falls below c. From definition (12) and equation (15) we can immediately see, that as a characteristic for reliability the "extended availability" A_0 takes the aspects presented in section 31 completely into account: also the failures causing only a partial reduction in the level of performance have a decreasing effect on the reliability of the system, but their effect is not, however, as total as the respective effect of failures which lead the system to complete inoperability.

Further, the ordinary availability A of a two-state operable-inoperable system stays as a particular case for A_0. For, when we set $c = c_1 = C$ (the capacity of the system; the syster has only two possible levels of performance, $c_0 = 0$ and c_1 = (in $A_0(c,t)$, we obtain for an operable-inoperable system

(16) $$\begin{cases} A_0(C,t) = \sum_{j=1}^{n_1} P_{s_1^j}(t) \\ \quad = P\{\text{the system is at time } t \text{ in one of the states of full operability}\} \\ \quad = P\{\text{the system is operable at time } t\} = A(t). \end{cases}$$

When the ordinary availability under the steady state[1] has an interpretation, according to which it gives the mean portion of time during which the system is in the functioning state[2], we also have for $A_0(c,t)$, for all values of $0<c \leq C$, quite an analogous interpretation under the steady state: it gives the mean portion of time during which the system is able to operat at least in the level c of performance. From figure 2, for instance, we obtain the steady state estimate for $A_0(c)$ as follows (the steady state quantities are denoted without the time variable t):

$$(17)\quad \begin{cases} A_0(c) = 1 - \dfrac{(t_2-t_1)+(t_4-t_3)}{T_2 - T_1} = 1 - \dfrac{2+1}{20} = 0.85, \text{ when } c_2<c \leq C \\ A_0(c) = 1 - \dfrac{t_4-t_3}{T_2-T_1} = 1 - \dfrac{1}{20} = 0.95, \text{ when } c_1<c\leq c_2 \\ A_0(c) = 1, \text{ when } 0<c\leq c_1. \end{cases}$$

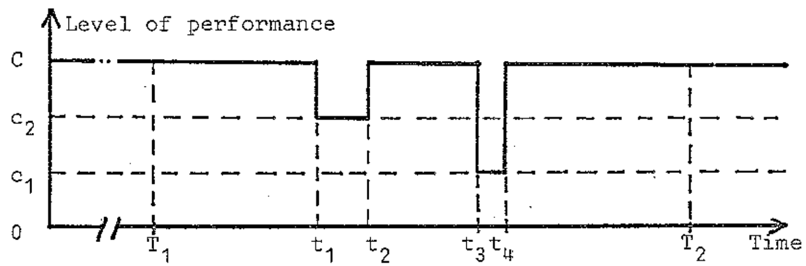

Figure 2. Estimation of $A_0(c)$ under the steady state

We have thus shown that the extended availability, availability of the levels of performance, meets with all the five requirements set in section 31 for the extended reliability concepts.

1 Steady state: state probabilities of the system and thus e.g. the availability of the system cease to depend on time
2 Gnedenko et al. (1969), p. 112

323 Mean availability of the capacity

When the availability of a traditional operable-inoperable system is considered, the states of the system are divided into two classes only. One class consists of the states of full operability (the level of performance is equal to the capacity) and the other of the states of total inoperability. These classes are denoted $S_1 = \{s_1^1, s_1^2, \ldots, s_1^{n_1}\}$ and $S_0 = \{s_0^1, s_0^2, \ldots, s_0^{n_0}\}$, respectively (i.e. we have K=1 in the notat ion of section 321).

When the system is in one of the states $s_1^j \in S_1$, its proportional level of performance is equal to 1, in the states $s_0^k \in S$ the proportional level of performance is = 0. Availability is thus obtained also as the probability

$$(18)\quad A(t) = P\{\text{the proportional level of performance of the system at time t is equal to 1}\}.$$

Taking the proportional level of performance at time t, denote $W(t)$, as a random variable, which can get the values 1 and 0 with probabilities $P_1(t) = P\{W(t)=1\}$ and $P_0(t) = P\{W(t)=0\}$, respectively, we obtain on the basis of equation (18):

$$(19)\quad \begin{cases} A(t) = P\{W(t)=1\} \\ \quad\;\; = P\{W(t)=1\}\cdot 1 + P\{W(t)=0\}\cdot 0 \\ \quad\;\; = E\{W(t)\}. \end{cases}$$

Thus we have found that availability $A(t)$ can also be given as the mean value of the proportional level of performance at time t.

For a generalized system with several possible levels of performance $c_0, c_1, \ldots, c_K$ (and with several possible proportional levels of performance $w_0, w_1, \ldots, w_K$ respectively), we can defin analogously to (18), an extended availability characteristic in the form of a distribution. The "availability" of the syste is now given by the probabilities of the positive proportional levels of performance:

$$(20)\quad \begin{cases} P_1(t) = P\{W(t)=w_1\} \\ P_2(t) = P\{W(t)=w_2\} \\ \vdots \\ P_K(t) = P\{W(t)=w_K\}. \end{cases}$$

Taking the relations between the states and the proportional levels of performance into account we can also express the new extended availability or availability function with the help of the state probabilities

$$(21)\quad \begin{cases} P_1(t) = P\{X(t) \in S_1\} = \sum_{j=1}^{n_1} P_{s_1 j}(t) \\ P_2(t) = P\{X(t) \in S_2\} = \sum_{j=1}^{n_2} P_{s_2 j}(t) \\ \vdots \\ P_K(t) = P\{X(t) \in S_K\} = \sum_{j=1}^{n_K} P_{s_K j}(t). \end{cases}$$

From the probability distribution (20) we can get into one specific characteristic when we turn, as in (19), to the expected value of the proportional level of performance. This new reliability characteristic, another generalized availability, we call the 'mean availability of the capacity'[1], and denote it by A_C:

$$(22)\quad A_C(t) = E\{W(t)\} = \sum_{i=0}^{K} w_i P\{W(t)=w_i\} = \sum_{i=0}^{K} w_i P_i(t).$$

Also $A_C(t)$ can be calculated with the help of the state probabilities of the system:

$$(23)\quad A_C(t) = \sum_{i=0}^{K} w_i P_i(t) = \sum_{i=0}^{K} w_i P\{X(t) \in S_i\} = \sum_{i=0}^{K} w_i \sum_{j=1}^{n_i} P_{s_i j}(t).$$

It is easy to notice that the mean availability of the capacity is a reliability characteristic in the sense of the general definition (3). In order to show this we define the functional Φ, for a fixed t, as follows:

$$(24)\quad \Phi_5(\hat{x}) = \begin{cases} w_0, \text{ if } \hat{x}(t) \in S_0 \\ w_1, \text{ if } \hat{x}(t) \in S_1 \\ \\ w_K, \text{ if } \hat{x}(t) \in S_K, \end{cases}$$

after which we have

1 The term 'mean availability of the capacity' comes from an important property, which A_C under the steady state posesses see considerations later on in this section

$$(25)\quad \begin{cases} E\{\Phi_5(\hat{X})\} = \sum_{i=0}^{K} w_i P\{\Phi_5(\hat{X})=w_i\} = \sum_{i=0}^{K} w_i P\{X(t) \in S_i\} \\ = \sum_{i=0}^{K} w_i P_i(t) = A_C(t), \end{cases}$$

i.e. $A_C(t)$ has been shown to be the expected value of the functional Φ_5 defined on the set $\hat{X}$ of the trajectories of the stochastic process X.

From the definition (22) and equation (23) we can immediately see, that A_C has a more general intension than the ordinary availability A: also the failures with only partial reduction in the level of performance have a decreasing effect on the value of A_C, in such a way, however, that this effect is smaller than the respective effect of failures with total inoperability.

Further, in the arguments for definition (22) it became apparent, that the availability of an operationally two-state operable-inoperable system stays as a particular case for the generalized characteristic A_C.

The analogy between the characteristics A and A_C holds also for their properties and interpretations under the steady state as we will show in the following. Under the steady state, the availability of a two-state operable-inoperable system (denote by A) gives the mean ratio of the actual system output to the output that would have been produced, if there had been no failures during the observation period. In figure 3 the steady state availability A thus is the ratio of the shaded area to the area of the rectangle with the base T_1T_2 and height 0C. In figure 4 the same situation is illustrated with relative units on both of the axises. Time is given as a portion of the interval T_1T_2 so that 0 corresponds to T_1 and 1 to T_2, and on the vertical axis the proportional level of performance of the system is shown. After the transformation of the units the steady state availability A is obtained as the shaded area in figure 4.

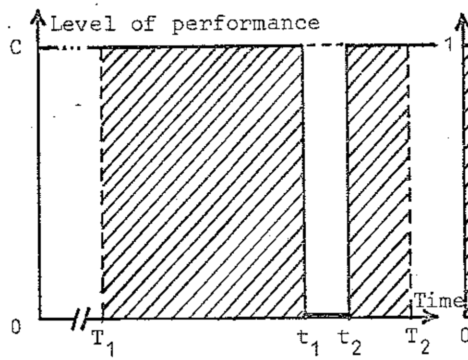

Figure 3. Steady state availability as a ratio of the actual and potential outputs of the system

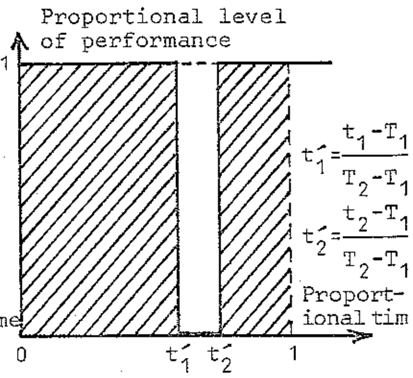

Figure 4. Steady state availability as an area under the proportional level of performance-curve

In reliability considerations the proportional levels of performance (or the levels of performance or the states) of the system during some interval (e.g. during the interval T_1T_2 in figures 3 and 4) are often illustrated in the form of a duration curve.[1] The proportional level of performance duration curve corresponding to figure 4 is presented in figure 5. From this figure we see, in addition to the area interpretatic of figure 4, the property of steady state availability, that it gives the mean portion of time during which the system is in the operable states. In figure 5 also the interpretation of availability as the mean proportional level of performance is illustrated.

In figures 3 to 5 we have four different steady state properties and interpretations for the traditional availability. In the following A_C, the mean availability of the capacity, is shown to posses quite analogous properties under the steady state as A was above shown to posses.

1 The duration curve of the proportional level of performance the proportional levels of performance appeared during the interval T_1T_2 are set into a descending order

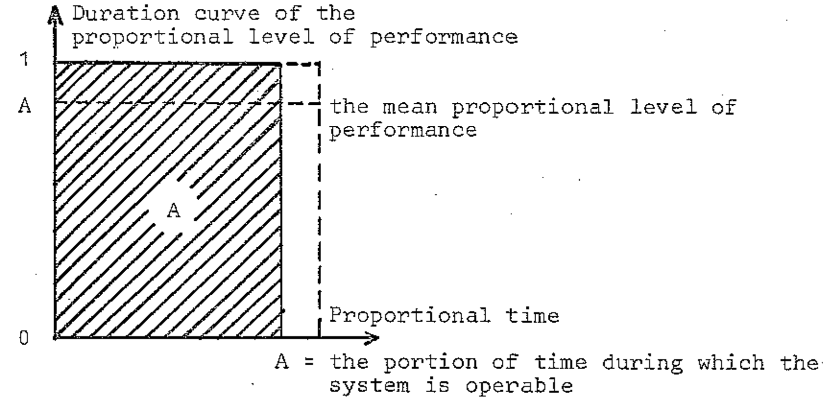

Figure 5. The steady state availability as the area under the duration curve of the proportional level of performance, as the mean proportional level of performance and as the portion of time during which the system is operable

First, in analogy with A (cf. figure 3), we can notice that under the steady state the mean availability of the capacity gives the ratio of the actual system output to the output that would have been possible without any failure during the observation period. This interpretation of A_C is illustrated in figure 6. A_C is the ratio of the shaded area to the area of the whole rectangle with the base T_1T_2 and height 0C. In figure 7 both time and performance quantities are expressed in relative units. In this case A_C is the shaded area under the proportional level of performance-curve (cf. figure 4).

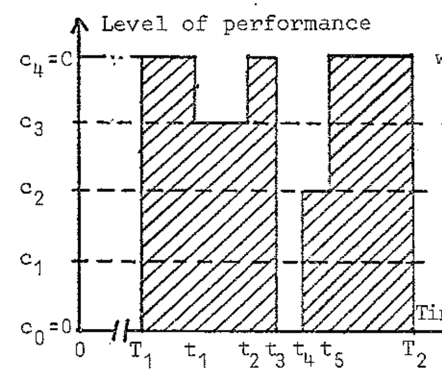

Figure 6. A_C as the ratio of the true and proportional outputs of the system

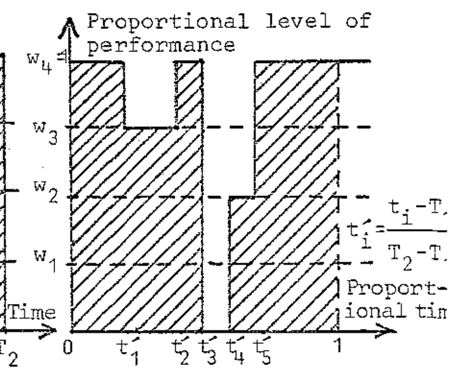

Figure 7. A_C as an area under the proportional lev of performance-curve

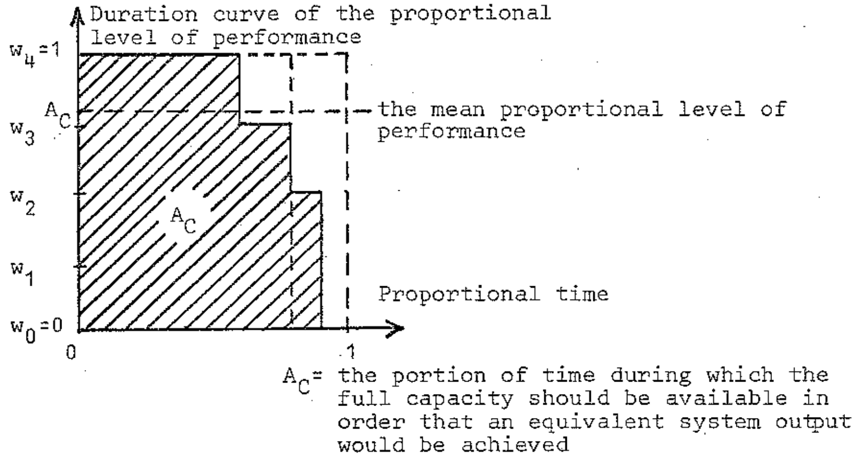

Figure 8. A_C as the area under the duration curve of the proportional level of performance, as the mean proportional level of performance and as the portion of tim for equivalent full capacity operation

In figure 8 the proportional level of performance-duration curve corresponding to figure 7 is shown. From this figure we can see, in analogy with figure 5, three different properties and interpretations for A_C: A_C is the mean proportional level of performance of the system (the definition of the characteristic A_C), A_C gives the area under the duration curve of the proportional level of performance (a derived property for A_C), and A_C equals to the portion of time during which the full capacity of the system should be available in order that an equivalent system output would be achieved (this is the interpretation of A_C, according to which the term 'mean availabilit of the capacity' has been chosen).

The extended availability concepts A_0 and A_C clearly emphasize on different points in the operational ability of the system. A_0 measures the reliability of the system in the sense of momentary operational ability (e.g. the ability of a power system to maintain different levels in the power production). A_C instead, measures the reliability of the system in the sense of production output (e.g. the amount of energy produced by the power system relative to the capacity of the system).

4 Conclusion

The extension of the concepts of reliability to cover also the systems with many possible levels of performance has been carried out above for the traditional characteristic 'availability' in detail. This characteristic has got as for its generalizations the more comprehensive characteristics 'availability of the levels of performance' and 'mean availability of the capacity'. On the basis of these examples and other account presented it is easy to carry out a corresponding generalization for any other conventional characteristic of reliability. Following the principles given, it is guaranteed that a natural connection with empirical interpretation as well as with the tradition of reliability theory will be preserved during the generalization.

REFERENCES

von Alven, W.H. (ed.) Reliability Engineering, Englewooc Cliffs (N.J.) 1964

Barlow, R.E., Proschan, F. Mathematical Theory of Reliabilit New York (N.Y.) 1965

Gnedenko, B.V., Belyayev, Y.K., Solovyev, A.D. Mathematical Methods of Reliabili Theory, New York (N.Y.) 1969

Jorgenson, D.W., McCall, J.J., Radner, R. Optimal Replacement Policy, Amste dam 1967

Lloyd, D.K., Lipow, M. Reliability: Management, Methods and Mathematics, Englewood Cliffs (N.J.) 1962

Polovko, A.M. Fundamentals of Reliability Theory, New York (N.Y.) 1968

Rau, J.G. Optimization and Probability in Systems Engineering, New York (N. 1970

Virtanen, I. On the Concepts and Derivation of Reliability in Stochastic Systems with States of Reduced Efficiency an Application of Supplementary Variables and Discrete Transforms (to appear in 1977)