THE LONG-TERM STABILITY AND STRUCTURAL INVARIANCE OF FINANCIAL RATIO PATTERNS

A comparative analysis between U.S. and Finnish firms on the aggregate level

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Financial statement analysis is an information-processing system developed to provide relevant data for decision makers. The great number of decision makers and their different objectives have caused that the ratios used in financial statement analysis have been numerous. Also many alternative categories of financial ratios have been proposed in litterature (Horrigan 1967: chapter 6, Foster 1978: 24-37, Courtis 1978: 372-375 and Tamari 1978: 24-44). However, there is no consensus on each ratio as to what the ratio primarly measures because of the differences in computation of financial ratios (Aho 1981: 16-19, Gibson 1982: 13 and Gombola and Ketz 1983: 105).

1.2. Review of prior research on topic

Remarkable insight into relationships between financial ratios was presented by the study of Pinches, Mingo and Caruthers (1973). They developed an empirically-based classification system for financial ratios using factor analysis. The approach introduced by Pinches, Mingo and Caruthers has been thereafter applied by many researchers, e.g. Courtis (1978), Johnson (1979), Aho (1980) and Laitinen (1983). Interesting results were also presented by Gombola and Ketz (1983) and Yli-Olli (1983) independently of each other. According to their results profitability ratios and cash-flow-ratios do not measure the same characteristic of firm performance (Gombola and Ketz 1983: 106 and Yli-Olli 1983: 40-50).

According to the results of Pinches, Mingo and Caruthers (1973) and Gombola and Ketz (1983) the classification patterns of the ratios where reasonable stable over time even when the magnitude of the ratios was undergoing change. Yli-Olli (1983) and Yli-Olli and Virtanen (1984 and 1985) used so called transformation analysis to measure the medium-term and the long-term stability of factor patterns. Compared with correlation or congruency analysis used in previous studies they got a more clear-cut picture about stability of factor patterns.

During the last two decades, a considerable amount of research has also been directed towards methodological issues in the use of financial ratios. The object of those papers is to provide insight into assumptions and limitations in the use of financial ratios (Gonedes 1973, Deakin 1976, Lev and Sunder 1979, Whittington 1980, and Frecka and Hopwood 1983).

1.3. The purpose of the study

The purposes of this research work are:

1. to develop, on the economy-wide level, empirically-based classification patterns for some commonly used financial ratios,

2. to compare, both on the theoretical and empirical levels, the usefulness of different aggregation methods in financial ratio analysis,

3. to measure, using transformation analysis, the long-term stability of financial ratios,

4. to measure the structural invariance of the financial ratio patterns between the U.S. and Finnish firms.

2. THE SELECTION OF FINANCIAL RATIOS AND SOME BASIC PROPERTIES OF THE RATIOS

In this chapter we present the litterature-based classification (called the a priori classification) of financial ratios examined. It will also be analyzed what each ratio, a priori, measures. The analysis is based on earlier researches and textbooks.

In this study, 12 different ratios are selected, which - **a priori** - measure **short-term solvency** (liquidity ratios), **long-term solvency** (leverage / capital structure ratios), **profitability** (profitability ratios) and **efficiency** (turnover ratios) of the firm (for the calculation of these ratios see e.g. Foster (1978: 43-48)). This classification is the most common in litterature (see e.g. Lev 1974: 12, Kettunen-Mäkinen-Neilimo 1979: 29, Foster 1978: 28 and Tamari 1978: 24-44) and it is oriented to the needs of users of these ratios (see more about the use of financial ratios Tamari 1978: 71-93 and 146-171).

2.1. Liquidity ratios

The ability of a firm to meet its short-term financial obligations is of prime interest to management, merchandise suppliers, lenders and investors. In the extreme case when the firm is not able to meet its short-term financial obligations those groups will be the losers.

The liquidity ratios examined in this study are the **current ratio** (CR), the **quick ratio** (QR) and the **defensive interval measure** (DI) (for calculation of liquidity ratios, see Foster 1978: 43-44). The current and quick ratios are, in principle, very similar. The denominator of both ratios consists of current liabilities. The numerator of the current ratio consists of current assets. The quick ratio includes in the numerator cash marketable securities and accounts receivable (current assets - (inventories + other current assets)). According to Lev (Lev 1974: 28) the quick ratio provides a stricter test of liquidity than the current ratio. In Gibson's inquiry ((Gibson (1982); a questionnaire was sent to the financial executives of the 500 largest industrial firms for 1979 listed in Fortune) there was a big consensus on each ratio as to what the ratio primarily measures: 94 % of the firms were of such opinion that the current ratio is a measure of liquidity. The corresponding number of quick ratio was 80 %.

The current and quick ratios have been criticized on the basis of their static structure (see Walter 1957: 38). These ratios reflect the surplus of current assets over current liabilities at a point in time.

This criticism led to the development of cash- and funds-flow-based liquidity ratios. Such a ratio is the defensive interval measure (see Davidson, Sorter and Kalle 1964: 23-26). This measure incorporates a dynamic element in liquidity evaluation. According to the results of Davidson's, Sorter's and Kalle's empirical study "there is substantial evidence that the interval measure and the traditional ratios (e.g. current ratio) produce differing impressions of the size and movement of a firm's defensive strength".

2.2. Long-term solvency ratios

A firm may finance its activities - as far as the external financing of the firm is concerned - either by using the funds borrowed or by investing the owners' money.

The selected long-term solvency ratios in this study are the **debt-to equity** (DE), **long-term debt to equity** (LTDE) and **times interest earned** (TIE) ratios.

Debt to equity and long-term debt to equity are very similar by nature and they are - like the current and quick ratios in liquidity measurement - static measures of the long-term solvency of the firm. They are measures for financial risk associated with the shareholders' equity.

The times interest earned ratio incorporates a dynamic element in long-term solvency evaluation.

2.3. Profitability ratios

It is presented in many textbooks that profitability may reflect different things to different users. Therefore many different indicators of profitability should be considered (see e.g. Tamari 1978: 25). In this study, three different ratios are used: **earnings to sales** (ES), **return on assets** (ROA) and **return on equity** (ROE).

The first ratio (ES) is a surrogate of operational efficiency of the firm and both the numerator and denominator of the ratio represent a flow over the entire period (i.e. a year). The second ratio (ROA) measures how efficiently total assets of the firm are being utilized. The third and most interesting ratio (ROE) indicates the profitability of the capital supplied by common stockholders.

2.4. Turnover ratios

Turnover ratios measure different aspects of firm's performance, i.e. the efficiency of the firm in using its assets in order to generate income. The

selected turnover ratios are: **total assets turnover** (TAT), **inventory turnover** (IT) and **accounts receivable turnover** (ART).

Total assets turnover together with earnings to sales (the first profitability ratio in this study) comprices the so-called DuPont system of ratio analysis (Foster 1978: 44). The inventory turnover ratio is supposed to indicate the efficiency of inventory management. The problem to be solved by inventory management is to determine and maintain an optimal inventory level. Accounts receivable turnover has been said to indicate efficiency of the credit department (Lev 1974: 28). So, the diminishing of this ratio may be due either to faulty collection system or to the weak financial position of the debtors. On the other hand, the reason can also be that the firm attempts to increase sales by granting more credit to customers. Irrespective of the cause, the decrease of this ratio from the target level indicates greater risk than the changes of default by customers.

3. DATA AND STATISTICAL METHODS

In this chapter, we present the data and give a brief description of the statistical methods used in the study. We also discuss the different aggregation methods to produce economy-wide data.

3.1. Data and empirical variables

The U.S. firms used for this study are selected from an Annual Industrial COMPUSTAT tape containing data for all December 31 fiscal year U.S. firms for the period 1947-75 (see Foster 1978: 156-160). The number of firms in the sample varies from year to year, increasing from about 450 in 1947 to about 1500 in 1975. The use of the same fiscal year firms gives a more clear-cut picture about different phases of economic cycles than the use of all firms regardles of the fiscal year. This is especially important in such cases - as in this study - where the analysis is mainly based on the first differences of the variables.

The Finnish firms used cover all the firms quoted on the Helsinki Stock Exchange (excluding bank and insurance companies). The number of the firms is 42 and the time period to be examined is 1974-84. In principle the ratios are calculated according to the recommendations of Yritystutkimusneuvottelukunta (see Yritystutkimusneuvottelukunta 1983).

The observations (rows) in the data matrix consist of the years 1947-75 (U.S. data) and of the years 1974-84 (Finnish data). The variables are the average values of the selected 12 financial ratios, the average values being computed across the individual firms. The average values have been computed in two different ways, as arithmetic (i.e. equal-weighted) averages and as value-weighted averages. In the following, these two ways to aggregate the individual firm-specific ratios into an economy-wide index are presented.

The ratios to be considered are defined in the traditional form, i.e. without the constant term. For firm i in year t we thus have

(3.1) $$r_{ti} = a_{ti}/b_{ti}, \qquad t = 1,2,\ldots, T;\ i = 1,2,\ldots, N_t,$$

where r_{ti} denotes the financial ratio in question for the ith firm in year t, and a_{ti} and b_{ti} are the relevant firm-specific accounting numbers in year t.

The usual arithmetic mean across the individual firms in year t is simply

(3.2) $$\bar{r}_t = (1/N_t) \sum_{i=1}^{N_t} (a_{ti}/b_{ti}), \qquad t = 1,2,\ldots, T.$$

The general form of the weighted average is

(3.3) $$\bar{r}_t^w = \sum_{i=1}^{N_t} w_{ti} r_{ti}, \qquad t = 1,2,\ldots, T,$$

where the weights w_{ti}, $t = 1,2,\ldots, T$, $i = 1,2,\ldots, N_t$, satisfy the conditions

(3.4) $$0 \leq w_{ti} \leq 1, \qquad \sum_{i=1}^{N_t} w_{ti} = 1.$$

In computing the weighted average (3.3), it is usual that the individual firm-specific ratios are weighted according to the size of the firms during the year in question. As the denominator of any financial ratio is typically a size or value variable (sales, equity etc.), the usual way to proceed is to use weights which are proportional to the denominators of the ratio. We thus have

(3.5) $$\begin{aligned} \bar{r}_t^w &= \sum_{i=1}^{N_t} w_{ti} r_{ti} \\ &= \sum_{i=1}^{N_t} (b_{ti} / \sum_{i=1}^{N_t} b_{ti})\, r_{ti} \\ &= \sum_{i=1}^{N_t} (b_{ti} / \sum_{i=1}^{N_t} b_{ti})\, (a_{ti}/b_{ti}) \\ &= (\sum_{i=1}^{N_t} a_{ti}) / (\sum_{i=1}^{N_t} b_{ti}), \qquad t = 1,2,\ldots, T. \end{aligned}$$

The last equation in (3.5) shows that the weighted average of the firm-specific values of a financial ratio, the weights being proportional to the size of the firms, can also be expressed as the ratio of the sums (over the individual firms) of the accounting numbers appearing in the numerator and in the denominator of the ratio, respectively.

3.2. Statistical methods

The empirical analysis in the study is based on multivariate time series data. The main statistical methods to be used are factor analysis and transformation analysis. Factor analysis can be regarded as a usual technique in business applications. Transformation analysis, on the contrary, has been largerly applied only in Finnish political and sosiological research. Therefore, the paper contains a short description of the main features of this multivariate method. And as transformation analysis uses the results of previous factor analyses, also a brief description of this technique, will also be presented.

3.2.1. Factor analysis

One of the specific purposes of the study is to develop from a set of 12 financial ratios classification patterns for the ratios in a lower dimension than the measurements in the data matrix have been presented. This is a typical problem to be handled via multivariate factor analysis.

Let us assume that we have p original variables $x_1,x_2,\ldots,x_p$ with mean values $\mu_1,\mu_2,\ldots,\mu_p$ and variances $\sigma_1^2,\sigma_2^2,\ldots,\sigma_p^2$, respectively. The common factor model postulates, that each x_i is linearly dependent upon a few unobservable variables $f_1,f_2,\ldots,f_r$ $(r < p)$, called common factors, and an additional source of variation u_i, called specific factor. The factor analysis model thus is (see e.g. Johnson and Wichern 1982: 402-407)

(3.6) $$x_i - \mu_i = l_{i1}f_1 + l_{i2}f_2 + \ldots + l_{ir}f_r + u_i, \quad i = 1,2,\ldots,p$$

or, in matrix notation

(3.7) $$\mathbf{x} - \mu = \mathbf{L}\mathbf{f} + \mathbf{u},$$

where $\mathbf{x}' = (x_1,x_2,\ldots,x_p)$, $\mu' = (\mu_1,\mu_2,\ldots,\mu_p)$, $\mathbf{L} = (l_{ij})_{pxr}$, $\mathbf{f}' = (f_1,f_2,\ldots,f_r)$ and $\mathbf{u}' = (u_1,u_2,\ldots,u_p)$. The coefficient l_{ij} is called the loading of the ith variable on the jth factor, so the matrix $\mathbf{L}$ is the matrix of factor loadings. Note that the ith specific factor u_i is associated only with the ith response x_i (u_i includes measurement error and quantities that are uniquely associated with the ith individual variable x_i). The p deviations $x_i - \mu_i$, $i = 1,2,..,p$, are expressed in terms of $r+p$ random variables $f_1,f_2,\ldots,f_r$, $u_1,u_2,\ldots,u_p$, all of which are unobservable.

Factor analysis contains three main phases: factoring, rotation and interpretation. The first phase, factoring, means the estimation of the factor matrix $\mathbf{L}$,

i.e. estimation of the number of factors r and the loadings l_{ij}. In this study the initial factor matrix is estimated by the principal component method. The number of factors is in the first hand determined according to the a priori hypothesis on the existence of four different classes of ratios (i.e. four factors). This criterion will be replenished by interpretative aspects and by eigenvalue and Cattell's scree test criteria.

There is always some inherent ambiquity associated with the factor model (3.7). For, if we have a nonsingular $r \times r$ matrix $\mathbf{T}$, and we denote

(3.8) $$\mathbf{L}^* = \mathbf{L}\mathbf{T} \quad \text{and} \quad \mathbf{f}^* = \mathbf{T}^{-1}\mathbf{f},$$

we can write

(3.9) $$\begin{aligned} \mathbf{x} - \mu &= \mathbf{L}\mathbf{f} + \mathbf{u} \\ &= \mathbf{L}\mathbf{T}\mathbf{T}^{-1}\mathbf{f} + \mathbf{u} \\ &= \mathbf{L}^*\mathbf{f}^* + \mathbf{u}. \end{aligned}$$

From (3.9) we can see that factor loadings $\mathbf{L}$ (and factors $\mathbf{f}$) are determined only up to a nonsingular matrix $\mathbf{T}$. Equations (3.8) represent the rotation phase of factor analysis. The initial loading matrix is rotated (multiplied by a nonsingular matrix), where the rotation is determined by some "simple-structure" or "ease-of-interpretation" criterion. The aim of the rotation thus is to provide a clearer resolution of the underlying factors.

Factor analysis contains several elements which have no unique solution (how many factors to extract?, how to choose the rotation matrix T?, etc.). In applications it is therefore important that these ambiguous quantities are fixed as to produce results which are based on some relevant theory and have meaningful empirical interpretations. Generally speaking, the interpretative phase is a proper part of the entire factoring process.

3.2.2. Transformation analysis

Another specific purpose of this study is to measure both the long-term stability and structural invariance in the factor analytical classification patterns. The degree of stability (both time-series and cross-sectional) in factor patterns has been traditionally measured with correlation coefficients (e.g. Pinches, Mingo and Caruthers 1973, Aho 1980) or with congruency coefficients (e.g. Johnson 1979, Gombola and Ketz 1983). Both of these measures give an index for the similarity of two different factor solutions in terms of the pattern of correlations among factor loadings across all variables in the reduced factor space. For the dissimilar part of these factor solutions these indices are, however, unable to describe and explain the reason for the non-invariant part prevailing in these factor solutions.

Recently, Yli-Olli (1983) introduced the use of transformation analysis for determining the degree and nature of medium-term stability exhibited by the factor patterns of the financial ratios. This approach was further applied and deepened by Yli-Olli and Virtanen (1984).

Transformation analysis was initiated by Ahmavaara (1954) and further developed by Ahmavaara (1963 and 1966), Ahmavaara and Nordenstreng (1970) and Mustonen (1966). The most applications of transformation analysis exist in the area of Finnish political and sociological research (e.g. Markkanen 1964, Nordenstreng 1968). Originally transformation analysis was developed to compare factor solutions between two (or more) different groups of objects, Yli-Olli (1983) and Yli-Olli and Virtanen (1984) have used the technique to compare two different factor solutions among the same group of objects, the two factor solutions being based on measurements made at different times (at two different time periods). In the following we sketch out the general idea behind transformation analysis (for a more detailed discussion, see e.g. Ahmavaara 1966, Mustonen 1966, Yli-Olli and Virtanen 1985).

Let's assume that we have two groups of observations G_1 and G_2 (two different groups of objects or one group measured at two different times) with the same variables, both by number and by content. Let L_1 and L_2 be the factor matrices (cf. equation (3.7)) for G_1 and G_2, respectively. Let's

further assume that the factor models used in deriving L_1 and L_2 are both orthogonal and have the same dimension, p x r, say.

If there exists invariance between the two factor structures, there exists a non-singular rxr-matrix T_{12} such that equation

$$L_2 = L_1 T_{12} \tag{3.10}$$

holds. Matrix T_{12} is called the transformation matrix (between L_1 and L_2, or in direction $G_1 \rightarrow G_2$). If equation (3.10) holds exactly, it means that the factor structures in groups G_1 and G_2 are, up to a linear transformation, invariant, all the variables have the same empirical meaning in different groups. Depending on the type of the transformation matrix T_{12}, the formation of the factors from the variables and thereby the interpretation of the factors either is preserved (T_{12} is the identity matrix I) or it changes (T_{12} has also non-zero off-diagonal elements).

In practice, situation (3.10) will not be reached, but, after matrix T_{12} has been estimated, we have $L_2 \neq L_1 T_{12}$. The goodness of fit criterion for the model (3.10) may be based on the residual matrix

$$E_{12} = L_1 T_{12} - L_2 \,. \tag{3.11}$$

Non-zero elements in E_{12} mean that the empirical meaning of the variables in question has changed. This is called abnormal transformation.

The main problem in transformation analysis is the estimation of the matrix T_{12}. The estimation methods are in general based on the minimization of the sum of squares of the residuals e_{ij} (the elements of the residual matrix E_{12}). This is the usual method of least squares. The problem is to minimize

$$\begin{aligned} \| E_{12} \| &= \| L_1 T_{12} - L_2 \| \\ &= \text{trace}\,((L_1 T_{12} - L_2)(L_1 T_{12} - L_2)') \,. \end{aligned} \tag{3.12}$$

Depending on additional constraints set for the matrix T_{12}, we have three different estimation methods (three transformation analysis models).

1. If there are no constraints for T_{12} in minimizing (3.12) we have the naive model.

2. If the transformation matrix has to obey the transitivity property $T_{kl}T_{lm} = T_{km}$, we get the relativistic model.

3. If the transformation matrix T_{12} is required to be orthogonal, i.e. $T_{12}^{-1} = T'_{12}$, we have the symmetric model. In this study the symmetric transformation analysis will be used (see Mustonen 1966).

Transformation analysis possesses several advantages in analysing the stability or the structural invariance of the factor patterns when compared for example with correlation or congruency analysis. With correlation and congruency coefficients one can only measure the degree of similarity of two factor solutions (correlations or congruencies among factor loadings across the variables in the factor space). This is also possible via transformation analysis (coefficients of coincidence on the main diagonal of the transformation matrix). In addition to this we obtain a regression type model for shifting of variables from one factor to another (normal or explained transformation). This is revealt by the non-zero off-diagonal elements in the transformation matrix and indicates interpretatively changes for the factors in question. And at last, large elements in the residual matrix, if any, indicate abnormal or unexplained transformation between the two factor solutions. This means that the empirical content of the corresponding variables has changed. Further, this abnormal transformation can be appointed to separate variables or to separate factors.

4. EMPIRICAL RESULTS

In this chapter we will develop empirically-based classification patterns for the presented financial ratios and measure the time-series stability and structural invariance of the ratios. The results are based on the value-weighted and equal-weighted averages of the selected financial ratios.

4.1. Financial ratio patterns using economy-wide ratio indices

Our factor-analytic derivation of the financial ratio patterns begins with the original level values of the aggregated ratios by using U.S. data. The four factors found via Kaiser's orthogonal varimax rotation are presented in Tables 1 and 2. The results in Table 1 are based on value-weighted averages and in Table 2, respectively, on the equal-weighted averages of the ratios. The number of factors to be extracted can be determined by using different criteria e.g. a priori knowledge, interpretative aspects, the eigenvalue criterion or Cattell's scree test. In this study, the number of factors extracted is mainly based on interpretative aspects and on a priori knowledge (i.e. the number of classes in the original classification). However, in most cases the eigenvalues associated with each factor exceed 1.

The form of all factor loading matrices to be presented in this study is the following. First, the columns (factors) appear in decreasing order of variance explained by the factors. The rows (variables) are rearranged so that, for each successive factor, loadings greater than 0.5 appear first. Loadings less than 0.25 are replaced by zero.

Table 1 shows that the four factor solution accounts for 97.1 per cent of the total variance in the original twelve financial ratios when value-weighted averages of the ratios are used. The corresponding value by using equal-weighted averages is 95.1 per cent. The communalities of all variables are also high. However, the interpretation of those factor solutions is not easy.

The financial ratios which achieve the highest factor loadings on the first factor in Table 1 are ROE (return on equity), ROA (return on assets), ART (accounts receivable turnover), TIE (time interest earned) and TAT (total assets turnover). The first factor can be interpreted as **a factor of profitability and efficiency.** However, the loading of ES (earnings to sales), supposed to be a profitability measure a priori, and the loading of IT (inventory turnover) an efficiency measure a priori, are quite low on the first factor. In addition, the loading of TIE (time interest earned), a long-term solvency variable a priori, is high on this factor.

Table 1. Varimax-rotated factor matrix for value-weighted averages (U.S. data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Communality h_i^2
ROEW	**0.974**	0.000	0.000	0.000	0.985
ROAW	**0.883**	0.290	-0.287	0.000	0.987
ARTW	**0.792**	0.253	-0.465	0.263	0.977
TIEW	**0.778**	0.324	-0.437	0.289	0.984
TATW	**0.775**	0.000	-0.471	0.338	0.953
LTDEW	0.000	**-0.889**	0.414	0.000	0.992
ESW	0.481	**0.862**	0.000	0.000	0.984
QRW	0.000	**0.750**	-0.453	0.379	0.967
DEW	-0.303	**-0.722**	0.493	-0.328	0.964
CRW	0.336	**0.659**	-0.589	0.278	0.972
ITW	0.000	-0.293	**0.899**	0.000	0.936
DIW	0.348	0.577	0.000	**0.684**	0.956
Variance explained by the factor	4.212	3.747	2.555	1.141	
Cumulative proportion of total variance	0.351	0.663	0.876	0.971	

Table 2. Varimax-rotated factor matrix for equal-weighted averages (U.S. data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Communality h_i^2
CRE	**0.920**	0.000	0.000	0.000	0.956
LTDEE	**-0.901**	0.000	0.000	0.000	0.962
DEE	**-0.865**	-0.396	0.000	0.000	0.966
ITE	**-0.861**	-0.380	0.000	0.000	0.898
ARTE	**0.766**	0.600	0.000	0.000	0.973
QRE	**0.749**	0.000	0.456	0.372	0.926
ROEE	0.000	**0.885**	0.000	0.339	0.914
ROAE	0.354	**0.863**	0.000	0.293	0.978
TATE	0.476	**0.810**	0.000	0.000	0.925
TIEE	0.521	**0.795**	0.259	0.000	0.981
DIE	0.283	0.307	**0.885**	0.000	0.983
ESE	0.000	0.397	0.000	**0.836**	0.950
Variance explained by the factor	5.069	3.804	1.324	1.215	
Cumulative proportion of total variance	0.422	0.739	0.850	0.951	

The second factor can be interpreted as a **factor of solvency**. The variable LTDE (long-term debt to equity), ES (earnings to sales), QR (quick ratio), DE (debt to equity) and CR (current ratio) have high loadings on this factor. All these variables, excluding ES, were, a priori, measures of either short- or long-term solvency.

The third factor indicates **the efficiency of the firms' inventory management.**

The fourth factor can be interpreted as a factor of **dynamic liquidity**.

The four factor solution, based on the equal-weighted averages of the ratios, is presented in Table 2. The interpretation of the first factor in Table 2 is, to some extent, similar to the interpretation of the second factor in Table 1. This factor describes, in the first place, the **solvency** of the firm. However, also the variables IT and ART (turnover measures a priori) achieve high loadings on this factor (the loadings being even provided with different signs).

The second factor in Table 2 can be interpreted as a **factor of profitability**. The third factor indicates **dynamic liquidity** of the firm.

The variable ES (earning to sales) creates a factor of its own in the solution of Table 2.

Summarizing, although the explained variances and communalities in Tables 1 and 2 are very high, the interpretation of the factors is not easy. This is especially true when the factor solution based on equal-weighted averages is concerned. In addition, the obtained factor solutions in Tables 1 and 2 differ a lot from each other. The reason for this numerically satisfactory but interpretatively confusing situation is quite clear: the high seeming correlations among the variables caused by time.

4.2. Financial ratio patterns using first differences of the ratios

The factor solutions found by using the first differences of the selected ratios are presented in Tables 3 and 4 (U.S. data). The four factor solutions account for

87.8 per cent (Table 3) and 86.4 per cent (Table 4) of the total variances in the original variables. The obtained values are slightly lower than those presented in Tables 1 and 2 because the trend is now removed from the variables.

The interpretation of the first factor in Table 3 is clear and unambigious. The financial ratios which achieve the highest loadings on this factor are DE, CR, LTDE and QR. This factor decribes **the solvency of the firms.** (In fact, the factor model being based on the first differences of the ratios, also the interpretation of the factor should be the **change** of the solvency of the firms. For the sake of simplicity and clarity, all the factors are named according to basic quantifies themselves, however.).

The second factor can be interpreted as a factor of **profitability.** The interpretation of this factor is also easy. Only the high loading of the variable TIE (times interest earned) on this factor requests an explanation. This variable was, according to the a priori classification, the measure of dynamic long-term solvency. However, Table 3 and also the following results show that TIE is rather the measure of profitability than that of long-term solvency.

The third factor describes the **efficiency** of the firm. The variables with the highest loadings on this factor are, in correspondence with a priori classification, the three turnover ratios selected for this study.

The fourth factor indicates the **dynamic short-term solvency** of the firm. This factor is a very pure one-variable (DI) factor, because the loadings of other ratios are very low on this factor.

We see that the interpretation of the financial ratio classification which is based on the first differences of the value-weighted indices of the ratios, becomes very clear-cut compared with the results obtained from the original level values of the financial ratios (Tables 1 and 2).

Table 4 shows the results based on the first differences of the equal-weighted averages of the financial ratios (U.S. data). The first factor in Table 4 is an indicator of **profitability and efficiency** of the firms. This factor includes the main parts (ART excluded) of the second and third factor presented in Table 3.

Table 3. Varimax-rotated factor matrix for the first differences of value-weighted averages (U.S. data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Communality h_i^2
DEWD	**-0.895**	0.000	0.000	0.000	0.832
CRWD	**0.834**	-0.272	-0.281	0.000	0.854
LTDEWD	**-0.834**	0.000	0.411	0.000	0.895
QRWD	**0.834**	0.000	0.000	0.454	0.935
ESWD	0.000	**0.942**	0.000	0.000	0.942
ROEWD	0.000	**0.874**	0.384	0.000	0.970
ROAWD	0.000	**0.813**	0.506	0.000	0.946
TIEWD	-0.394	**0.709**	0.000	0.358	0.785
TATWD	-0.338	0.000	**0.821**	0.000	0.850
ITWD	-0.271	0.613	**0.621**	0.000	0.858
ARTWD	-0.454	0.412	**0.590**	0.000	0.725
DIWD	0.000	0.000	0.000	**0.928**	0.943
Variance explained by the factor	3.607	3.568	2.124	1.235	
Cumulative proportion of total variance	0.301	0.598	0.775	0.878	

Table 4. Varimax-rotated factor matrix for the first differences of equal-weighted averages (U.S. data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Communality h_i^2
ROAED	**0.901**	0.000	-0.282	0.000	0.899
ESED	**0.875**	0.000	0.000	0.000	0.776
ROEED	**0.857**	0.000	0.000	0.000	0.787
TIEED	**0.837**	0.000	-0.345	0.000	0.882
ITED	**0.703**	-0.590	0.000	0.000	0.885
TATED	**0.548**	-0.398	-0.336	-0.370	0.708
DEED	0.000	**-0.913**	0.000	0.000	0.891
LTDEED	0.000	**-0.877**	-0.375	0.000	0.923
QRED	0.000	0.328	**0.851**	0.269	0.948
CRED	-0.407	0.411	**0.759**	0.000	0.928
ARTED	0.000	0.000	0.000	**-0.906**	0.877
DIED	0.427	0.000	0.455	**0.691**	0.867
Variance explained by the factor	4.235	2.430	2.077	1.629	
Cumulative proportion of total variance	0.353	0.555	0.728	0.864	

The first factor in the value-weighted average solution in Table 3 (the solvency factor), is in Table 4 divided into two factors: a factor of **long-term solvency** (the second factor) and a factor of **short-term solvency** (the third factor).

The fourth factor can be interpreted - as in Table 3 - as a factor of **dynamic liquidity.** However, that interpretation proves difficult, because of the high negative loading of the variable ART on this factor.

The inconsistency in the behaviour of the variable ART between Tables 3 and 4 derives its origin probably from the very different role of accounts receivable when we have computed the values of variable DI among small and big firms. Thus, different aggregation methods lead to the very different results. Also other differences, caused by different aggregation methods can be found between the factor patterns presented in Tables 3 and 4. These differences are analyzed in a more detailed way in Section 4.3. The presented results (Tables 1-4) confirm that the use of the variables in first-difference form lead to a more valid classification pattern than the classification pattern of original ratios. Further, the results also suggest that the value-weighted indices give, in addition to that they are theoretically more accurate, more clear-cut classifications of the financial ratios than the equally-weighted indices. The differences between these two classifications are analyzed in greater detail in Section 4.3. by using transformation analysis.

It is difficult to find clear theoretical arguments why the factors extracted from financial ratios should be uncorrelated. Therefore, the results presented in Tables 3 and 4 were verified with a non-ortogonal rotation method. The result showed that the non-orthogonal factor pattern was very similar to the orthogonal factor pattern. The solvency factor seemed to be non-orthogonal both to the profitability and efficiency factors. The correlations between those factors were -.227 and -.445, respectively. This result can be interpreted as follows: maintenance of high solvency has a slightly decreasing effect on efficiency and profitability. The correlation between profitability factor and efficiency factor was .421. The positive correlation coefficient suggests that although the profitability and efficiency ratios measure different dimensions of the firms' performance those dimensions are not independent. The fourth factor (dynamic short-term solvency) clearly measures quite an independent dimension in the firms' behaviour.

The oblique factor solution mainly supported the findings for the corresponding orthogonal solution. On the other hand, it gave valuable additional information about the interdependencies existing between the main dimensions of the financial ratios. Due to the strong similarity between these two solutions, the subsequent analysis will be restricted, however, to orthogonal factor models only.

4.3. The long-term stability of financial ratio patterns (U.S. data)

One of the objectives in this study was to measure, using transformation analysis, the long-term stability of factor patterns obtained. For this analysis, the whole period concerning U.S. firms is divided into two sub-periods of equal length: sub-period 1 includes the years 1947-61 and sub-period 2 the years 1962-75.

Sections 4.1. and 4.2. show that empirical results based on the variables in the first-difference form are very clear-cut compared to those in ratio forms. This indicates that the difference-formed models should be preferred to the level-formed models. Therefore, stability analysis presented in this section is based only on the variables in the first-difference form.

Tables 5 and 6 show, value-weighted variables and U.S. data being used, the four factor solutions for sub-period 1 and for sub-period 2, respectively. The four factor solutions account for 91.2 per cent (Table 5) and 91.3 per cent (Table 6) of the total variances in the original variables. The corresponding number for the whole period was 87.8 (Table 3).

The four factor solution for sub-period 1 is quite similar to that of the whole period. The major difference between those two solutions is in the loadings of the variables IT and ART. Those variables have the highest loading on the profitability/efficiency factor in Table 5. In Table 3 those variables, together with the variable TAT, created more clearly an efficiency factor of their own (this efficiency factor still exists, however, in the solution of Table 5).

Table 5. Varimax-rotated factor matrix for the ratios (for the value-weighted averages in the first difference form) in sub-period 1 (U.S. data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Communality h_i^2
ESWD	**0.952**	0.000	0.000	0.000	0.943
ROEWD	**0.920**	-0.266	0.000	0.000	0.981
ROAWD	**0.879**	0.000	0.421	0.000	0.979
TIEWD	**0.755**	-0.432	0.000	0.295	0.876
ITWD	**0.623**	-0.433	0.547	0.000	0.915
ARTWD	**0.604**	-0.574	0.328	0.000	0.803
DEWD	0.000	**-0.913**	0.000	0.000	0.837
CRWD	-0.284	**0.867**	-0.291	0.000	0.924
LTDWD	0.000	**-0.861**	0.335	0.000	0.908
QRWD	0.000	**0.799**	0.000	0.500	0.928
TATWD	0.332	-0.385	**0.792**	0.000	0.886
DIWD	0.000	0.000	0.000	**0.955**	0.959
Variance explained by the factor	4.137	3.943	1.553	1.307	
Cumulative proportion of total variance	0.345	0.673	0.803	0.912	

Table 6. Varimax-rotated factor matrix for the ratios (for the value-weighted averages in the first difference form) in sub-period 2 (U.S. data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Communality h_i^2
ROEWD	**0.930**	0.000	0.279	0.000	0.970
ESWD	**0.897**	0.000	0.000	0.286	0.939
ITWD	**0.896**	0.000	0.359	0.000	0.953
ROAWD	**0.888**	0.000	0.328	0.000	0.967
TIEWD	**0.579**	0.571	-0.321	-0.293	0.850
QRWD	0.000	**0.961**	0.000	0.000	0.970
CRWD	0.000	**0.924**	0.000	0.000	0.900
DEWD	0.000	**-0.840**	0.000	-0.263	0.814
LTDEWD	0.278	**-0.802**	0.356	0.000	0.903
ARTWD	0.000	0.000	**0.920**	0.000	0.909
TATWD	0.371	-0.257	**0.654**	-0.525	0.906
DIWD	0.000	0.543	0.000	**0.695**	0.870
Variance explained by the factor	3.963	3.922	1.918	1.150	
Cumulative proportion of total variance	0.330	0.657	0.817	0.913	

The factor solution for sub-period 2 is more close to the solution of the whole period than that for sub-period 1. Now only the variable IT changes the factor. It transfers from efficiency factor to profitability factor (in the whole period vs. sub-period 2).

Table 7 presents the transformation matrix between the factors for sub-period 1 (Table 5) and sub-period 2 (Table 6). The factors were calculated on the basis of value-weighted indices, and in the first-difference form they display considerable long-term stability. This conclusion is based on the coefficients of coincidence on the main diagonal of the transformation matrix. The numerical values of those coefficients are very close to 1. In addition, the transformation matrix shows a slight transference between the first and fourth factors. This result confirms apparent differences between the variable DI (dynamic liquidity) and other liquidity or short-term solvency measures (CR and QR). The variable DI loads on a separate and distinct factor which has a weak connection with the profitability factor.

Table 8 presents the residual matrix for sub-period 2 (matrix E_{12}). Zero elements in residual matrix mean that the variables in question measure the same characteristic of the firms' performance during different periods. Non-zero elements in residual matrix mean that the empirical meaning of the variables in question has changed.

The residual matrix shows that there are only two variables with a moderately high abnormal transformation. The abnormal transformation of the variable TIE can be designated to the factors 1, 2 and 4. This variable was in the a priori classification the measure of long-term solvency. However, during the first sub-period, this ratio was, in the first place, the measure of profitability. The great negative numerical value -0.850 on the second factor in residual matrix shows that the feature measuring long-term solvency increases considerably in variable TIE during the second sub-period. The abnormal transformation of the variable ART is high on the first, second and third factors. It means that the empirical content of this variable has changed. However, the transformation and residual matrices in Tables 7 and 8 indicate a very high long-term stability of the factor pattern.

Table 7. Transformation matrix between the factor patterns of ratios in sub-period 1 and sub-period 2 (factors based on the first differences of the value-weighted averages of the ratios; U.S. data).

		Sub-period 2			
	Factor	1	2	3	4
Sub-period 1	1	**0.954**	0.145	-0.080	-0.249
	2	-0.178	**0.978**	-0.039	-0.100
	3	0.056	0.042	**0.994**	-0.080
	4	0.234	0.143	0.058	**0.960**

Table 8. Residual matrix E_{12} and abnormal transformation for sub-period 2 (factors based on the first differences of the value-weighted averages of the ratios; U.S. data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Abnormal transformation t_i^2
CRWD	-0.227	-0.118	-0.350	0.161	0.214
QRWD	-0.171	-0.133	-0.102	0.230	0.111
DIWD	0.105	-0.249	0.277	0.165	0.177
DEWD	0.190	-0.057	-0.122	0.327	0.161
LTDEWD	0.119	0.009	-0.008	0.240	0.072
TIEWD	0.277	-0.850	0.115	0.446	**1.010**
ESWD	0.007	0.106	0.226	-0.510	0.322
ROAWD	0.019	0.197	0.031	0.026	0.041
ROEWD	0.002	0.032	-0.095	-0.197	0.049
TATWD	0.058	-0.039	0.122	0.413	0.190
ITWD	-0.147	-0.151	0.164	-0.035	0.072
ARTWD	0.480	-0.360	-0.618	-0.026	**0.742**
Abnormal transformation s_j^2	0.474	1.024	0.722	0.941	3.161

Tables 9 and 10 present for sub-period 1 and for sub-period 2, respectively, the four factor solutions based on the equal-weighted indices in the first-difference form (U.S. data). The factor pattern in Table 9 is very similar to that obtained by using value-weighted averages of variables in the first difference form (Tables 3, 5 and 6). Table 9 includes four different factors: profitability, solvency, dynamic liquidity and efficiency.

Table 9. Varimax-rotated factor matrix for the ratios (for the equal-weighted averages in the first difference form) in sub-period 1 (U.S. data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Communality h_i^2
ROAED	**0.934**	0.000	0.000	0.000	0.937
TIEED	**0.912**	-0.284	0.000	0.000	0.954
ESED	**0.902**	0.000	0.000	0.000	0.845
ROEED	**0.896**	0.000	0.000	0.293	0.932
LTDEED	0.000	**-0.949**	0.000	0.000	0.922
DEED	0.000	**-0.873**	0.000	0.000	0.860
QRED	-0.366	**0.839**	0.377	0.000	0.980
CRED	-0.470	**0.831**	0.000	0.000	0.963
ARTED	0.000	0.000	**-0.925**	0.000	0.903
DIED	0.315	0.292	**0.844**	0.000	0.930
ITED	0.626	0.000	0.000	**0.719**	0.960
TATED	0.383	-0.509	0.000	**0.705**	0.937
Variance explained by the factor	4.356	3.594	1.865	1.307	
Cumulative proportion of total variance	0.363	0.663	0.818	0.927	

Table 10. Varimax-rotated factor matrix for the ratios (for the equal-weighted averages in the first difference form) in sub-period 2 (U.S. data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Communality h_i^2
QRED	**0.946**	0.000	0.000	0.000	0.942
CRED	**0.859**	0.000	0.279	-0.308	0.928
DIED	**0.786**	0.000	0.000	0.445	0.819
TIEED	0.000	**0.909**	0.000	0.000	0.870
ROAED	-0.474	**0.745**	0.000	0.323	0.921
TATED	-0.630	**0.703**	0.000	0.000	0.896
ESED	0.293	**0.650**	0.000	0.613	0.893
ARTED	-0.415	**0.542**	0.350	0.000	0.617
DEED	0.000	0.000	**-0.967**	0.000	0.938
LTDEED	0.000	0.000	**-0.953**	0.000	0.955
ITED	0.000	0.485	**-0.628**	0.316	0.739
ROEED	0.000	0.000	0.000	**0.902**	0.887
Variance explained by the factor	3.232	2.887	2.538	1.749	
Cumulative proportion of total variance	0.269	0.510	0.721	0.867	

On the contrary, the factor solution in Table 10 differs considerably from the solutions presented in Tables 3, 5, 6 and 9. The first factor describes the short-term solvency of the firms. The variables, which according to the a priori classification serve as the measures of liquidity, have the highest loadings on this factor (exceptionally also the variable DI). The second factor can be interpreted as a factor of profitability and efficiency. The third factor describes the long-term solvency of the firms. Finally, the variable ROE creates a factor of its own.

Table 11 presents the transformation matrix between the factors given in Tables 9 and 10. The transformation matrix shows that the long-term stability between factor patterns is very low. The first factor given in Table 9 is divided in the first place to the second and fourth factors during the second period. The second factor is divided to the first and third factors etc. Table 12 presentes the residual matrix involved. The residual matrix shows that, in spite of the considerable instability associated with the factors, any remarkable abnormal transformation does not exist. The empirical meaning has changed only among factors, not among variables. Only the variables TIE and ROE have some noticeable abnormal transformation.

Table 7 indicates a very high long-term stability between factor patterns when the variables are value-weighted indices in the first-difference form. On the other hand, the results presented in Table 11 give evidence of considerable instability between factor patterns when the variables are equally-weighted indices in the first-difference form. These results prove that the role of the aggregation method is very important, when we consider and calculate some industry-wide or economy-wide norms to be "target financial ratios" for firms.

Table 11. Transformation matrix between the factor patterns of ratios in sub-period 1 and sub-period 2 (factors based on the first differences of the equal-weighted averages of the ratios; U.S. data).

		Sub-period 2			
	Factor	1	2	3	4
Sub-period 1	1	-0.101	**0.556**	0.038	**0.824**
	2	**0.622**	0.175	**0.759**	-0.077
	3	**0.768**	-0.186	**-0.562**	0.246
	4	0.114	**0.791**	-0.327	**-0.504**

Table 12. Residual matrix E_{12} and abnormal transformation for sub-period 2 (factors based on the first differences of the equal-weighted averages of the ratios; U.S. data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Abnormal transformation t_i^2
CRED	-0.177	-0.133	0.283	-0.028	0.130
QRED	-0.095	-0.106	0.229	-0.158	0.097
DIED	0.033	0.214	-0.234	-0.093	0.110
DEED	-0.344	-0.087	0.141	-0.156	0.170
LTDEED	-0.336	-0.105	0.165	0.043	0.153
TIEED	-0.240	-0.462	-0.194	0.690	**0.785**
ESED	-0.353	-0.013	0.190	0.040	0.162
ROAED	0.308	-0.060	0.086	0.333	0.216
ROEED	0.005	0.492	-0.162	-0.290	**0.352**
TATED	0.214	0.013	-0.463	0.010	0.260
ITED	0.118	0.375	0.204	-0.114	0.209
ARTED	-0.210	-0.176	0.226	-0.387	0.276
Abnormal transformation s_j^2	0.654	0.724	0.649	0.896	2.922

4.4. On structural invariance of the ratio pattern between U.S. and Finnish firms

The resulting factor pattern for U.S. data (Table 3) displayed a very high long-term stability, when the variables are value-weighted indices and in the first difference form.

The corresponding factor solution for the Finnish data (the first differences of the value-weighted averages is presented in Table 13.

Table 13. Varimax-rotated factor matrix for the first differences of value-weighted indices (Finnish data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Communality h_i^2
ROEWD	**0.970**	0.000	0.000	0.000	0.981
ESWD	**0.966**	0.000	0.000	0.000	0.974
TIEWD	**0.964**	0.000	0.000	0.000	0.987
ROAWD	**0.945**	0.000	0.000	0.000	0.966
ITWD	**0.900**	0.000	0.000	0.000	0.824
LTDEWD	0.000	**-0.976**	0.000	0.000	0.964
DEWD	-0.326	**-0.913**	0.000	0.000	0.993
CRWD	0.541	**0.642**	0.000	0.444	0.910
DIWD	0.000	0.000	**0.969**	0.000	0.981
TATWD	**0.633**	0.347	**-0.656**	0.000	0.960
ARTWD	0.000	0.000	-0.341	**0.893**	0.929
QRWD	0.402	**0.501**	0.381	**0.642**	0.970
Variance explained by the factor	5.481	2.770	1.685	1.502	
Cumulative proportion of total variance	0.457	0.688	0.828	0.953	

The initial factor extraction - like was the case with U.S. data - was based on the principal component method. The eigenvalues associated with the principal components were 6.786, 2.083, 1.685, 0.885, 0.290, 0.250, 0.014, 0.005, 0.003, 0, 0, and 0. These might suggest that only three factors should be extracted. All the other criteria, a priori hypothesis (based both on theory and on earlier empirical results), the scree test, and interpretative aspects (after rotation), motivated, however, strongly that four factors should be used.

The explanatory power of the factor model is high. The solution accounts for 95.3 per cent of the total variance associated with the original variables, the individual communalities varying from 0.824 (ITWD) to 0.993 (DEWD). Also the interpretation of the solution is quite clear-cut and analogous to that of the U.S. model.

The first factor, explaining 45.7 per cent of total variance, describes the **profitability** of the Finnish firms. This interpretation can be based on the very

high loadings of the "profitability" ratios ROEWD, ESWD, TIEWD and ROAWD. These four ratios are the same which formed the second, profitability, factor in the U.S. model. The pure loading of the ratio times interest earned (TIEWD) on the profitability factor is also a confirmation to the similar finding in U.S. data (Table 3). The usual classification of the ratio TIE is to consider it as a long-term solvency (leverage/capital structure) ratio (see e.g. Foster 1978: 31). The factor also has a high loading by inventory turnover (ITWD) and moderate loadings by total assets turnover (TATWD) and current ratio (CRWD). The latter cause a slight difference between these two factors, both having, however, a very clear interpretation of profitability.

The second factor (explaining 23.1 per cent of the total variance) has high loadings by long-term debt to equity (LTDEWD), debt to equity (DEWD), current ratio (CRWD) and quick ratio (QRWD). The factor describes the **solvency** of the firms. It is almost identical to the first factor in the U.S. model. It is worth to note here again the interesting feature that the short-term solvency (or liquidity) ratios CR and QR and the long-term solvency ratios DE and LTDE are empirically classified into the same dimension in the behaviour of the firms. Traditionally the short-term solvency and long-term solvency measures have been considered as different ratio classes in the ratio analysis (e.g. Lev 1974: 12, Kettunen - Mäkinen - Neilimo 1976: 29, Foster 1978: 28 and Tamari 1978: 24-44).

The third factor (with 14.0 per cent contribution to the variance) is mainly formed by the ratio defensive interval measure (DIWD). It indicates the **dynamic short-term solvency** of the firms. The factor has a good coincidence with the corresponding factor of U.S. firms (factor four in Table 5). Some dissimilarity exists in the loadings of the turnover ratios (especially TATWD).

The fourth factor (with 12.5 per cent contribution to the variance) is the least satisfactory one. In the U.S. classification the corresponding factor (the third factor) was a very clear efficiency factor, all the turnover ratios having high loadings on it. On the basis of the highest loading (0.893 by ARTWD) the factor might also now be simply named as an efficiency factor. Rebembering, however, the role of the ratio ART and taking the moderate loadings of the variables CRWD and QRWD into account, we can specify the factor as a measure for the **efficiency of the credit management.** The factor is thus of a more specific nature than the corresponding factor of "general efficiency" of the U.S. firms.

Thus far we have the following results and findings. First, we have the U.S. model which has been shown to posses a high degree of long-term stability (i.e. time-invariance). And second, we have the Finnish model derived in this section which seems quite similar to the U.S. model. Comparing these two models, which are based on data originating from different countries in different time periods, we can analyze the structural invariance of the two models, in fact the general invariance of the whole classification procedure.

The transformation matrix between the factors for U.S. data (Table 3) and Finnish data (Table 13) are presented in Table 14. We see that the analysis displays a considerable invariance between the two factor patterns. The coefficients of coincidence for the four factors are 0.954 (solvency factor), 0.924 (profitability), 0.860 (dynamic short-term solvency) and 0.815 (efficiency) which all can be regarded as high. A certain amount of tranference of loadings can be seen, however, to exist between profitability and efficiency factors (elements -0.337 and 0.290 in the transformation matrix) and between dynamic short-term solvency and efficiency factors (elements -0.487 and 0.398). The result is as expected. The two similar factor patterns result in a near-to-unity transformation matrix indicating a high degree of structural invariance in the classification with minor elements of non-invariance (caused mainly by the turnover ratios), however.

The residual matrix shows that three of the ratios, viz. CRWD, ARTWD and TATWD have a moderate high abnormal transformation. These ratios thus measure, to some extent, different aspects in the firm's behaviour among U.S. firms than among Finnish firms. The abnormal transformation can be mainly designated to the profitability factor (CRWD and ARTWD), to the solvency factor (TATWD) and to the efficiency factor (ARTWD). The result is not surprising, cf. for example the comments given in connection with the presentation of factor loadings matrices. As a whole, however, the amount of abnormal transformation can be regarded quite tolerable: the total residual is 5.499 when it eg. in transformation between the two sub-periods for U.S. data was 3.161 (and in the latter case the models were obtained for the **same** group in two different time periods).

Table 14. Transformation matrix between the factor patterns of the financial ratios in USA and Finland (factors based on the first differences of the value-weighted averages of the ratios).

	Factor	Finland 1	2	3	4
	Interpretation	Profitability	Solvency	Dynamic short-term solvency	Efficiency of credit management
USA	1 Solvency	-0.078	**0.954**	0.141	0.252
	2 Profitability	**0.924**	0.173	-0.057	-0.337
	3 Efficiency	0.290	-0.120	-0.487	**0.815**
	4 Dynamic short-term solvency	0.238	-0.212	**0.860**	0.398

Table 15. Residual matrix E_{12} and abnormal transformation between U.S. and Finnish data (factors based on the first differences of the value-weighted averages of the ratios).

Ratio	Factor 1	Factor 2	Factor 3	Factor 4	Abnormal transformation of the ratios
CRWD	-0.923	0.127	0.236	-0.345	**1.043**
QRWD	-0.509	0.194	0.199	-0.321	0.439
DIWD	0.392	-0.092	-0.169	0.559	0.503
DEWD	0.366	0.040	-0.134	0.103	0.164
LTDEWD	0.410	0.168	-0.427	0.062	0.382
TIEWD	-0.197	-0.515	0.260	-0.349	0.494
ESWD	-0.057	0.108	-0.099	-0.171	0.054
ROAWD	-0.021	-0.318	-0.127	0.104	0.128
ROEWD	-0.041	-0.316	-0.311	-0.061	0.202
TATWD	-0.151	-0.717	0.161	0.390	**0.715**
ITWD	-0.096	-0.332	-0.167	0.243	0.206
ARTWD	0.679	-0.532	-0.004	-0.651	**1.169**
Abnormal transformation of the factors	2.103	1.478	0.568	1.350	5.499

The output from transformation analysis (Tables 14 and 15) thus is that the empirical aggregate-level classification pattern possesses, when it is based on the value-weighted averages of the ratios in the difference form, a high degree of structural invariance between different countries (USA and Finland, explicitely) and different time periods. The invariance analysis can be further deepened when the Finnish classification pattern obtained for the years 1974-84 is compared with the two sub-models of U.S. data (years 1947-61 and 1962-75, respectively).

The fit between the model for sub-period 1 (Table 5) and the Finnish model is about the same level as that between the total U.S. model and the Finnish model: the coefficients of coincidence in the transformation matrix vary from 0.839 (dynamic short-term solvency) to 0.966 (solvency), the off-diagonal elements in the transformation matrix are of the same magnitude as in Table 7, and the total amount of abnormal transformation is 6.412 (the main sources of this abnormal transformation being again ARTWD, CRWD and TATWD). The corresponding figures for the comparison between sub-model 2 (Table 6) and the Finnish model are the following. The coefficients of coincidence are now even higher, varying from 0.895 (efficiency) to 0.990 (profitability). The off-diagonal elements are near to zero, the largest element being -0.357 (efficiency/dynamic short-term solvency). The residual matrix shows no considerable abnormal transformation, the total residual is only 3.856.

The analysis confirms that there exists a high degree of structural invariance between the four-factor U.S. and Finnish models. The invariance is the better, the more adjacent the time-periods to be compared are, but it is sustained also to a period more far away.

4.5. Alternative empirical patterns for ratio classification

The analyses carried out in Sections 4.3. and 4.4. show that the empirical aggregate-level classification pattern possesses, when based on the value-weighted averages of the ratios in the difference form, a high structural invariance between different time periods and between different countries. On the other hand we saw, when the U.S. data was concerned, that the use of the

data in alternative forms caused problems in the classification procedure. The classification pattern was either difficult to interprete (the use of the ratio indices in the original level form) or it was not stable enough over time (the use of equal-weighted averages as the primary data). The objective of this section is to find out whether the same problems arise when comparisons across countries are made. The results will give valuable insight into the problems concerning both the elimination of the harmfull "multicollinear" trend-effect and the choice of an adequate aggregation technique.

4.5.1. Classification pattern based on the first differences of the equal-weighted ratio indices

The classification pattern based on the differences of the equal-weighted ratio indices for U.S. data (Table 4) was also quite clear-cut and easy to interprete. The classification pattern showed, however, no considerable time-series stability, wherefore preference was given to the model based on the value-weighted averages.

The corresponding factor matrix for Finnish data is given in Table 16. The resulting model is a clear four-factor solution, because the eigenvalues of the correlation matrix are 6.000, 2.739, 1.349, 1.056, 0.535, 0.176, 0.134, 0.010 and four 0's. In addition, the explanatory power of the model is reasonable high (92.9 per cent in total). Also the communalities are high (with one exception: the communality for ARTWD is only 0.598!). And further, the model can be given quite a clear empirical interpretation.

The first factor in Table 16 is an indicator of **profitability** of the firms. It also includes elements for efficiency of capital invested (the high loading of TATWD). The factor is quite similar to that of the corresponding U.S. model (Table 4) and also to that of the Finnish model based on value-weighted averages (Table 13).

The second factor describes in the first hand the **long-term solvency** of firms. It includes, however, short-term elements (DIWD, ARTWD), which are not easy to interprete (especially the minus sign for DIWD). The factor is formed differently than both in the corresponding U.S. model and in the value-weighted Finnish model.

Table 16. Varimax-rotated factor matrix for the first differences of equal-weighted ratio indices (Finnish data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Communality
ROEED	**0.955**	0.000	0.000	0.000	0.977
ROAED	**0.948**	0.000	0.000	0.000	0.976
ESED	**0.946**	0.000	0.262	0.000	0.964
TIEED	**0.852**	0.000	0.453	0.000	0.956
TATED	**0.792**	0.470	0.000	0.000	0.902
LTDEED	0.000	**-0.973**	0.000	0.000	0.982
DEED	0.000	**-0.887**	-0.359	0.000	0.968
DIED	0.000	**-0.827**	0.485	0.000	0.949
ARTED	0.345	**0.547**	0.424	0.000	0.598
QRED	0.332	0.000	**0.917**	0.000	0.974
CRED	0.389	0.340	**0.802**	0.000	0.914
ITED	0.000	0.000	0.000	**0.971**	0.982
Variance explained by the factor	4.465	3.097	2.423	1.159	
Cumulative proportion of total variance	0.500	0.728	0.841	0.929	

The third factor is unambiguously a factor of **short-term solvency.** It possesses similarities with the corresponding factor in the U.S. model but it does not, as such, exist in the value-weighted Finnish model.

The fourth factor is a pure one-variable factor, which describes the **efficiency** of inventory management. The factor is totally different from the fourth factor in the U.S. model and also from the fourth factor in value-weighted Finnish model (in the latter comparison, however, both factors are indicators of efficiency, either for credit management or for inventory management).

It is perhaps worth to note that the solution above produces, when interpreted liberally, the four categories common in litterature: profitability, long-term solvency/capital structure, short-term solvency/liquidity and efficiency. The structure of the categories is not, however, as supposed. Some categories are wider by content (e.g. profitability), others more narrow and specific (the fourth class e.g. describes efficiency only from the point of view of inventory

management).

Until now we have seen that the classification pattern based on the first differences of the equal-weighted averages of the ratios might, as far as the specific Finnish data is concerned, be acceptable. The final decision about the quality of the pattern can not, however, be made until we have seen whether or not the pattern possesses structural invariance over time and across countries.

The transformation matrix between the factor matrices of U.S. and Finnish data (Tables 4 and 16) is given in Table 17. The transformation matrix shows that the degree of invariance is not satisfactory. Only the first factor (ratio category), i.e. profitability, can be regarded as the same in both data. The three other factors are almost uniformly transfered across each others in the two solutions. The classification procedure based on the equal-weighted ratio indices thus possesses neither time-series stability nor structural invariance across countries.

Table 17. Transformation matrix between the factor matrices of U.S. and Finnish equal-weighted ratio indices (factors based on the first differences of the equal-weighted averages of the ratios).

	Factor	Finland 1	Finland 2	Finland 3	Finland 4
USA	1	**0.963**	-0.060	0.201	0.167
	2	0.026	**0.605**	**0.539**	**-0.586**
	3	-0.261	0.008	**0.713**	**0.651**
	4	0.055	**0.794**	**-0.402**	**0.452**

The Finnish data included (with respect to the ratio IT) two outliers. Excluding of those outliers had no noticeable effect on the value-weighted ratio indices but it changed the equal-weighted indices, especially the index IT. Also the classification pattern became a little different. On the lack of structural invariance this re-examination had, however, no noticeable effect.

As a summary from our analysis, when the first differences of the ratio indices are used, we can conclude that the aggregation method based on equal-weighted averages seems to be very sensitive both for outliers and for heterogeneity in the data. The results found out strongly support the use of value-weighted indices instead of equal-weighted indices in aggregate level ratio analysis.

4.6. Implications of the analysis for selection of financial ratios

Finally we will determine the good financial ratios on the basis of our analysis. When utilizing the financial ratios, it is valuable to know the theoretical relationships between the classes of ratios under consideration. After choosing a small subset of ratios to be used, it is important to know the empirical behavior of these ratios. In this respect, we have four requirements for the good ratios. First, the factor solution should be clear-cut and easy to interprete. Second, the ratios should have high loading on one factor and low loadings on all the other factors. Third, the communality of the variable should be close to one, i.e. the factor solution examined should in practice explain, as much as possible, the total variation of the ratios in question. Fourth, the coefficient of coincidence in transformation matrix should be close to 1, and all elements in residual matrix close to zero; in that case the stability and structural invariance of the financial ratio pattern is high.

The final financial ratio pattern deduced through the analysis, the four factor solution found by using the first differences of the value-weighted averages of the ratios, was presented in Tables 3 and 13. The classification of the ratios differed to some extent from the a priori classification. We found the following four factors: solvency, profitability, efficiency and dynamic liquidity.

The best solvency measures were in U.S. data DE (debt to equity) and QR (quick ratio) and in Finnish data DE (debt of equity) and LTDE (long term debt to equity). Respectively, the best profitability measure was ROE (return on equity) in both countries. ROA (return on assets) and ES (earnings to sales) were also quite good measures of profitability. TAT (total assets turnover) was very clearly the best U.S. efficiency ratio and quite good Finnish efficiency ratio was ART (account receivable turnover). The second best was U.S. efficiency ratio IT (inventory turnover) and the worst ART (accounts receivable turnover). The fourth factor, dynamic liquidity was not included in the a priori classification. Only the variable DI (defensive interval) loaded strongly on this factor in both countries. DI measured very well this characteristic of firms' performance.

5. SUMMARY

The purpose of this study was to develop, on the economy-wide level, an empirically-based classification pattern for 12 commonly used financial ratios. The selected ratios were according to a priori classification the measures of short-term solvency, long-term solvency, profitability and efficiency of the firms. The U.S. firms used for this study were selected from an annual industrial COMPUSTAT tape containing data for all December 31 fiscal year U.S. firms for the period 1947-75. The Finnish firms used for this study cover all the firms quated on the Helsinki Stock Exchange (excluding bank and insurance companies for the period 1974-84). The empirical results were based on both the value- and equal-weighted indices of the selected ratios. Classification patterns of financial ratios were developed via factor analysis using indices (variables) both in the level and in the first-difference form.

The number of factors - i.e. the number of financial ratio classes - extracted was determined in the first place by a priori knowledge and interpretative aspects.

The empirical analysis showed that the resulting empirically-based classification was not fully equivalent to the a priori classification. We found the following factors: solvency, profitability, efficiency and dynamic liquidity. An interesting feature was that the short-term and long-term solvency did not differ from each other. The above mentioned result was obtained using the first differences of the value-weighted averages of the ratios. The use of the first differences of the ratios was necessary because of the very clear positive or negative trend in the time series. The use of first-differences in the analysis made it also possible to overcome the open problem concerning the role of the constant term in financial ratio analysis. Further, the empirical analysis showed that different aggregation methods led to different results. The theoretically better value-weighted indices gave more accurate empirical results which were also more easy to interprete.

After that we analyzed, using transformation analysis, the long-term stability and structural invariance of the factor patterns obtained. The resulting factor pattern - based on value weighted averages of the selected ratios (in the difference form) - displayd very clear time series stability and strong structural invariance between U.S. and Finnish data. On the other hand, the results gave evidence of considerable instability and slight structural invariance when the variables were equal-weighted indices. Equal-weighted averages were especially sensitive both for outliers and for the heterogeneity in the data. These results confirmed the great importance of aggregation method in the financial ratio analysis when we use aggregate data.

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