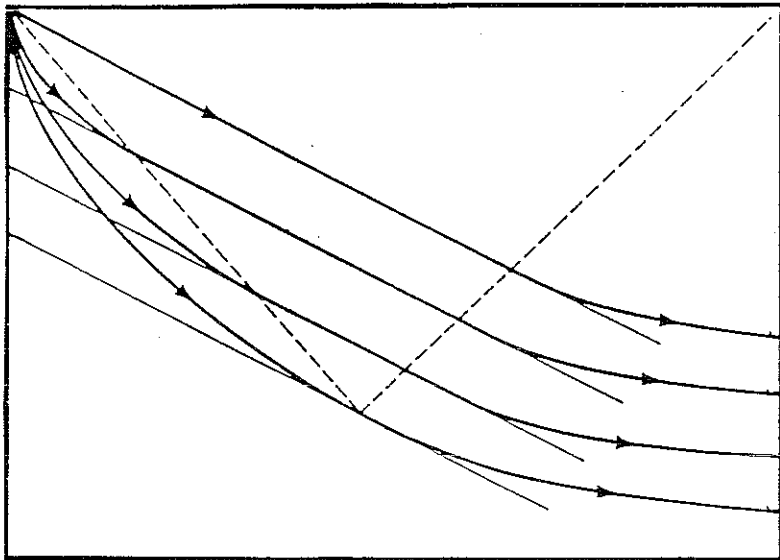


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FINANCIAL RATIO PATTERNS

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ON THE LONG-TERM STABILITY AND CROSS-COUNTRY INVARIANCE OF FINANCIAL RATIO PATTERNS

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ABSTRACT

The purpose of this study is to develop, on the economy-wide level, empirically-based classification patterns for twelve commonly used financial ratios and to measure the long-term stability and structural invariance of these patterns. The data are based on annual reports of U.S. and Finnish industrial firms for the periods 1947-75 and 1974-84, respectively. The selected financial ratios are, according to a priori classification, the measures of short-term solvency, long term-solvency, profitability and efficiency. Classification patterns of the financial ratios are developed via factor analysis and the stability and invariance analyses are carried out via transformation analysis.

The results show that empirically-based classifications are not fully equivalent to the a priori classification. The following factors are found: solvency, profitability, efficiency and dynamic liquidity. The empirical results are based both on the value- and equal-weighted indices of the ratios. Classification patterns are developed using ratio indices in the first-difference form. The use of the first differences becomes necessary because of the clear trend in the time series. Further, empirical results show that different aggregation methods lead to different results. The theoretically better value-weighted indices (in the first-difference form) give more accurate and easy-to-interpretate empirical results. Factor patterns based on these indices display also very clear time-series stability and cross-sectional invariance. This result confirms the great importance of aggregation method in ratio analysis.

KEYWORDS

Classification of financial ratios, long-term stability, cross-sectional invariance, aggregation of financial ratios, factor analysis, transformation analysis.

1. INTRODUCTION

1.1 Financial statement analysis

Financial statements serve as the primary financial reporting mechanism of a firm, both internally and externally. Financial statements are the method by which management communicates financial information to decision makers. Those decision makers are investors, lenders, labor unions, researchers and other interested parties.

Financial statement analysis is an information-processing system developed to provide relevant data for decision makers. The great number of decision makers

and their different objectives have caused that the ratios used in financial statement analysis have been numerous. Also many alternative categories of financial ratios have been proposed in literature [15, 10 pp. 24-37, 7 pp. 372-375, 25 pp. 372-375]. However, there is no consensus on each ratio as to what the ratio primarily measures because of the differences in computation of financial ratios [6 pp. 16-18, 12 p. 13, 13 p. 105].

1.2 Review of prior research on topic

Remarkable insight into relationships between financial ratios was presented by the study of [24]. They developed an empirically-based classification system for financial ratios using factor analysis. The approach introduced by Pinches, Mingo and Caruthers has been thereafter applied by many researchers, e.g. [7, 16, 5, 18]. Interesting results were also presented by Gombola and Ketz [13] and Yli-Olli [28] independently of each other. According to their results profitability ratios and cash-flow-ratios do not measure the same characteristic of firm performance [13 p. 106, 28 pp. 40-50].

According to the results of [24, 13] the classification patterns of the ratios were reasonable stable over time even when the magnitude of the ratios was undergoing change. Yli-Olli [28] and Yli-Olli and Virtanen [29, 30] used so called transformation analysis to measure the medium-term and the long-term stability of factor patterns. Compared with correlation or congruency analysis used in previous studies they got a more clear-cut picture about stability of factor patterns.

During the last two decades, a considerable amount of research has also been directed towards methodological issues in the use of financial ratios. The object of those papers is to provide insight into assumptions and limitations in the use of financial ratios [14, 9, 20, 27, 11].

1.3 The purpose of the study

The purposes of this research work are:

1. to develop, on the economy-wide level, empirically-based classification patterns for some commonly used financial ratios,
2. to compare, both on the theoretical and empirical levels, the usefulness of different aggregation methods in financial ratio analysis,
3. to measure, using transformation analysis, the long-term stability of financial ratios,
4. to measure the structural invariance of the financial ratio patterns between the U.S. and Finnish firms.

2. THE SELECTION OF FINANCIAL RATIOS AND SOME BASIC PROPERTIES OF THE RATIOS

In this chapter we present the literature-based classification (called the a priori classification) of financial ratios examined. It will also be analyzed what each ratio, a priori, measures. The analysis is based on earlier researches and textbooks.

In this study, 12 different ratios are selected, which - a priori - measure **short-term solvency** (liquidity ratios), **long-term solvency** (leverage / capital structure ratios), **profitability** (profitability ratios) and **efficiency** (turnover ratios) of the firm (for the calculation of these ratios see e.g. [10, pp. 43-48]). This classification is the most common in literature (see e.g. [19 p. 12, 17 p. 29, 10 p. 28, 25 pp. 24-44]) and it is oriented to the needs of users of these ratios (see more about the use of financial ratios [25 pp. 71-93 and 146-171]).

2.1 Liquidity ratios

The ability of a firm to meet its short-term financial obligations is of prime interest to management, merchandise suppliers, lenders and investors. In the extreme case when the firm is not able to meet its short-term financial obligations those groups will be the losers.

The liquidity ratios examined in this study are the **current ratio** (CR), the **quick ratio** (QR) and the **defensive interval measure** (DI) (for calculation of liquidity ratios, see [10, pp. 43-44]). The current and quick ratios are, in principle, very similar. The denominator of both ratios consists of current liabilities. The numerator of the current ratio consists of current assets. The quick ratio includes in the numerator cash marketable securities and accounts receivable (current assets - (inventories + other current assets)). According to Lev [19, p. 28] the quick ratio provides a stricter test of liquidity than the current ratio. In Gibson's inquiry [12] (a questionnaire was sent to the financial executives of the 500 largest industrial firms for 1979 listed in Fortune) there was a big consensus on each ratio as to what the ratio primarily measures: 94 % of the firms were of such opinion that the current ratio is a measure of liquidity. The corresponding number of quick ratio was 80 %.

The current and quick ratios have been criticized on the basis of their static structure [26, p. 38]. These ratios reflect the surplus of current assets over current liabilities at a point in time.

This criticism led to the development of cash- and funds-flow-based liquidity ratios. Such a ratio is the defensive interval measure [8, pp. 23-26]. This measure incorporates a dynamic element in liquidity evaluation. According to the results of Davidson's, Sorter's and Kalle's empirical study "there is substantial evidence that the interval measure and the traditional ratios (e.g. current ratio) produce differing impressions of the size and movement of a firm's defensive strength".

2.2 Long-term solvency ratios

A firm may finance its activities - as far as the external financing of the firm is concerned - either by using the funds borrowed or by investing the owners' money.

The selected long-term solvency ratios in this study are the **debt-to equity** (DE), **long-term debt to equity** (LTDE) and **times interest earned** (TIE) ratios.

Debt to equity and long-term debt to equity are very similar by nature and they are - like the current and quick ratios in liquidity measurement - static measures of the long-term solvency of the firm. They are measures for financial risk associated with the shareholders' equity.

The times interest earned ratio incorporates a dynamic element in long-term solvency evaluation.

2.3 Profitability ratios

It is presented in many textbooks that profitability may reflect different things to different users. Therefore many different indicators of profitability should be considered (see [25, p. 25]). In this study, three different ratios are used: **earnings to sales** (ES), **return on assets** (ROA) and **return on equity** (ROE).

The first ratio (ES) is a surrogate of operational efficiency of the firm and both the numerator and denominator of the ratio represent a flow over the entire period (i.e. a year). The second ratio (ROA) measures how efficiently total assets of the firm are being utilized. The third and most interesting ratio (ROE) indicates the profitability of the capital supplied by common stockholders.

2.4 Turnover ratios

Turnover ratios measure different aspects of firm's performance, i.e. the efficiency of the firm in using its assets in order to generate income. The selected turnover ratios are: **total assets turnover** (TAT), **inventory turnover** (IT) and **accounts receivable turnover** (ART).

Total assets turnover together with earnings to sales (the first profitability ratio in this study) comprises the so-called DuPont system of ratio analysis [10, p. 44]. The inventory turnover ratio is supposed to indicate the efficiency of inventory management. The problem to be solved by inventory management is to determine and maintain an optimal inventory level. Accounts receivable turnover has been said to indicate efficiency of the credit department [19, p. 28]. So, the diminishing of this ratio may be due either to faulty collection system or to the weak financial position of the debtors. On the other hand, the reason can also be that the firm attempts to increase sales by granting more credit to customers. Irrespective of the cause, the decrease of this ratio from the target level indicates greater risk than the changes of default by customers.

3. DATA AND STATISTICAL METHODS

The U.S. firms used for this study are selected from an Annual Industrial COMPUSTAT tape containing data for all December 31 fiscal year U.S. firms for the period 1947-75 (see [10, pp. 156-160]). The number of firms in the sample varies from year to year, increasing from about 450 in 1947 to about 1500 in 1975. The use of the same fiscal year firms gives a more clear-cut picture about different phases of economic cycles than the use of all firms regardless of the fiscal year. This is especially important in such cases - as in this study - where the analysis is mainly based on the first differences of the variables.

The Finnish firms used cover all the firms quoted on the Helsinki Stock Exchange (excluding bank and insurance companies). The number of the firms is 42 and the time period to be examined is 1974-84. In principle the ratios are calculated according to the recommendations of Yritystutkimusneuvottelukunta [31].

The observations (rows) in the data matrix consist of the years 1947-75 (U.S. data) and of the years 1974-84 (Finnish data). The variables are the average values of the selected 12 financial ratios, the average values being computed across the individual firms. The average values have been computed in two different ways, as arithmetic (i.e. equal-weighted) averages and as value-weighted averages. On the properties of these two ways to aggregate the individual firm-specific ratios into an economy-wide index see [30, pp. 18-21].

The empirical analysis in the study is based on multivariate time series data. The main statistical methods to be used are factor analysis and transformation analysis. Factor analysis can be regarded as a usual technique in business applications. Transformation analysis, on the contrary, has been largely applied only in Finnish political and sociological research. Therefore, the paper contains a short description of the main features of this multivariate method.

The degree of stability (both time-series and cross-sectional) in factor patterns has been traditionally measured with correlation coefficients [24, 5] or with congruency coefficients [16, 13]. Both of these measures give an index for the similarity of two different factor solutions in terms of the pattern of correlations among factor loadings across all variables in the reduced factor space. For the dissimilar part of these factor solutions these indices are, however, unable to describe and explain the reason for the non-invariant part prevailing in these factor solutions.

Yli-Olli [28] introduced the use of transformation analysis for determining the degree and nature of medium-term stability exhibited by the factor patterns of the financial ratios. This approach was further applied and deepened by Yli-Olli and Virtanen [29, 30].

Transformation analysis was initiated by Ahmavaara [1] and further developed by Ahmavaara [2, 3], Ahmavaara and Nordenstreng [4] and Mustonen [22]. The most applications of transformation analysis exist in the area of Finnish political and sociological research [21, 23]. Originally transformation analysis was developed to compare factor solutions between two (or more) different groups of objects, Yli-Olli [28] and Yli-Olli and Virtanen [29] have used the technique to compare two different factor solutions among the same group of objects, the two factor solutions being based on measurements made at different times (at two different time periods). In the following we sketch out the general idea behind transformation analysis (for a more detailed discussion, see e.g. [3, 22, 30]).

Let's assume that we have two groups of observations G_1 and G_2 (two different groups of objects or one group measured at two different times) with the same variables, both by number and by content. Let L_1 and L_2 be the factor matrices for G_1 and G_2 , respectively. Let's further assume that the factor models used in deriving L_1 and L_2 are both orthogonal and have the same dimension, $p \times r$, say.

If there exists invariance between the two factor structures, there exists a non-singular $r \times r$ -matrix T_{12} such that equation

$$L_2 = L_1 T_{12} \quad (3.1)$$

holds. Matrix T_{12} is called the transformation matrix (between L_1 and L_2 , or in direction $G_1 \rightarrow G_2$). If equation (3.1) holds exactly, it means that the factor structures in groups G_1 and G_2 are, up to a linear transformation, invariant, all the variables have the same empirical meaning in different groups. Depending on the type of the transformation matrix T_{12} , the formation of the factors from the variables and thereby the interpretation of the factors either is preserved (T_{12} is the identity matrix I) or it changes (T_{12} has also non-zero off-diagonal elements).

In practice, situation (3.1) will not be reached, but, after matrix T_{12} has been estimated, we have $L_2 \neq L_1 T_{12}$. The goodness of fit criterion for the model (3.1) may be based on the residual matrix

$$E_{12} = L_1 T_{12} - L_2 \quad (3.2)$$

Non-zero elements in E_{12} mean that the empirical meaning of the variables in question has changed. This is called abnormal transformation.

The main problem in transformation analysis is the estimation of the matrix T_{12} . The estimation methods are in general based on the minimization of the sum of squares of the residuals e_{ij} (the elements of the residual matrix E_{12}). This is the usual method of least squares. The problem is to minimize

$$\begin{aligned} \|E_{12}\| &= \|L_1 T_{12} - L_2\| \\ &= \text{trace}((L_1 T_{12} - L_2)(L_1 T_{12} - L_2)') \end{aligned} \quad (3.3)$$

Depending on additional constraints set for the matrix T_{12} , we have three different estimation methods (three transformation analysis models).

1. If there are no constraints for T_{12} in minimizing (3.3) we have the naive model.
2. If the transformation matrix has to obey the transitivity property $T_{kl} T_{lm} = T_{km}$, we get the relativistic model.
3. If the transformation matrix T_{12} is required to be orthogonal, i.e. $T_{12}^{-1} = T_{12}'$, we have the symmetric model. In this study the symmetric transformation analysis will be used (see [22]).

Transformation analysis possesses several advantages in analysing the stability or the structural invariance of the factor patterns when compared for example with correlation or congruency analysis. With correlation and congruency coefficients one can only measure the degree of similarity of two factor solutions (correlations or congruencies among factor loadings across the variables in the factor space). This is also possible via transformation analysis (coefficients of coincidence on the main diagonal of the transformation matrix). In addition to this we obtain a regression type model for shifting of variables from one factor to another (normal or explained transformation). This is revealed by the non-zero off-diagonal elements in the transformation matrix and indicates interpretatively changes for the factors in question. And at last, large elements in the residual matrix, if any, indicate abnormal or unexplained transformation between the two factor solutions. This means that the empirical content of the corresponding variables has changed. Further, this abnormal transformation can be appointed to separate variables or to separate factors.

4. EMPIRICAL RESULTS

In this chapter we will develop empirically-based classification patterns for the presented financial ratios and measure the time-series stability and structural invariance of the ratios. The results are based on the value-weighted and equal-weighted averages of the selected financial ratios.

The number of factors to be extracted can be determined by using different criteria e.g. a priori knowledge, interpretative aspects, the eigenvalue criterion or Cattell's scree test. In this study, the number of factors extracted is mainly based on interpretative aspects and on a priori knowledge (i.e. the number of classes in the original classification). However, in most cases the eigenvalues associated with each factor exceed 1.

The form of all factor loading matrices to be presented in this study is the following. First, the columns (factors) appear in decreasing order of variance explained by the factors. The rows (variables) are rearranged so that, for each successive factor, loadings greater than 0.5 appear first. Loadings less than 0.25 are replaced by zero.

Our factor-analytic derivation of the financial ratio patterns began with the original level values of the aggregated ratios by using U.S. data. The four-factor solutions accounted for 97.1 per cent and 95.1 per cent of the total variance in the original twelve financial ratios when value-weighted and equal-weighted averages, respectively, were used. The communalities of all variables were also high.

However, the interpretation of the factors was not easy. This was especially true when the factor solution based on equal-weighted averages was concerned. In addition, the obtained factor solutions differed a lot from each other. The reason for this numerically satisfactory but interpretatively confusing situation is quite clear: the high seeming correlations among the variables caused by time. Therefore the models based on the original level values of the aggregated ratio indices are abandoned.

4.1. Financial ratio patterns using first differences of the ratios

The factor solutions found by using the first differences of the selected ratios are presented in Tables 1 and 2 (U.S. data). The four factor solutions account for 87.8 per cent (Table 1) and 86.4 per cent (Table 2) of the total variances in the original variables. The interpretation of the first factor in Table 1 is clear and unambiguous. The financial ratios which achieve the highest loadings on this factor are DE, CR, LTDE and QR. This factor describes the **solvency of the firms**. (In fact, the factor model being based on the first differences of the ratios, also the interpretation of the factor should be the **change** of the solvency of the firms. For the sake of simplicity and clarity, all the factors are named according to basic quantifies themselves, however.).

The second factor can be interpreted as a factor of **profitability**. The interpretation of this factor is also easy. Only the high loading of the variable TIE (times interest earned) on this factor requests an explanation. This variable was, according to the a priori classification, the measure of dynamic long-term solvency. However, Table 1 and also the following results show that TIE is rather the measure of profitability than that of long-term solvency.

The third factor describes the **efficiency** of the firm. The variables with the highest loadings on this factor are, in correspondence with a priori classification, the three turnover ratios selected for this study.

The fourth factor indicates the **dynamic short-term solvency** of the firm. This factor is a very pure one-variable (DI) factor, because the loadings of other ratios are very low on this factor.

Table 2 shows the results based on the first differences of the equal-weighted averages of the financial ratios (U.S. data). The first factor in Table 2 is an indicator of **profitability and efficiency** of the firms. This factor includes the main parts (ART excluded) of the second and third factor presented in Table 1.

The first factor in the value-weighted average solution in Table 1 (the solvency factor), is in Table 2 divided into two factors: a factor of **long-term solvency** (the second factor) and a factor of **short-term solvency** (the third factor).

Table 1. Varimax-rotated factor matrix for the first differences of value-weighted averages (U.S. data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Communality h_i^2
DEWD	-0.895	0.000	0.000	0.000	0.832
CRWD	0.834	-0.272	-0.281	0.000	0.854
LTDEWD	-0.834	0.000	0.411	0.000	0.895
QRWD	0.834	0.000	0.000	0.454	0.935
ESWD	0.000	0.942	0.000	0.000	0.942
ROEWD	0.000	0.874	0.384	0.000	0.970
ROAWD	0.000	0.813	0.506	0.000	0.946
TIEWD	-0.394	0.709	0.000	0.358	0.785
TATWD	-0.338	0.000	0.821	0.000	0.850
ITWD	-0.271	0.613	0.621	0.000	0.858
ARTWD	-0.454	0.412	0.590	0.000	0.725
DIWD	0.000	0.000	0.000	0.928	0.943
Variance explained by the factor	3.607	3.568	2.124	1.235	
Cumulative proportion of total variance	0.301	0.598	0.775	0.878	

Table 2. Varimax-rotated factor matrix for the first differences of equal-weighted averages (U.S. data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Communality h_i^2
ROAED	0.901	0.000	-0.282	0.000	0.899
ESED	0.875	0.000	0.000	0.000	0.776
ROEED	0.857	0.000	0.000	0.000	0.787
TIEED	0.837	0.000	-0.345	0.000	0.882
ITED	0.703	-0.590	0.000	0.000	0.885
TATED	0.548	-0.398	-0.336	-0.370	0.708
DEED	0.000	-0.913	0.000	0.000	0.891
LTDEED	0.000	-0.877	-0.375	0.000	0.923
QRED	0.000	0.328	0.851	0.269	0.948
CRED	-0.407	0.411	0.759	0.000	0.928
ARTED	0.000	0.000	0.000	-0.906	0.877
DIED	0.427	0.000	0.455	0.691	0.867
Variance explained by the factor	4.235	2.430	2.077	1.629	
Cumulative proportion of total variance	0.353	0.555	0.728	0.864	

The fourth factor can be interpreted - as in Table 1 - as a factor of **dynamic liquidity**. However, that interpretation proves difficult, because of the high negative loading of the variable ART on this factor.

The inconsistency in the behaviour of the variable ART between Tables 1 and 2 derives its origin probably from the very different role of accounts receivable when we have computed the values of variable DI among small and big firms. Thus, different aggregation methods lead to the very different results. Also other differences, caused by different aggregation methods can be found between the factor patterns presented in Tables 1 and 2. These differences are analyzed in a more detailed way in Section 4.2. The presented results (Tables 1-2) confirm that the use of the variables in first-difference form lead to a more valid classification pattern than the classification pattern of original ratios. Further, the results also suggest that the value-weighted indices give, in addition to that they are theoretically more accurate, more clear-cut classifications of the financial ratios than the equally-weighted indices. The differences between these two classifications are analyzed in greater detail in Section 4.2. by using transformation analysis.

It is difficult to find clear theoretical arguments why the factors extracted from financial ratios should be uncorrelated. Therefore, the results presented in Tables 1 and 2 were verified with a non-orthogonal rotation method. The result showed that the non-orthogonal factor pattern was very similar to the orthogonal factor pattern. Due to the strong similarity between these two solutions, the subsequent analysis will be restricted, however, to orthogonal factor models only.

4.2. The long-term stability of financial ratio patterns (U.S. data)

One of the objectives in this study was to measure, using transformation analysis, the long-term stability of factor patterns obtained. For this analysis, the whole period concerning U.S. firms is divided into two sub-periods of equal length: sub-period 1 includes the years 1947-61 and sub-period 2 the years 1962-75.

The empirical results indicate that the difference-formed models should be preferred to the level-formed models. Therefore, stability analysis presented in this section is also based on the variables in the first-difference form only.

Tables 3 and 4 show, value-weighted variables and U.S. data being used, the four factor solutions for sub-period 1 and for sub-period 2, respectively. The four factor solutions account for 91.2 per cent (Table 3) and 91.3 per cent (Table 4) of the total variances in the original variables. The corresponding number for the whole period was 87.8 (Table 1).

The four factor solution for sub-period 1 is quite similar to that of the whole period. The major difference between those two solutions is in the loadings of the variables IT and ART. Those variables have the highest loading on the profitability/efficiency factor in Table 3. In Table 1 those variables, together with the variable TAT, created more clearly an efficiency factor of their own (this efficiency factor still exists, however, in the solution of Table 3).

The factor solution for sub-period 2 is more close to the solution of the whole period than that for sub-period 1. Now only the variable IT changes the factor. It transfers from efficiency factor to profitability factor (in the whole period vs. sub-period 2).

Table 3. Varimax-rotated factor matrix for the ratios (for the value-weighted averages in the first difference form) in sub-period 1 (U.S. data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Communality h_i^2
ESWD	0.952	0.000	0.000	0.000	0.943
ROEWD	0.920	-0.266	0.000	0.000	0.981
ROAWD	0.879	0.000	0.421	0.000	0.979
TIEWD	0.755	-0.432	0.000	0.295	0.876
ITWD	0.623	-0.433	0.547	0.000	0.915
ARTWD	0.604	-0.574	0.328	0.000	0.803
DEWD	0.000	-0.913	0.000	0.000	0.837
CRWD	-0.284	0.867	-0.291	0.000	0.924
LTDWD	0.000	-0.861	0.335	0.000	0.908
QRWD	0.000	0.799	0.000	0.500	0.928
TATWD	0.332	-0.385	0.792	0.000	0.886
DIWD	0.000	0.000	0.000	0.955	0.959
Variance explained by the factor	4.137	3.943	1.553	1.307	
Cumulative proportion of total variance	0.345	0.673	0.803	0.912	

Table 4. Varimax-rotated factor matrix for the ratios (for the value-weighted averages in the first difference form) in sub-period 2 (U.S. data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Communality h_i^2
ROEWD	0.930	0.000	0.279	0.000	0.970
ESWD	0.897	0.000	0.000	0.286	0.939
ITWD	0.896	0.000	0.359	0.000	0.953
ROAWD	0.888	0.000	0.328	0.000	0.967
TIEWD	0.579	0.571	-0.321	-0.293	0.850
QRWD	0.000	0.961	0.000	0.000	0.970
CRWD	0.000	0.924	0.000	0.000	0.900
DEWD	0.000	-0.840	0.000	-0.263	0.814
LTDWD	0.278	-0.802	0.356	0.000	0.903
ARTWD	0.000	0.000	0.920	0.000	0.909
TATWD	0.371	-0.257	0.654	-0.525	0.906
DIWD	0.000	0.543	0.000	0.695	0.870
Variance explained by the factor	3.963	3.922	1.918	1.150	
Cumulative proportion of total variance	0.330	0.657	0.817	0.913	

Table 5 presents the transformation matrix between the factors for sub-period 1 (Table 3) and sub-period 2 (Table 4). The factors were calculated on the basis of value-weighted indices, and in the first-difference form they display considerable long-term stability. This conclusion is based on the coefficients of coincidence on the main diagonal of the transformation matrix. The numerical values of those coefficients are very close to 1. In addition, the transformation matrix shows a slight transference between the first and fourth factors. This result confirms apparent differences between the variable DI (dynamic liquidity) and other liquidity or short-term solvency measures (CR and QR). The variable DI loads on a separate and distinct factor which has a weak connection with the profitability factor.

Table 5. Transformation matrix between the factor patterns of ratios in sub-period 1 and sub-period 2 (factors based on the first differences of the value-weighted averages of the ratios; U.S. data).

	Factor	Sub-period 2			
		1	2	3	4
Sub-period 1	1	0.954	0.145	-0.080	-0.249
	2	-0.178	0.978	-0.039	-0.100
	3	0.056	0.042	0.994	-0.080
	4	0.234	0.143	0.058	0.960

Table 6 presents the residual matrix for sub-period 2 (matrix E_{12}). Zero elements in residual matrix mean that the variables in question measure the same characteristic of the firms' performance during different periods. Non-zero elements in residual matrix mean that the empirical meaning of the variables in question has changed.

The residual matrix shows that there are only two variables with a moderately high abnormal transformation. The abnormal transformation of the variable TIE can be designated to the factors 1, 2 and 4. This variable was in the a priori classification the measure of long-term solvency. However, during the first sub-period, this ratio was, in the first place, the measure of profitability. The great negative numerical value -0.850 on the second factor in residual matrix shows that the feature measuring long-term solvency increases considerably in variable TIE during the second sub-period. The abnormal transformation of the variable ART is high on the first, second and third factors. It means that the empirical content of this variable has changed. However, the transformation and residual matrices in Tables 5 and 6 indicate a very high long-term stability of the factor pattern.

Tables 7 and 8 present for sub-period 1 and for sub-period 2, respectively, the four factor solutions based on the equal-weighted indices in the first-difference form (U.S. data). The factor pattern in Table 7 is very similar to that obtained by using value-weighted averages of variables in the first difference form (Tables 1, 3 and 4). Table 7 includes four different factors: profitability, solvency, dynamic liquidity and efficiency.

On the contrary, the factor solution in Table 8 differs considerably from the solutions presented in Tables 1, 3, 4 and 7. The first factor describes the short-term solvency of the firms. The variables, which according to the a priori classification serve as the measures of liquidity, have the highest loadings on this factor (exceptionally also the variable DI). The second factor can be interpreted as a factor of profitability and efficiency. The third factor describes the long-term solvency of the firms. Finally, the variable ROE creates a factor of its own.

Table 6. Residual matrix E_{12} and abnormal transformation for sub-period 2 (factors based on the first differences of the value-weighted averages of the ratios; U.S. data).

Variable	Factor	Factor	Factor	Factor	Abnormal transformation
	1	2	3	4	t_i^2
CRWD	-0.227	-0.118	-0.350	0.161	0.214
QRWD	-0.171	-0.133	-0.102	0.230	0.111
DIWD	0.105	-0.249	0.277	0.165	0.177
DEWD	0.190	-0.057	-0.122	0.327	0.161
LTDEWD	0.119	0.009	-0.008	0.240	0.072
TIEWD	0.277	-0.850	0.115	0.446	1.010
ESWD	0.007	0.106	0.226	-0.510	0.322
ROAWD	0.019	0.197	0.031	0.026	0.041
ROEWD	0.002	0.032	-0.095	-0.197	0.049
TATWD	0.058	-0.039	0.122	0.413	0.190
ITWD	-0.147	-0.151	0.164	-0.035	0.072
ARTWD	0.480	-0.360	-0.618	-0.026	0.742
Abnormal transformation s_j^2	0.474	1.024	0.722	0.941	3.161

Table 7. Varimax-rotated factor matrix for the ratios (for the equal-weighted averages in the first difference form) in sub-period 1 (U.S. data).

Variable	Factor	Factor	Factor	Factor	Communality
	1	2	3	4	h_i^2
ROAED	0.934	0.000	0.000	0.000	0.937
TIEED	0.912	-0.284	0.000	0.000	0.954
ESED	0.902	0.000	0.000	0.000	0.845
ROEED	0.896	0.000	0.000	0.293	0.932
LTDEED	0.000	-0.949	0.000	0.000	0.922
DEED	0.000	-0.873	0.000	0.000	0.860
QRED	-0.366	0.839	0.377	0.000	0.980
CRED	-0.470	0.831	0.000	0.000	0.963
ARTED	0.000	0.000	-0.925	0.000	0.903
DIED	0.315	0.292	0.844	0.000	0.930
ITED	0.626	0.000	0.000	0.719	0.960
TATED	0.383	-0.509	0.000	0.705	0.937
Variance explained by the factor	4.356	3.594	1.865	1.307	
Cumulative proportion of total variance	0.363	0.663	0.818	0.927	

Table 8. Varimax-rotated factor matrix for the ratios (for the equal-weighted averages in the first difference form) in sub-period 2 (U.S. data).

Variable	Factor	Factor	Factor	Factor	Communality h_i^2
	1	2	3	4	
QRED	0.946	0.000	0.000	0.000	0.942
CRED	0.859	0.000	0.279	-0.308	0.928
DIED	0.786	0.000	0.000	0.445	0.819
TIED	0.000	0.909	0.000	0.000	0.870
ROAED	-0.474	0.745	0.000	0.323	0.921
TATED	-0.630	0.703	0.000	0.000	0.896
ESED	0.293	0.650	0.000	0.613	0.893
ARTED	-0.415	0.542	0.350	0.000	0.617
DEED	0.000	0.000	-0.967	0.000	0.938
LTDEED	0.000	0.000	-0.953	0.000	0.955
ITED	0.000	0.485	-0.628	0.316	0.739
ROEED	0.000	0.000	0.000	0.902	0.887
Variance explained by the factor	3.232	2.887	2.538	1.749	
Cumulative proportion of total variance	0.269	0.510	0.721	0.867	

Table 9 presents the transformation matrix between the factors given in Tables 7 and 8. The transformation matrix shows that the long-term stability between factor patterns is very low. The first factor given in Table 7 is divided in the first place to the second and fourth factors during the second period. The second factor is divided to the first and third factors etc.

Table 5 indicates a very high long-term stability between factor patterns when the variables are value-weighted indices in the first-difference form. On the other hand, the results presented in Table 9 give evidence of considerable instability between factor patterns when the variables are equally-weighted indices in the first-difference form. These results prove that the role of the aggregation method is very important, when we consider and calculate some industry-wide or economy-wide norms to be "target financial ratios" for firms.

Table 9. Transformation matrix between the factor patterns of ratios in sub-period 1 and sub-period 2 (factors based on the first differences of the equal-weighted averages of the ratios; U.S. data).

Sub-period	Factor	Sub-period 2			
		1	2	3	4
1	1	-0.101	0.556	0.038	0.824
1	2	0.622	0.175	0.759	-0.077
1	3	0.768	-0.186	-0.562	0.246
1	4	0.114	0.791	-0.327	-0.504

4.3 On structural invariance of the ratio pattern between U.S. and Finnish firms

The resulting factor pattern for U.S. data (Table 1) displayed a very high long-term stability, when the variables are value-weighted indices and in the first difference form.

The corresponding factor solution for the Finnish data (the first differences of the value-weighted averages is presented in Table 10).

Table 10. Varimax-rotated factor matrix for the first differences of value-weighted indices (Finnish data).

Variable	Factor	Factor	Factor	Factor	Communality h_i^2
	1	2	3	4	
ROEWD	0.970	0.000	0.000	0.000	0.981
ESWD	0.966	0.000	0.000	0.000	0.974
TIEWD	0.964	0.000	0.000	0.000	0.987
ROAWD	0.945	0.000	0.000	0.000	0.966
ITWD	0.900	0.000	0.000	0.000	0.824
LTDEWD	0.000	-0.976	0.000	0.000	0.964
DEWD	-0.326	-0.913	0.000	0.000	0.993
CRWD	0.541	0.642	0.000	0.444	0.910
DIWD	0.000	0.000	0.969	0.000	0.981
TATWD	0.633	0.347	-0.656	0.000	0.960
ARTWD	0.000	0.000	-0.341	0.893	0.929
QRWD	0.402	0.501	0.381	0.642	0.970
Variance explained by the factor	5.481	2.770	1.685	1.502	
Cumulative proportion of total variance	0.457	0.688	0.828	0.953	

The explanatory power of the factor model is high. The solution accounts for 95.3 per cent of the total variance associated with the original variables, the individual communalities varying from 0.824 (ITWD) to 0.993 (DEWD). Also the interpretation of the solution is quite clear-cut and analogous to that of the U.S. model.

The first factor, explaining 45.7 per cent of total variance, describes the profitability of the Finnish firms. This interpretation can be based on the very high loadings of the "profitability" ratios ROEWD, ESWD, TIEWD and ROAWD. These four ratios are the same which formed the second, profitability, factor in the U.S. model. The pure loading of the ratio times interest earned (TIEWD) on the profitability factor is also a confirmation to the similar finding in U.S. data (Table 1). The usual classification of the ratio TIE is to consider it as a long-term solvency (leverage/capital structure) ratio (see e.g. Foster 1978: 31). The factor also has a high loading by inventory turnover (ITWD) and moderate loadings by total assets turnover (TATWD) and current ratio (CRWD). The latter cause a slight difference between these two factors, both having, however, a very clear interpretation of profitability.

The second factor (explaining 23.1 per cent of the total variance) has high loadings by long-term debt to equity (LTDEWD), debt to equity (DEWD), current ratio (CRWD) and quick ratio (QRWD). The factor describes the **solvency** of the firms. It is almost identical to the first factor in the U.S. model. It is worth to note here again the interesting feature that the short-term solvency (or liquidity) ratios CR and QR and the long-term solvency ratios DE and LTDE are empirically classified into the same dimension in the behaviour of the firms. Traditionally the short-term solvency and long-term solvency measures have been considered as different ratio classes in the ratio analysis [19 p. 12, 17 p. 29, 10 p. 28, 25 pp. 24-44].

The third factor (with 14.0 per cent contribution to the variance) is mainly formed by the ratio defensive interval measure (DIWD). It indicates the **dynamic short-term solvency** of the firms. The factor has a good coincidence with the corresponding factor of U.S. firms (factor four in Table 3). Some dissimilarity exists in the loadings of the turnover ratios (especially TATWD).

The fourth factor (with 12.5 per cent contribution to the variance) is the least satisfactory one. In the U.S. classification the corresponding factor (the third factor) was a very clear efficiency factor, all the turnover ratios having high loadings on it. On the basis of the highest loading (0.893 by ARTWD) the factor might also now be simply named as an efficiency factor. Remembering, however, the role of the ratio ART and taking the moderate loadings of the variables CRWD and QRWD into account, we can specify the factor as a measure for the **efficiency of the credit management**. The factor is thus of a more specific nature than the corresponding factor of "general efficiency" of the U.S. firms.

Thus far we have the following results and findings. First, we have the U.S. model which has been shown to possess a high degree of long-term stability (i.e. time-invariance). And second, we have the Finnish model derived in this section which seems quite similar to the U.S. model. Comparing these two models, which are based on data originating from different countries in different time periods, we can analyze the structural invariance of the two models, in fact the general invariance of the whole classification procedure.

The transformation matrix between the factors for U.S. data (Table 1) and Finnish data (Table 10) are presented in Table 11. We see that the analysis displays a considerable invariance between the two factor patterns. The coefficients of coincidence for the four factors are 0.954 (solvency factor), 0.924 (profitability), 0.860 (dynamic short-term solvency) and 0.815 (efficiency) which all can be regarded as high. A certain amount of transference of loadings can be seen, however, to exist between profitability and efficiency factors (elements -0.337 and 0.290 in the transformation matrix) and between dynamic short-term solvency and efficiency factors (elements -0.487 and 0.398). The result is as expected. The two similar factor patterns result in a near-to-unity transformation matrix indicating a high degree of structural invariance in the classification with minor elements of non-invariance (caused mainly by the turnover ratios), however.

The residual matrix (Table 12) shows that three of the ratios, viz. CRWD, ARTWD and TATWD have a moderate high abnormal transformation. These ratios thus measure, to some extent, different aspects in the firm's behaviour among U.S. firms than among Finnish firms. The abnormal transformation can be mainly designated to the profitability factor (CRWD and ARTWD), to the solvency factor (TATWD) and to the efficiency factor (ARTWD). The result is not surprising, cf. for example the comments given in connection with the presentation of factor loadings matrices. As a whole, however, the amount of abnormal transformation can be regarded quite tolerable: the total residual is 5.499 when it eg. in transformation between the two sub-periods for U.S. data was 3.161 (and in the latter case the models were obtained for the same group in two different time periods).

Table 11. Transformation matrix between the factor patterns of the financial ratios in USA and Finland (factors based on the first differences of the value-weighted averages of the ratios).

Factor		Finland			
		1	2	3	4
Inter-pretation		Profit-ability	Solvency	Dynamic short-term solvency	Efficiency of credit management
U S A	1 Solvency	-0.078	0.954	0.141	0.252
	2 Profitability	0.924	0.173	-0.057	-0.337
	3 Efficiency	0.290	-0.120	-0.487	0.815
	4 Dynamic short-term solvency	0.238	-0.212	0.860	0.398

Table 12. Residual matrix E_{12} and abnormal transformation between U.S. and Finnish data (factors based on the first differences of the value-weighted averages of the ratios).

Ratio	Factor				Abnormal transformation of the ratios
	1	2	3	4	
CRWD	-0.923	0.127	0.236	-0.345	1.043
QRWD	-0.509	0.194	0.199	-0.321	0.439
DIWD	0.392	-0.092	-0.169	0.559	0.503
DEWD	0.366	0.040	-0.134	0.103	0.164
LTDEWD	0.410	0.168	-0.427	0.062	0.382
TIEWD	-0.197	-0.515	0.260	-0.349	0.494
ESWD	-0.057	0.108	-0.099	-0.171	0.054
ROAWD	-0.021	-0.318	-0.127	0.104	0.128
ROEWD	-0.041	-0.316	-0.311	-0.061	0.202
TATWD	-0.151	-0.717	0.161	0.390	0.715
ITWD	-0.096	-0.332	-0.167	0.243	0.206
ARTWD	0.679	-0.532	-0.004	-0.651	1.169
Abnormal transformation of the factors	2.103	1.478	0.568	1.350	5.499

The output from transformation analysis (Tables 11 and 12) thus is that the empirical aggregate-level classification pattern possesses, when it is based on the value-weighted averages of the ratios in the difference form, a high degree of structural invariance between different countries (USA and Finland, explicitly) and different time periods. The invariance analysis can be further deepened when the Finnish classification pattern obtained for the years 1974-84 is compared with the two sub-models of U.S. data (years 1947-61 and 1962-75, respectively).

The fit between the model for sub-period 1 (Table 3) and the Finnish model is about the same level as that between the total U.S. model and the Finnish model: the

coefficients of coincidence in the transformation matrix vary from 0.839 (dynamic short-term solvency) to 0.966 (solvency), the off-diagonal elements in the transformation matrix are of the same magnitude as in Table 5, and the total amount of abnormal transformation is 6.412 (the main sources of this abnormal transformation being again ARTWD, CRWD and TATWD). The corresponding figures for the comparison between sub-model 2 (Table 4) and the Finnish model are the following. The coefficients of coincidence are now even higher, varying from 0.895 (efficiency) to 0.990 (profitability). The off-diagonal elements are near to zero, the largest element being -0.357 (efficiency/dynamic short-term solvency). The residual matrix shows no considerable abnormal transformation, the total residual is only 3.856.

The analysis confirms that there exists a high degree of structural invariance between the four-factor U.S. and Finnish models. The invariance is the better, the more adjacent the time-periods to be compared are, but it is sustained also to a period more far away.

4.4 Alternative empirical patterns for ratio classification

The analyses carried out in Sections 4.2. and 4.3. show that the empirical aggregate-level classification pattern possesses, when based on the value-weighted averages of the ratios in the difference form, a high structural invariance between different time periods and between different countries. On the other hand we saw, when the U.S. data was concerned, that the use of the data in alternative forms caused problems in the classification procedure. The classification pattern was either difficult to interpret (the use of the ratio indices in the original level form) or it was not stable enough over time (the use of equal-weighted averages as the primary data). The objective of this section is to find out whether the same problems arise when comparisons across countries are made. The results will give valuable insight into the problems concerning both the elimination of the harmful "multicollinear" trend-effect and the choice of an adequate aggregation technique.

The classification pattern based on the differences of the equal-weighted ratio indices for U.S. data (Table 2) was also quite clear-cut and easy to interpret. The classification pattern showed, however, no considerable time-series stability, wherefore preference was given to the model based on the value-weighted averages. The corresponding factor matrix for Finnish data is given in Table 13.

The first factor in Table 13 is an indicator of **profitability** of the firms. It also includes elements for efficiency of capital invested (the high loading of TATWD). The factor is quite similar to that of the corresponding U.S. model (Table 2) and also to that of the Finnish model based on value-weighted averages (Table 10).

The second factor describes in the first hand the **long-term solvency** of firms. It includes, however, short-term elements (DIWD, ARTWD), which are not easy to interpret (especially the minus sign for DIWD). The factor is formed differently than both in the corresponding U.S. model and in the value-weighted Finnish model.

The third factor is unambiguously a factor of **short-term solvency**. It possesses similarities with the corresponding factor in the U.S. model but it does not, as such, exist in the value-weighted Finnish model.

The fourth factor is a pure one-variable factor, which describes the **efficiency** of inventory management. The factor is totally different from the fourth factor in the U.S. model and also from the fourth factor in value-weighted Finnish model (in the latter comparison, however, both factors are indicators of efficiency, either for credit management or for inventory management).

Table 13. Varimax-rotated factor matrix for the first differences of equal-weighted ratio indices (Finnish data).

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Communality
ROEED	0.955	0.000	0.000	0.000	0.977
ROAED	0.948	0.000	0.000	0.000	0.976
ESED	0.946	0.000	0.262	0.000	0.964
TIEED	0.852	0.000	0.453	0.000	0.956
TATED	0.792	0.470	0.000	0.000	0.902
LTDEED	0.000	-0.973	0.000	0.000	0.982
DEED	0.000	-0.887	-0.359	0.000	0.968
DIED	0.000	-0.827	0.485	0.000	0.949
ARTED	0.345	0.547	0.424	0.000	0.598
QRED	0.332	0.000	0.917	0.000	0.974
CRED	0.389	0.340	0.802	0.000	0.914
ITED	0.000	0.000	0.000	0.971	0.982
Variance explained by the factor	4.465	3.097	2.423	1.159	
Cumulative proportion of total variance	0.500	0.728	0.841	0.929	

It is perhaps worth to note that the solution above produces, when interpreted liberally, the four categories common in literature: profitability, long-term solvency/capital structure, short-term solvency/liquidity and efficiency. The structure of the categories is not, however, as supposed. Some categories are wider by content (e.g. profitability), others more narrow and specific (the fourth class e.g. describes efficiency only from the point of view of inventory management).

Until now we have seen that the classification pattern based on the first differences of the equal-weighted averages of the ratios might, as far as the specific Finnish data is concerned, be acceptable. The final decision about the quality of the pattern can not, however, be made until we have seen whether or not the pattern possesses structural invariance over time and across countries.

The transformation matrix between the factor matrices of U.S. and Finnish data (Tables 2 and 13) is given in Table 14. The transformation matrix shows that the degree of invariance is not satisfactory. Only the first factor (ratio category), i.e. profitability, can be regarded as the same in both data. The three other factors are almost uniformly transferred across each others in the two solutions. The classification procedure based on the equal-weighted ratio indices thus possesses neither time-series stability nor structural invariance across countries.

The Finnish data included (with respect to the ratio IT) two outliers. Excluding of those outliers had no noticeable effect on the value-weighted ratio indices but it changed the equal-weighted indices, especially the index IT. Also the classification pattern became a little different. On the lack of structural invariance this re-examination had, however, no noticeable effect.

Table 14. Transformation matrix between the factor matrices of U.S. and Finnish equal-weighted ratio indices (factors based on the first differences of the equal-weighted averages of the ratios).

Factor	Finland				
	1	2	3	4	
U	1	0.963	-0.060	0.201	0.167
S	2	0.026	0.605	0.539	-0.586
A	3	-0.261	0.008	0.713	0.651
	4	0.055	0.794	-0.402	0.452

As a summary from our analysis, when the first differences of the ratio indices are used, we can conclude that the aggregation method based on equal-weighted averages seems to be very sensitive both for outliers and for heterogeneity in the data. The results found out strongly support the use of value-weighted indices instead of equal-weighted indices in aggregate level ratio analysis.

4.5 Implications of the analysis for selection of financial ratios

Finally we will determine the good financial ratios on the basis of our analysis. When utilizing the financial ratios, it is valuable to know the theoretical relationships between the classes of ratios under consideration. After choosing a small subset of ratios to be used, it is important to know the empirical behavior of these ratios. In this respect, we have four requirements for the good ratios. First, the factor solution should be clear-cut and easy to interpret. Second, the ratios should have high loading on one factor and low loadings on all the other factors. Third, the communality of the variable should be close to one, i.e. the factor solution examined should in practice explain, as much as possible, the total variation of the ratios in question. Fourth, the coefficient of coincidence in transformation matrix should be close to 1, and all elements in residual matrix close to zero; in that case the stability and structural invariance of the financial ratio pattern is high.

The final financial ratio pattern deduced through the analysis, the four factor solution found by using the first differences of the value-weighted averages of the ratios, was presented in Tables 1 and 10. The classification of the ratios differed to some extent from the a priori classification. We found the following four factors: solvency, profitability, efficiency and dynamic liquidity.

The best solvency measures were in U.S. data DE (debt to equity) and QR (quick ratio) and in Finnish data DE (debt of equity) and LTDE (long term debt to equity). Respectively, the best profitability measure was ROE (return on equity) in both countries. ROA (return on assets) and ES (earnings to sales) were also quite good measures of profitability. TAT (total assets turnover) was very clearly the best U.S. efficiency ratio and quite good Finnish efficiency ratio was ART (account receivable turnover). The second best was U.S. efficiency ratio IT (inventory turnover) and the worst ART (accounts receivable turnover). The fourth factor, dynamic liquidity was not included in the a priori classification. Only the variable DI (defensive interval) loaded strongly on this factor in both countries. DI measured very well this characteristic of firms' performance.

5. SUMMARY

The purpose of this study was to develop, on the economy-wide level, an empirically-based classification pattern for 12 commonly used financial ratios. The selected ratios were according to a priori classification the measures of short-term solvency, long-term solvency, profitability and efficiency of the firms. The U.S. firms used for this study were selected from an annual industrial COMPUSTAT tape containing data for all December 31 fiscal year U.S. firms for the period 1947-75. The Finnish firms used for this study cover all the firms quoted on the Helsinki Stock Exchange (excluding bank and insurance companies for the period 1974-84). The empirical results were based on both the value- and equal-weighted indices of the selected ratios. Classification patterns of financial ratios were developed via factor analysis using indices (variables) both in the level and in the first-difference form.

The number of factors - i.e. the number of financial ratio classes - extracted was determined in the first place by a priori knowledge and interpretative aspects.

The empirical analysis showed that the resulting empirically-based classification was not fully equivalent to the a priori classification. We found the following factors: solvency, profitability, efficiency and dynamic liquidity. An interesting feature was that the short-term and long-term solvency did not differ from each other. The above mentioned result was obtained using the first differences of the value-weighted averages of the ratios. The use of the first differences of the ratios was necessary because of the very clear positive or negative trend in the time series. The use of first-differences in the analysis made it also possible to overcome the open problem concerning the role of the constant term in financial ratio analysis. Further, the empirical analysis showed that different aggregation methods led to different results. The theoretically better value-weighted indices gave more accurate empirical results which were also more easy to interpret.

After that we analyzed, using transformation analysis, the long-term stability and structural invariance of the factor patterns obtained. The resulting factor pattern - based on value weighted averages of the selected ratios (in the difference form) - displayed very clear time series stability and strong structural invariance between U.S. and Finnish data. On the other hand, the results gave evidence of considerable instability and slight structural invariance when the variables were equal-weighted indices. Equal-weighted averages were especially sensitive both for outliers and for the heterogeneity in the data. These results confirmed the great importance of aggregation method in the financial ratio analysis when we use aggregate data.

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