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OPTIMAL MAINTENANCE POLICY AND SALE DATE FOR A MACHINE WITH RANDOM DETERIORATION AND SUBJECT TO RANDOM CATASTROPHIC FAILURE

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The decision problem concerning the optimization of the maintenance policy and the selection of the planned sale date for a machine subject to random deterioration and random catastrophic failure is considered from a control-theoretic viewpoint. The formulation of the problem is based on the well-known model by Thompson [2], which is, however, completely deterministic: the machine cannot fail and its deterioration with age obeys a given deterministic law. Alam et al. [1] have presented a model where the random nature of both the deterioration and the life-time of the machine have been taken into account. In their model, however, the maintenance policy only is optimized, the option of selling a still operable, but perhaps already worthless and unprofitable machine is not taken into account. In order to avoid such an improper optimum for the problem, simultaneous optimization of both the maintenance policy and the sale date of the machine must be carried out also in the case of random deterioration and random life-time.

In the present model both the deterioration rate and the life-time of the machine are considered as random processes. This means that also the salvage value of the machine and the net present value to be maximized become stochastic processes. The salvage value and the deterioration rate are treated as state variables and the maintenance expenditure as a control variable. Because of the stochastic state equations the model gets the form of a stochastic optimal control problem.

The stochastic maximum principle is applied to derive the conditions for the optimal maintenance policy and for the optimal planned sale date to maximize the expected net present value of the machine. An analytic solution for the problem is found in the special case when some of the random processes are independent of time and thus simply random variables.

The case of one particular life-time probability distribution, the exponential case, is analyzed in full detail. The parameter of the distribution, i.e. the failure rate of the machine, is shown to have an interesting and important economic interpretation. It may be interpreted as a risk premium which can be used to adjust both the production rate and the discount rate to a higher level in order to get for the stochastic problem a deterministic certainty-equivalent problem.

REFERENCES

1. Alam, M. et al., Int. J. Syst. Sci., 7 (1976), No. 9, 1071-1080.
2. Thompson, G.L., Mgmt Sci., 14 (1968), No. 9, 543-550.