## VAASAN KORKEAKOULUN JULKAISUJA

**TUTKIMUKSIA No 142** 

Business Administration 50
Accounting and Finance

Paavo Yli-Olli - Ilkka Virtanen - Teppo Martikainen

ON THE LONG-TERM STABILITY AND CROSS-COUNTRY
SIMILARITY OF THE FACTOR PATTERNS IN
THE ARBITRAGE PRICING MODEL

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The purpose of this paper is to test the Arbitrage Pricing Theory (APT) using monthly data for Finnish and Swedish stock returns during the 1977-1986 period. The first stage involves estimating the systematic risk components for each asset using factor analysis. The second stage involves testing by transformation analysis if the number and structure of factors which influence the security returns remain unchanged across various time periods and across different samples in two Scandinavian countries.

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#### 1. INTRODUCTION

#### 1.1 Background

The Capital Asset Pricing Model (CAPM), developed by Treynor (1961), Sharpe (1963 and 1964), Lintner (1965), and Mossin (1966) is a simple and elegant model for pricing risky assets. The CAPM is an equilibrium model, and in the CAPM, the systematic risk of an asset is defined to be the covariance of the asset with the market portfolio divided by the variance of the market portfolio. The CAPM has been the central topic in the empirical work in finance over the past twenty years. Empirical tests of the CAPM have produced mixed results. The most powerful evidence in support of the CAPM are the early findings of Black, Jensen and Scholes (1972), Fama and McBeth (1973) and Foster (1978). They found that portfolios with higher estimated betas also have higher realized returns. The critical point in the estimation of the CAPM is the difficulty of measuring the true market portfolio (more about the importance of a relevant data base in empirical research see Ball and Foster 1982). Stock market indices are usually used as a proxy of true market portfolio. Miller and Scholes (1972) found that the results are not very sensitive to the choice of stock market index. The result seems to be directly contrary to the result presented by Roll (1977). Roll obtained that the CAPM is extremely sensitive to the choice of a market proxy (see also Ball 1978: 110-126). Roll's critique goes further. He casts serious doubts on the testability of the CAPM itself. The CAPM is not testable unless the exact composition of the true market portfolio is used in the tests.

Several other equilibrium models and extensions of the CAPM have also been presented in literature. Fama (1971) showed that the CAPM can be extented into stable non-normal distributions and Black (1972) generalized the model into restricted borrowing. Furthermore, for example, Merton (1973) formulated a two-factor equilibrium model in the spirit of the CAPM and Kraus and Litzenberger (1976) created their third-moment equilibrium model where securities are priced according to their contributions to the variance and skewness of the market portfolio. However, the most frequently tested equilibrium model in recent literature is obviously the model based on the Arbitrage Pricing Theory (APT) formulated by Ross (1976). The APT is based on similar intuition as the CAPM, but it is more general. The CAPM assumes that a return of any security will be linearly related to a single common factor, to a return of the market portfolio,

whereas APT assumes that a security return is a linear function, not only of one, but of a set of common factors. The normal empirical procedure to test the APT is the following:

First, a factor analysis procedure is used to identify the number of factors and the factor loadings from daily, weekly or monthly time series. Second, the estimated factor loadings are used to explain the cross-sectional variation of estimated expected returns.

Unfortunately, there are many problems in testing of the APT. An intensively discussed problem is how to decide the correct number of priced factors. It has been found that the number of significant factors is an increasing function of the size of the groups analyzed (Dhrymes, Friend and Gultekin 1984, and Dhrymes, Friend, Gultekin and Gultekin 1985). There are also some additional methodological problems with the use of factor analysis (Elton and Gruber 1987: 343 - 354). First, the decision as how many factors to extract has been made subjectively. Second, there is no guarantee that factors are produced in a particular order. Third, there is no meaning to the signs of the parameters.

In our opinion, the most relevant questions in testing the APT is neither the question how to determine the "correct number" of the priced factors in different samples nor the question in what kind of order factors are produced in those samples. We may get the same number of factors in different samples or in the same sample in different time periods but the content or empirical interpretation of the factors do not necessarily remain as the same in those groups. Therefore, it is very important to find such common factors which are the same across different samples during the same time period (cross-sectional studies) or across different time periods in the same sample irrespective of what is the number of those factors or in what order the factors are produced. Transformation analysis offers us a versatile methodology with which it is possible to study the stability and invariance existing among different factor structures (for business applications of transformation analysis see Yli-Olli and Virtanen 1990).

### 1.2 The purposes of the study

The purposes of this study are:

- 1. to describe briefly the APT,
- 2. to test the APT using monthly time series data of the Scandinavian firms quoted on the Helsinki and Stockholm Stock Exchanges,

in testing the APT, the main effort is made to test, using transformation analysis, the stability of the factor structure over time and across samples. That means: transformation analysis tells us if the content of the factors remains the same in different time periods and in different samples (The empirical content or interpretation of the factors could be for example market portfolio, unexpected inflation etc. Quite clearly, however, the transformation analysis can tell us if the content of the factors remains the same but it does not tell what the content explicitely is). Analogously, we can find, using transformation analysis, if there are the same common factors (the empirical interpretation of the factors being the same) in different samples. In our analysis, therefore, it is not a problem if the factors in different samples are not produced in a particular order.

#### 2. THE APT-MODEL

The Arbitrage Pricing Theory, originally formulated by Ross (1976) predicts that on the perfectly competitive and frictionless stock markets the stock return is a linear function of a certain number, say k, economic factors. So, the APT starts with the assumption that returns on any stock, R<sub>it</sub>, are generated by a k-factor model of the form (see e.g. Roll and Ross 1980, 1076-1082):

(2.1) 
$$R_{it} = E(R_i) + b_{i1} \delta_{1t} + b_{i2} \delta_{2t} + ... + b_{ik} \delta_{kt} + \epsilon_{it}$$
,

where  $E(R_i)$ , i=1,2,...n, is the expected return of the stock  $i, \delta_j$ , j=1,2,...,k, are unobserved economic factors,  $b_{ij}$  is the sensitivity of the security i to the economic factor j and  $\epsilon_i$  are the idiosyncratic risks of the stocks. In addition, we assume that  $E(\delta_j)=0$  for j=1,2,...,k,  $E(\epsilon_i)=0$  for i=1,2,...,n,  $E(\epsilon_i\epsilon_k)=0$  for  $i\neq h$ , and  $E(\epsilon_i^2)=\sigma_i^2<\infty$ .

Ross (1976) has shown that if the number of stocks is sufficiently large the following linear risk-return relationship can be written:

(2.2) 
$$E(R_i) = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + ... + \lambda_k b_{ik}$$

where  $\lambda_0$  is a constant riskless rate of return (the common return on all zero-beta stocks)  $\lambda_j$ , j = 1, 2, ..., k, represents, in equilibrium, the risk premium for the jth factor.

In equation (2.1) each stock i has a unique sensitivity  $b_{ij}$  to each factor  $\delta_j$  but any factor  $\delta_j$  has a value that is the same for all stocks. These common factors capture the systematic components of risk in equation (2.1). Therefore, any  $\delta_j$  affects necessarily more than one security return. In the other case it would have been compounded in the unsystematic component of the risk, i.e. in the residual term  $\epsilon_j$ .

In order to test the Arbitrage Pricing Theory we have in principle two alternative approaches to test the model (2.1):

First, we could try to specify a priori, on the basis of the theory, the general factors that explain pricing in the stock market. Such macroeconomic variables could be e.g. the spread between long-term and short-term interest rates, expected and unexpected

inflation, industrial production and spread between high- and low-grade bonds (see Chen, Roll and Ross 1986). In the thin Finnish stock market such variables could be e.g. aggregate future cash-flow of the firms, interest rate of bank deposits or return of the state bonds, the supply of money, and inflation (see Virtanen and Yli-Olli 1987). In the case we have factors based on economic theory the estimation procedure should be as follows. In the first stage, time series regressions are run for each series of stocks (portfolios) to estimate each stock's (portfolio's) sensitivities  $b_{ij}$  to macroeconomic variables. Then the risk premia  $\lambda_j$  are estimated by running a cross-sectional regression for each time period examined. In every cross-sectional regression the average return of stocks is used as the dependent variable and the sensitivities of the securities as independent variables.

The more general and also much more problematic approach is to estimate the  $b_{ij}$  and unknown factors  $\underline{\delta}_j$  simultaneously by factor analysis. In that case a theory does not tell, a priori, what is the exact content or even the number of relevant factors. Without any theory a decision how many factors to extract from the data has to be made subjectively or by statistical criteria. When we have obtained systematic components of the risk,  $b_{ij}$ , the risk premia  $\lambda_i$  are estimated again using cross-sectional regressions.

In a factor analysis approach we have many methodological problems. First, there are no meaning to the signs of the factors produced by factor analysis. Second, the scaling of  $b_{ij}$ 's and  $\lambda_j$ 's is arbitrary. Third, there is no guarantee that factors are produced in a particular order when analysis is performed on separate samples (see Elton and Gruber 1987:336-352). In addition, we have serious difficulties when we try to decide what is the correct number of priced factors. Dhrymes, Friend, Gultekin and Gultekin (1985) used samples of different sizes (30, 60 and 90 stocks) and they found that the number of significant factors is an increasing function of the size of the group analyzed.

In our opinion, a very important but non-discussed and non-analyzed problem is the question if the contents of the factor structures in different samples during the same time period or the contents of the factors in the same sample in different time periods are the same. In this paper we use a method which makes it possible to analyze the stability of factor structures across different samples in the same time period or across different time periods in the same sample. It is not important in this method whether the factors in different samples are produced in a particular order or not. The only limitation is that we have the same number of factors in different samples. After that we can find such common factors which have the same contents, i.e. the same empirical interpretation for different samples.

#### 3. DATA AND STATISTICAL METHODS

The purpose of this study was to test, using Finnish and Swedish data, the Arbitrage Pricing Theory, especially the stability of factor patterns between different time periods and cross-sectionally across different samples. In Finland, on the Helsinki Stock Exchange, we have three commonly used indices, Unitas and KOP stock market indices published by two Finnish commercial banks and a return index developed by Berglund, Wahlroos and Grandell (1983). From the theoretical point of view, the return index developed by Berglund, Wahlroos and Grandell is the best measure and also selected for our research. This index also includes the dividend component whereas Unitas and KOP indices are pure price indices. When no trade has occured, we have proxied the true price by the bid quotation. In Sweden, on the Stockholm Stock Exchange, the indices have been calculated using the same method as in Finland. Using indices from these two closely related economies is a fruitful starting point to study the cross-country invariances in the APT since the previous research indicate that the stock price behaviour in these two countries is relatively similar to each other (compare Virtanen and Yli-Olli 1987).

The empirical verification of the APT and the stability analysis require both a large sample in terms of number of securities and also a long time period. We have in use monthly values of selected indices from January 1977 to December 1986. The Finnish sample consists of the shares of 30 firms (Sample 1, Appendix 1). The sample includes shares which have been quoted on the Helsinki Stock Exchange during the entire sample period and have been most frequently traded in period 1970-1986. For the stability analysis the whole period is divided into two subperiods: subperiod 1 includes years 1977-1981, subperiod 2 years 1982-1986. The Swedish data is analyzed in two samples (Appendices 2 and 3). The first Swedish sample (Sample 2, Appendix 2) consists of the 30 most frequently traded firms during the research period and the second Swedish sample (Sample 3, Appendix 3) represents the next 30 stocks ranked by the trading frequency (in both countries only one serie for each firm has been selected). The reason why only one sample from Finland has been selected is simply the fact the number of listed Finnish firms at the end of the period was only 59 and we are not able to select two samples of 30 stocks due to thin trading.

Using the most frequently traded stocks in thin security markets is a reasonable approach due to the nonsyncronous bias caused by the infrequent trading. Typically, due to

infrequent trading, in thin stock markets returns will be measured from return intervals of different lengths. This leads to situation where the measured variances will overstate the "true" unobservable variances and understate the "true" covariances when the indices presented in this study are used. Thus, the estimated betas in the CAPM and factor loadings in the APT will be biased downward. This makes the use of the most frequently traded stocks reasonable. The effects of thin trading are also decreased by selecting monthly return intervals in the study. An interesting point will be to study how the two Swedish samples are associated with each other. That is due to the fact that because of the sample formulation the thin trading bias in the second Swedish sample, i.e. in sample 3 is expected to be higher than in sample 2. This enables us to study the effects of trading frequency on the stability of factor loadings. The trading frequencies of the selected Swedish stocks are plotted in Appendix 5. The figure shows that the problems of irregular trading are obvious in the third sample.

The main statistical methods used in the study are factor analysis, regression analysis and transformation analysis. Factor analysis and regression analysis are usual techniques in business applications. Transformation analysis, on the contrary, has been mainly applied only in Finnish sosiological research. Therefore, this paper contains a short description of this multivariate method.

The degree of stability in factor patterns has been traditionally measured with correlation or congruence coefficients (the same coefficients are used in measuring the stability of estimated betas; see. e.g. Blume 1971). Both of these measures give an index for the similarity of two different factor solutions in terms of the pattern of correlations among factor loadings across all variables in the reduced factor space. For the dissimilar part of these factor solutions these indices are, however, unable to describe and explain the reason for the non-invariant part prevailing in these factor solutions (see Yli-Olli and Virtanen 1985: 25).

Yli-Olli (1983) introduced the use of transformation analysis for determining the degree and nature of medium-term stability exhibited by the factor patterns of the financial ratios. This approach was further applied and deepened by Yli-Olli and Virtanen (1985).

Originally transformation analysis (initiated by Ahmavaara (1963) and further developed by Ahmavaara (1966) and Mustonen (1966); most applications exist in the area of Finnish sosiological research) was developed to compare factor solutions between two different groups of objects. Yli-Olli (1983) and Yli-Olli and Virtanen (1985 and 1990) have used

the technique to compare two different factor solutions among the same group of objects, the two factor solutions being based on measurements made during two different time periods. In the following we sketch out the general idea behind transformation analysis (according to the papers of Yli-Olli and Virtanen 1985 and 1990).

Let us assume that we have two groups of observations  $G_1$  and  $G_2$  with the same variables, both by number and content. Let  $L_1$  and  $L_2$  be the factor matrices for  $G_1$  and  $G_2$ , respectively. Let us further assume that the factor models used in deriving  $L_1$  and  $L_2$  are both orthogonal and have the same dimension, pxr, say.

If there exists invariance between the two factor structures, there exists a nonsingular rxr-matrix T such that equation

$$(3.1) L_2 = L_1 T_{12}$$

holds. Matrix  $T_{12}$  is called the transformation matrix (between  $L_1$  and  $L_2$ , or in direction  $G_1 \rightarrow G_2$ ). If equation (3.1) holds exactly, it means that the factor structures in groups  $G_1$  and  $G_2$  are, up to a linear transformation, invariant, all the variables have the same empirical meaning in different groups. Depending on the type of the transformation matrix  $T_{12}$ , the formation of the factors from the variables and thereby the interpretation of the factors either is preserved ( $T_{12}$  is the identity matrix I) or it changes ( $T_{12}$  has also non-zero off-diagonal elements).

In practice, situation (3.1) will not be reached, but, after matrix  $T_{12}$  has been estimated, we have  $L_2 \neq L_1 T_{12}$ . The goodness of fit criterion for the model (3.1) may be based on the residual matrix

$$\mathbf{E}_{12} = \mathbf{L}_1 \, \mathbf{T}_{12} - \mathbf{L}_2 \, .$$

Non-zero elements in  $E_{12}$  mean that the empirical meaning of the variables in question has changed. This is called abnormal transformation.

The main problem in transformation analysis is the estimation of the matrix  $T_{12}$ . The estimation methods are in general based on the minimization of the sum of squares of the residuals  $e_{ij}$  (the elements of the residual matrix  $E_{12}$ ). This is the common method of least squares. The problem is to minimize

(3.3) 
$$\|\mathbf{E}_{12}\| = \|\mathbf{L}_1 \mathbf{T}_{12} - \mathbf{L}_2\|$$
  
= trace (( $\mathbf{L}_1 \mathbf{T}_{12} - \mathbf{L}_2$ ) ( $\mathbf{L}_1 \mathbf{T}_{12} - \mathbf{L}_2$ )').

Depending on additional constrains set for the matrix  $T_{12}$ , we have three different estimation methods, i.e. three transformation analysis models (see e.g. Yli-Olli and Virtanen 1985). Of these three techniques, the symmetric transformation analysis is the most popular one. It is also applied in this study.

With correlation and congruence coefficients one can only measure the degree of similarity of two factor solutions (correlations or congruences among factor loadings). This is also possible via transformation analysis (coefficients of coincidence on the main diagonal of the transformation matrix). In addition to this we obtain a regression type model for shifting of variables from one factor to another (normal or explained transformation). This is revealed by non-zero off-diagonal elements in the transformation matrix and indicates interpretatively changes for the factors in question. And finally, large elements in the residual matrix indicate abnormal or unexplained transformation between the two factor solutions. This means that the empirical content of the corresponding variables has changed. Further, this abnormal transformation can be appointed to separate variables or to separate factors.

#### 4. EMPIRICAL RESULTS

The empirical analysis in this paper is divided in two phases. In the first phase we study the long-term stability of the factor loadings using three different samples collected from the Helsinki Stock Exchange (sample 1) and the Stockholm Stock Exchange (samples 2 and 3). In the second phase we study the cross-sectional invariance of these three samples, i.e. we aim to find out if the contents of factors in different samples are similar to each other.

#### 4.1 Long-term stability of factor patterns

The first step in our empirical analysis is to use factor analysis procedure to identify the number of factors affecting equilibrium returns. This procedure has been very problematic because it has been shown that the number of factors discovered depends e.g. on the size of the groups of securities one deals with (see Dhrymes, Friend and Gultekin 1984: 345-346). The estimation of factors can be carried out by different factor analytic methods. In this study we use the principal component method based on the covariance matrices of stock returns and varimax rotation thereafter.

As stated above, the whole period was divided into two subperiods. In this section we try to find such common factors which are stable for different subperiods. For this purpose we first extracted two, three, four, five, six and seven factor solutions for each subperiod. Cumulative proportions of total variance explained (of the unrotated factor patterns) are presented in Table 1. These results indicate that the cumulative proportions of the Swedish samples are in each case higher than the proportions of the Finnish sample and the first Swedish sample (sample 2) always outperforms the second (sample 3), respectively.

Cattell's scree tests (Appendix 4, Figures 1-6) show that we can find 2-5 different factors for each subperiod. However, we have no absolute guarantee that the factors extracted have the same interpretation when the analysis is performed on separate subperiods, i.e. we can not be sure that the presented factors are those common factors we try to identify.

Table 1. Cumulative proportions of total variance explained.

	Sample 1		Sample 2		Sample 3	
	1977-81	1982-86	1977-81	1982-86	1977-81	1982-86
FACTOR1	0.326	0.311	0.438	0.453	0.368	0.419
FACTOR2	0.449	0.414	0.558	0.544	0.487	0.525
FACTOR3	0.520	0.495	0.629	0.603	0.560	0.525
FACTOR4	0.582	0.565	0.693	0.655	0.626	0.631
FACTOR5	0.633	0.625	0.735	0.693	0.676	0.669
FACTOR6	0.680	0.670	0.770	0.730	0.715	0.705
FACTOR7	0.715	0.714	0.801	0.764	0.752	0.738

Next we measure, using transformation analysis, the stability of factor patterns over time. The conclusion about stability is based on the coefficients on the main diagonal of the transformation matrix provided that factors in different samples are produced in the same order. The numerical values of those coefficients are very close to one when the factor structure over time is stable. Tables 2, 3 and 4 present the transformation matrices between three-factor solutions of subperiods. We use the three-factor solutions as a starting point due to the observation carried out by Yli-Olli and Virtanen (1989) who found the three-factor solution to be relatively stable in Finnish factor patterns. The results show that the stability of factors concerning the first two samples is very high during different subperiods. This means we have found at least three very stable factors in these two samples. Tables 2 and 3 also show that the two successive subperiods for these two samples have produced the first and the second factor in different order. That means the first and second factor have changed their positions in the second subperiod as compared to the first subperiod.

However, the transformation matrix for the third sample indicates that the factors in this sample have not been as stable as in the two first samples. The factors between successive subperiods seem to be relatively unstable and we cannot find stable common factors in this three-factor solution as proposed in the APT. One potential reason for that could be the effect of infrequent trading stated above. The factor loadings may be biased due to nonsyncronous trading especially in this sample because of the fact that it consists of relatively infrequently traded stocks. Since we were not able to find stable factors from the three factor-solution, we applied the transformation analysis for the two-factor solutions in the third sample. The results from that analysis are reported in Table 5. They indicate that there exists two stable common factors in that sample. Similar kind of conclusion can also be drawn from the scree test in Figures 5 and 6 in Appendix 4.

**Table 2.** Transformation matrix between the factor patterns of returns, three-factor solution. Sample 1.

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	Factor	1	2	3
Sub-	1	0.127	0.987	0.094
period	2	0.992	-0.127	-0.002
1	3	-0.011	-0.094	0.996

Table 3. Transformation matrix between the factor patterns of returns, three-factor solution. Sample 2.

Subperiod 2

	Factor	1	2	3
Sub-	1	0.365	0.931	0.003
period	2	0.929	-0.365	0.059
1	3	-0.056	0.019	0.998

Table 4. Transformation matrix between the factor patterns of returns, three-factor solution. Sample 3.

Subperiod 2

	Factor	1	2	3
Sub-	1	0.597	-0.009	0.802
period	2	0.530	0.755	-0.386
1	3	-0.602	0.656	0.456

**Table 5.** Transformation matrix between the factor patterns of returns, two-factor solution. Sample 3.

Subperiod 2

	Factor	1	2
Sub-	1 2	0.998	-0.070
period 1		0.070	0.998

As stated above, the stability of the factors in the two first samples was found to be high in the three-factor solutions. So, it is naturally interesting to study how the stability is affected when four- and five-factor solutions for these samples are studied. The results are reported in Tables 6, 7, 8 and 9. The results from the four- and five-factor solutions for the third sample give support, as expected, to the finding of instability in factor patterns.

Table 6. Transformation matrix between the factor patterns of returns, four-factor solution. Sample 1.

Subperiod 2

Subperiod 2

	· · · · · · · · · · · · · · · · · · ·					
	Factor	1	2	3	4	
Sub-	1	-0.186	0.863	0.467	-0.040	
period	2	0.867	0.368	-0.328	0.068	
1	3	0.440	-0.339	0.816	0.160	
~	4	-0.139	0.064	-0.091	0.984	
****						

Table 7. Transformation matrix between the factor patterns of returns, four-factor solution. Sample 2.

	Suspende 2					
-swape	Factor	1	2	3	4	
Sub-	1	0.276	0.954	0.116	0.021	
period	2	0.174	-0.170	0.958	0.150	
1	3	0.945	-0.247	-0.214	-0.010	
•	4	-0.023	0.003	-0.150	0.988	

Using the four-factor approach in the two first samples, the results are quite encouraging. There seems to exist at least three stable factors for both samples. The factors have again changed their positions, but the contents of the factors seem to been somewhat the same. However, if one compares the obtained results with the results of the three-factor solutions, the three factor solutions seem to outperform the four-factor solutions. This is especially true for the the Finnish sample, i.e. for sample 1.

Table 8. Transformation matrix between the factor patterns of returns, five-factor solution. Sample 1.

#### Subperiod 2

	Factor	1	2	3	4	5
Sub-	1	0.356	0.840	-0.096	-0.166	0.362
period	2	-0.233	0.435	0.656	0.365	-0.440
1	3	0.359	-0.291	0.742	-0.301	0.381
	4	0.830	-0.117	-0.097	0.342	-0.414
	5	-0.041	-0.083	0.002	0.795	0.599

Table 9. Transformation matrix between the factor patterns of returns, five-factor solution. Sample 2.

#### Subperiod 2

4	Factor	1	2	3	4	5
Sub-	1	0.295	0.713	0.618	0.012	-0.151
period	2	0.786	0.188	-0.585	0.056	0.032
1	3	-0.119	0.237	-0.002	0.341	0.902
	4	0.526	-0.602	0.515	-0.133	0.279
	5	0.068	-0.194	0.102	0.929	-0.291

The results from five-factor solutions indicate that the five-factor solution seems somewhat inappropriate in the sense that stability of the factors is relatively poor. Thus, concerning the first two samples, the transformation matrices suggest that there exist at least three common factors in these samples. Concerning the third sample, the number of stable common factors was reported to be two.

Tables 10, 11 and 12 present the residual matrices between subperiods 1 and 2 for the three samples of the three-factor solutions. The residual matrices show that any remarkable abnormal transformation does not exist (there are no large non-zero elements in the matrices).

Table 10. Residual matrix  $E_{12}$  and abnormal transformation for subperiod 2, three-factor solution. Sample 1.

Abnormal transformation

Firm	Factor 1	Factor 2	Factor 3	ti <sup>2</sup>
	1 40.01 1	1 8001 2	Tactor 5	4°
KOP	0.222	0.139	-0.071	0.074
SYP	0.004	0.033	-0.066	0.005
POHJOLA	0.097	0.349	0.087	0.139
EFFOA	-0.524	0.051	0.000	0.278
KESKO	0.125	0.020	-0.138	0.035
STOCK.	0.262	0.197	0.561	0.422
TAMRO	-0.142	0.028	-0.431	0.207
ENSO	-0.478	0.068	0.217	0.280
FISK.	-0.269	0.238	0.155	0.153
HUHTAM.	0.063	0.086	-0.304	0.104
KAJAANI	-0.370	0.624	-0.191	0.563
KEMI	-0.316	0.137	-0.146	0.140
KONE	0.426	-0.613	0.378	0.700 ~
KYMMENE	-0.221	0.020	0.156	0.073
LASSILA	0.503	-0.135	-0.079	0.227
LOHJA	-0.082	-0.187	-0.182	0.075
METSÄL.	-0.050	-0.157	-0.047	0.029
NOKIA	-0.041	-0.201	0.004	0.042
OTAVA	0.138	-0.047	0.374	0.161
PARTEK	-0.101	0.043	-0.058	0.015
RAUMA-R.	0.139	-0.071	0.088	0.032
ROSENLEW	0.033	-0.066	0.183	0.039
SCHAUMAN	0.349	0.087	-0.155	0.153
SERLACHIUS	0.051	0.000	0.172	0.032
SUOMEN S.	0.020	-0.138	-0.014	0.020
SUOMEN TR.	0.197	0.561	-0.512	0.616
ΓAMFELT	0.028	-0.431	0.326	0.293
<b>TAMPELLA</b>	0.068	0.217	-0.186	0.086
WÄRTSILÄ	0.238	0.155	-0.343	0.198
YHTYNEET	0.086	-0.304	0.230	0.153
Abnormal				
ransformation sj <sup>2</sup>	1.756	1.859	1.778	5.393

Table 11. Residual matrix  $E_{12}$  and abnormal transformation for subperiod 2, three-factor solution. Sample 2.

Abnormal transformation

Firm	Factor 1	Factor 2	Factor 3	ti <sup>2</sup>
AGA	0.033	-0.152	0.081	0.031
ALFA	-0.058	-0.055	0.325	0.112
ASEA	0.128	-0.133	0.388	0.185
ASTRA	0.171	-0.022	-0.103	0.040
ATLAS	-0.046	0.101	0.028	0.013
BOLIDEN	0.415	0.065	-0.300	0.267
ABV	-0.263	-0.241	0.168	0.155
ELECTRO	0.117	0.027	0.259	0.081
ERIC	0.391	0.116	-0.223	0.216
ESAB	0.071	0.206	-0.291	0.132
EUROC	-0.084	0.143	-0.016	0.028
INDUSTRI	-0.250	0.121	0.073	0.083
INVEST	-0.135	0.068	0.020	0.023
MODO	-0.111	-0.030	0.181	0.046
SONES	0.364	-0.057	-0.496	0.381
PHARMA	-0.051	-0.094	-0.086	0.019
PLM	0.115	-0.064	-0.030	0.018
PROVE	-0.118	0.094	0.100	0.033
SAAB	-0.134	0.002	0.264	0.087
SANDVIK	-0.150	0.470	-0.330	0.353
SCA	-0.152	0.081	-0.009	0.030
SE-BANKEN	-0.055	0.325	-0.313	0.207
SKANDIA	-0.133	0.388	-0.021	0.169
SKANSKA	-0.022	-0.103	-0.078	0.017
SKF	0.101	0.028	0.282	0.091
SKÅNE	0.065	-0.300	0.014	0.095
STORA	-0.241	0.168	0.024	0.087
SHB	0.027	0.259	-0.205	0.110
SWEDISH	0.116	-0.223	0.359	0.192
VOLVO	0.296	-0.291	0.124	0.142
Abnormal				
transformation sj <sup>2</sup>	0.941	1.054	1.448	3.443

**Table 12.** Residual matrix  $E_{12}$  and abnormal transformation for subperiod 2, three-factor solution. Sample 3.

Abnormal transformation

Firm	Factor 1	Factor 2	Factor 3	t <sub>i</sub> 2
ATI_A	0.065	-0.078	-0.121	0.025
FLÄKT	-0.321	-0.102	-0.118	0.128
GARB	-0.001	0.056	-0.179	0.035
GUNNEBO	0.191	0.216	-0.201	0.124
ÅKER	-0.166	-0.065	-0.087	0.039
IGGES.	-0.208	-0.052	-0.199	0.085
MUNKSJÖ	-0.337	-0.295	-0.379	0.345
CARNE	0.302	-0.230	-0.087	0.152
EDSTRA	0.196	0.111	-0.389	0.202
HENNES	0.407	0.001	-0.415	0.338
HÜFVUD	0.436	-0.367	-0.127	0.341
HÖGANÄS	0.019	-0.118	-0.013	0.014
INCEN -	-0.274	0.110	0.362	0.218
MARABOU	-0.283	0.460	-0.205	0.334
MARIEB.	0.023	0.036	-0.145	0.023
NOBEL	-0.192	0.179	0.098	0.078
TRANS	0.409	0.270	-0.415	0.413
TRELLE	-0.272	0.112	-0.337	0.200
RATOS	-0.045	0.143	-0.217	0.070
CUSTOS	0.181	-0.089	0.120	0.055
EXPO	-0.078	-0.121	-0.044	0.023
FÖRETAG	-0.102	-0.118	-0.024	0.025
HEVEA	0.056	-0.179	-0.288	0.118
RANG	0.216	-0.201	0.137	0.106
ÖRESUND	-0.065	-0.087	0.213	0.057
GÖTA	-0.052	-0.199	0.080	0.049
NORD	-0.295	-0.379	0.200	0.271
SKÅNSKA	0.230	-0.087	0.196	0.099
WERM	0.111	-0.389	0.164	0.190
ÖSTGÖTA	0.001	-0.415	0.451	0.376
Abnormal		,		
transformation s <sub>j</sub> <sup>2</sup>	1.509	1.369	1.655	4.532

Also the other residual matrices showed only minor abnormal transformation. The cumulative abnormal transformations for three-, four-, five- and six-factor solutions are presented in Table 13. These numbers indicate that the abnormal transformation has been highest for the Finnish sample. This might be due to the significant structural changes in the Finnish stock market during the research period, for example in the form of rapid changes in the trading volume in the Helsinki Stock Exchange.

Table 13. Cumulative abnormal transformation for subperiod 2.

	3-factor solution	4-factor solution	5-factor solution	6-factor solution
Sample 1	5.393	8.963	10.399	11.164
Sample 2	3.443	5.672	7.582	7.421
Sample 3	4.532	4.651	6.173	6.029

The analysis presented thus shows that the stability of factor structure over time is best for three-factor solution (and also quite good for four-factor solutions) for the two first samples. First, the factor structures are very stable according to the transformation matrices. Second, the Cattel's scree-tests and eigenvalues also support the results obtained by the transformation analysis (Appendix 4). Concerning the third sample, the best results were obtained by the two-factor solution.

The following step involved examining the effect of factors on equilibrium returns. In cross-sections the dependent variable is the monthly mean return and the independent variables are factor loadings from factor analysis. The OLS regression coefficients would be the estimated risk premia. In factor analysis there is no absolute meaning to the signs of the parameters and the scaling of the factors and then also the signs of regression coefficients are arbitrary. Therefore, only the statistical significance of regression coefficients is relevant instead of their numerical values.

The results of the cross-sectional regressions are presented in Tables 14 - 17. They show that in the first subperiod at least two different factors are priced, and the fourth factor has only a bit more explanatory power compared to the three-factor solution (F-values are usually lower in the four-factor solution). On the other hand, concerning the first two samples, the transformation analysis showed that we can extract three or four factors with the same content in different subperiods. The seeming inconsistency of the results rises from the fact that in the two first samples transformation analysis gives the number of the factors which have the same content in different subperiods. Regression analysis on the other hand gives the number of priced common factors. So, the transformation analysis gives the maximum number of priced common factors, i.e. the content of factors is the same in different time periods. It is possible that some very stable factors extracted by factor analysis are so firm- or industry-specific that their t-statistic is so low in cross-sectional regression that they are not common. However, it is very important to remember

that transformation analysis is necessary in testing if the contents of factors in different subperiods are the same.

In the second subperiod the rates of determinations in the regression equations are obvoiusly lower than in the first subperiod. An interesting observation is the stability of the results in this sense. The APT seems not to work as well in the second subperiod than it did in the first subperiod in any of the samples. This might be due to the fact that the price behaviour in the second subperiod was relatively instable in both countries.

The results in this chapter indicate that there exists three relatively stable factors over time in samples as well in Finland as in Sweden. However, it is relevant to emphasize that the number of stable factors in sample 3, i.e. in the second Swedish sample is not as clear. This is because of the problems of measuring returns when they are due to infrequent trading (on the trading frequencies of stocks in the two Swedish samples see Appendix 5).

Table 14. Regression analysis estimates. Subperiod 1.

dependent variable: average monthly return for security independent variables: factor loadings (k=3)

(t-values in parantheses)

#### Coefficients of

Sample	Constant	Fact1	Fact2	Fact3	R-square	F
1	0.0179 (3.898)	-0.0162 (-2.448)	-0.0036 (-0.584)	0.0089 (1.431)	0.395	5.663
2	0.0145 (2.104)	0.0099 (1.280)	-0.0066 (-0.789)	-0.0208 (-2.617)	0.392	5.598
3	0.0076 (0.927)	0.0026 (2.496)	-0.0214 (-2.116)	-0.0233 (-2.637)	0.613	13.217

Table 15. Regression analysis estimates. Subperiod 2.

dependent variable: average monthly return for security

independent variables: factor loadings (k=3)

(t-values in parantheses)

#### Coefficients of

Sample	Constant	Fact1	Fact2	Fact3	R-square	F
1	0.0156 (2.179)	0.0150 (1.574)	0.0123 (1.371)	0.0065 (0.629)	0.114	1.075
2	0.0144 (2.854)	0.0076 (1.244)	0.0119 (2.120)	0.0037 (0.596)	0.160	1.647
3	0.0280 (3.034)	0.0052 (0.525)	-0.0048 (-0.409)	-0.0104 (-1.009)	0.074	0.695

Table 16. Regression analysis estimates. Subperiod 1.

dependent variable: average monthly return for security independent variables: factor loadings (k=4)

-

(t-values in parantheses)

#### Coefficients of

Sample	Constant	Fact1	Fact2	Fact3	Fact4	R-square	F
1	0.0175 (3.931)	-0.0150 (-2.671)	-0.0095 (-1.556)	0.0085 (1.481)	0.0032 (0.527)	0.453	5.180
2	0.0171 (3.584)	0.0051 (-3.912)	-0.0098 (-2.241)	0.0019 (-1.391)	-0.0258 (0.740)	0.454	5.191
3	0.0085 (0.992)	0.0159 (1.543)	-0.0250 (-2.506)	0.0012 (0.113)	-0.0286 (-3.817)	0.617	9.644

**Table 17.** Regression analysis estimates. Subperiod 2.

dependent variable: average monthly return for security

independent variables: factor loadings (k=4)

(t-values in parantheses)

#### Coefficients of

Sample	Constant	Fact1	Fact2	Fact3	Fact4	R-square	F
1	0.0168 (2.241)	0.107 (1.180)	0.0152 (1.660)	0.0043 (0.443)	0.0030 (0.309)	0.129	0.884
2	0.0106 (2.130)	0.0048 (0.935)	0.0134 (2.584)	0.0167 (2.658)	0.0083 (1.398)	0.303	2.719
3	0.0282 (3.009)	-0.0068 (-0.463)	0.0075 (0.901).	-0.0080 (-1.021)	0.0042 (0.526)	0.084	0.576

## 4.2 Cross-sectional similarity of factor patterns

#### 4.2.1 Cross-country similarity of factor patterns

As stated above, transformation analysis also enables us to compare the factor loadings in cross-sectional samples. In the first phase we studied the cross-country similarity between the factor patterns of the most frequently traded stocks, i.e. the similarity between factor patterns of samples 1 and 2.

Again, we started from the three-factor solutions (see Tables 17 and 18). In the first subperiod, it seems that there exists at least two common factors produced in different order, but the third factor seems to differ across countries. In the second subperiod, the results give support to three common factors across countries. From these three factors, the first and second factors have changed their positions between different samples.

Table 18. Transformation matrix between the factor patterns of returns, three-factor solution. Sample 1 vs. Sample 2. Subperiod 1

Sample	1
Sallible	L

	Factor	1	2	3
	1	0.313	0.493	0.812
Sample	2	0.462	0.668	-0.584
1	3	0.830	-0.557	0.019

**Table 19.** Transformation matrix between the factor patterns of returns, three-factor solution. Sample 1 vs. Sample 2. Subperiod 2.

Sample 2

	Factor	1	2	3
	1	0.014	0.948	0.317
Sample	2	0.907	0.122	-0.404
1	3	0.422	-0.293	0.858

Since the three-factor solutions seem not to entirely support three common factors across countries, we also studied the two-factor solutions in this context. These two-factor solutions in Tables 20 and 21 suggest that there has existed two common factors in Finland and Sweden in both subperiods. In the first subperiod these factors have been produced in different order than in the second period, respectively. Thus, the results strongly support two stable common factors across the two Scandinavian countries. How these two stable factors are produced in different samples in different time periods are presented in Figure 1.

Table 20. Transformation matrix between the factor patterns of returns, two-factor solution. Sample 1 vs. Sample 2. Subperiod 1.

Sample 2

	Factor	1	2
Sample 1	1 2	0.391 0.920	0.920 -0.391

Table 21. Transformation matrix between the factor patterns of returns, two-factor solution. Sample 1 vs Sample 2. Subperiod 2.

		Samp	le 2
· · · · · · · · · · · · · · · · · · ·	Factor	1	2
Sample 1	1 2	0.960 0.281	-0.281 0.960

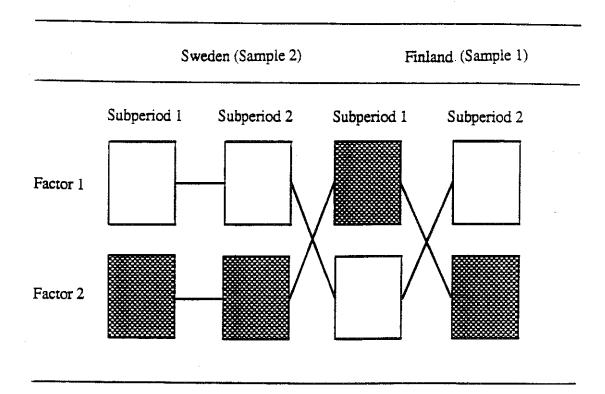


Figure 1. Cross-country similarity of factor patterns. Two-factor solutions.

#### 4.2.2 Intra-country similarity of factor patterns

In the second cross-sectional phase we studied the stability of intra-country factor patterns, i.e. the stability of factors loadings between samples 2 and 3. The results from that cross-sectional analysis are reported in Tables 22-25.

Transformation matrix between the factor patterns of returns, three-factor solution. Sample  $2\ vs.$  Sample 3. Subperiod 1.Table 22.

Sample 3			
1		2	3
<del></del>			

	Factor	1	2	3
Sample 2	1	0.442	0.524	0.728
	2	0.888	-0.137	-0.440
	3	-0.130	0.841	-0.525

Transformation matrix between the factor patterns of returns, three-factor Table 23. solution. Sample 2 vs. Sample 3. Subperiod 2.

Samp	le 3
------	------

	Factor	1	2	3
	1	0.502	-0.031	0.864
Sample	2	0.754	0.505	-0.420
2	3	-0.043	0.863	0.277

Table 24. Transformation matrix between the factor patterns of returns, two-factor solution. Sample 2 vs. Sample 3. Subperiod 1.

Sample 3

	Factor	1	2	
Sample	1	0.157	0.988	
2	2	0.988	-0.157	

Transformation matrix between the factor patterns of returns, two-factor Table 25. solution. Sample 2 vs Sample 3. Subperiod 2.

Sample 3

	Factor	1	2	
Sample	1 2	0.995	0.098 0.995	

The transformation matrices above indicate that there seems not to exist three stable factors among the two Swedish samples. This may well be due to the thin trading in the third sample which apparently causes problems when measuring returns and estimating factor loadings for this sample. On the other hand, the transformation matrices based on two-factor solutions presented in Tables 24 and 25 indicate quite high level of similarity in these factor patterns.

#### 5. SUMMARY

The main purpose of this study was to test the APT using monthly time series data of Scandinavian firms quoted on the Helsinki Stock Exchange and on the Stockholm Stock Exchange, as a part of this, especially to test, using transformation analysis, the stability of the factor structure over time and across different samples. That means: transformation analysis tells us if the content of the factors remains the same in different time periods and also in different samples during the same time period.

The empirical verification of the APT involved in the first stage the estimation of the systematic risk components for each asset using factor analysis. The second stage involved testing by transformation analysis if the number and structure of factors remained unchanged or stable across different time periods and between different samples in two Scandinavian stock exchanges. For stability analysis the whole period was divided into three samples (one Finnish, two Swedish) and two subperiods: 1977-1981, 1982-1986. The factor and transformation analysis showed that we found at least three very stable factors for the two most frequently traded samples. For the sample consisted of more infrequently traded stocks we could only find two stable factors as well in Finland as in Sweden. This might well be due to the problems measuring returns and estimating factor loadings for the infrequently traded stocks. In addition, we also carried out an analysis where the similarity of factor patterns across different samples was tested. When using the most frequently traded stocks, about three stable factors were found. Again, for the sample consisted of more infrequently traded stocks we could only find two stable factors. Our results indicate that infrequent trading apparently causes significant problems when testing the Arbitage Pricing Theory.

In addition, we also examined the effects of factors on equilibrium returns. The results of the cross-sectional regression showed the relatively high level of similarity between factor patterns in the same subperiods. In the first subperios, the APT seemed to perform relatively well, but in the second subperiod the results were not as encouraging. As a summary we can state that transformation analysis gave us the maximum number of stable factors which preserved the same content across different time periods and could thus serve as the common priced factors. Regression analysis then gave us the final number of common priced factors.

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## APPENDIX 1. STOCKS INCLUDED IN THE STUDY

YHTYNEET

Sample 1.	(Fi	nland)
KOP	=	KANSALLIS-OSAKE-PANKKI
SYP	=	UNION BANK OF FINLAND
POHJOLA	=	POHJOLA
EFFOA	=	EFFOA
KESKO	=	KESKO
STOCK.	=	STOCKMANN
TAMRO	=	TAMRO
ENSO	=	ENSO-GUTZEIT
FISK.	=	FISKARS
HUHTAM.	=	HUHTAMÄKI
KAJAANI	=	KAJAANI
KEMI	=	KEMI
KONE	=	KONE
KYMMENE	=	KYMMENE
LASSILA	=	LASSILA & TIKANOJA
LOHJA	=	LOHJA
METSÄL.	=	METSÄLIITTO
NOKIA	=	NOKIA
OTAVA	=	OTAVA
PARTEK	=	PARTEK
RAUMA-R.	=	RAUMA-REPOLA
ROSENLEW	=	W. ROSENLEW
SCHAUMAN	=	SCHAUMAN
SERLA	=	G.A. SERLACHIUS
SUOMEN S.	=	FINNISH SUGAR
SUOMEN TR.	=	SUOMEN TRIKOO
TAMFELT	=	TAMFELT
TAMPELLA	=	TAMPELLA
WÄRTSILÄ	=	WÄRTSILÄ

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## APPENDIX 2. STOCKS INCLUDED IN THE STUDY

Sample 2.	(Sv	veden)
AGA	-	AGA
ALFA	=	ALFA-LAVAL
ASEA	=	ASEA
ASTRA	=	ASTRA
ATLAS	=	ATLAS COPCO
BOLIDEN	=	BOLIDEN
ABV	=	ABV
ELECTRO	=	ELECTROLUX
ERIC	=	ERICSSON
ESAB	=	ESAB
EUROC	=	EUROC
INDUSTRI	=	INDUSTRIVÄRDEN
INVEST	=	INVESTOR
MODO	=	MO OCH DOMSJÖ
SONES	=	SONESSON
PHARMA	=	PHARMACIA
PLM	=	PLM
PROVE	=	PROVENTUS
SAAB	=	SAAB-SCANIA
SANDVIK	=	SANDVIK
SCA	=	SCA
SE-BANKEN	=	SE-BANKEN
SKANDIA	=	SKANDIA
SKANSKA	=	SKANSKA (CEMENTGJ.)
SKF	=	SKF
SKÅNE	=	SKÅNE GRIPEN
STORA	=	STORA KOPPARBERG
SHB	=	SHB
SWEDISH	=	SWEDISH MATCH
T 107 T 10		*****

VOLVO

VOLVO

## APPENDIX 3. STOCKS INCLUDED IN THE STUDY

Sample 3.	(Sw	veden)
ATLA	=	ATLANTICA
FLÄKT	=	FLÄKT
GARB	=	GARBHYTTAN
<b>GUNNEBO</b>	=	GUNNEBO
ÅKER	=	ÅKERMANS
IGGES.	=	IGGESUND
MUNKSJÖ	=	MUNKSJÖ
CARNE	=	CARNEGIE & CO
EDSTRA	=	EDSTRAND
HENNES	=	HENNES & MAURITZ
HUFVUD	=	HUFVUDSTADEN
HÖGANÄS	=	HÖGANÄS
INCEN -	===	INCENTIVE
MARABOU	=	MARABOU
MARIEB.	-	MARIEBERG
NOBEL	=	NOBEL IND. SVERIGE (BOFORS)
TRANS	=	TRANSATLANTIC
TRELLE	=	TRELLEBORG
RATOS	=	RATOS
CUSTOS	=	CUSTOS
EXPO	=	EXPORT-INVEST
FÖRETAG	=	FÖRETAGSFINANS
HEVEA	=	HEVEA
RANG	=	RANG INVEST
ÖRESUND	=	ÖRESUND
GÖTA	=	GÖTABANKEN
NORD	=	NORDBANKEN (SUNDSVALLSB.)
SKÅNSKA	=	SKÅNSKA BANKEN
WERM	=	WERMLANDSBANKEN
ÖSTGÖTA	=	ÖSTGÖTABANKEN

## APPENDIX 4. CATTEL'S SCREE-TESTS

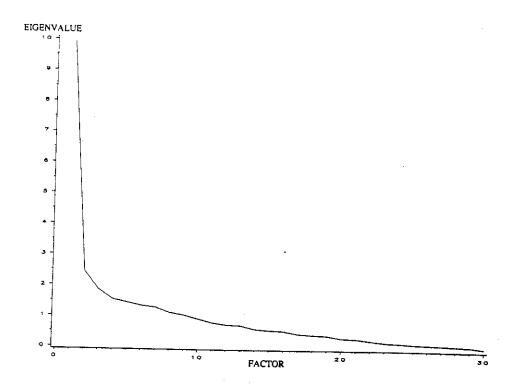


Figure 1. Scree plot of eigenvalues. Sample 1. Subperiod 1.

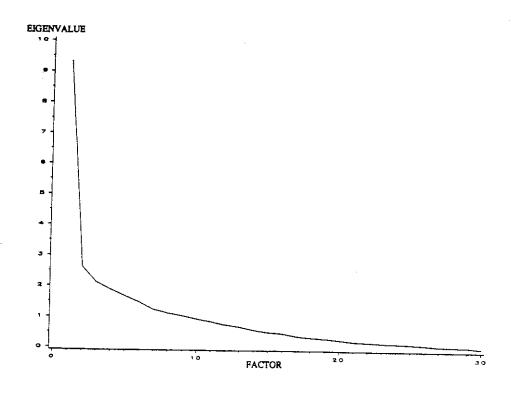


Figure 2. Scree plot of eigenvalues. Sample 1. Subperiod 2.

## APPENDIX 4. CATTEL'S SCREE-TESTS

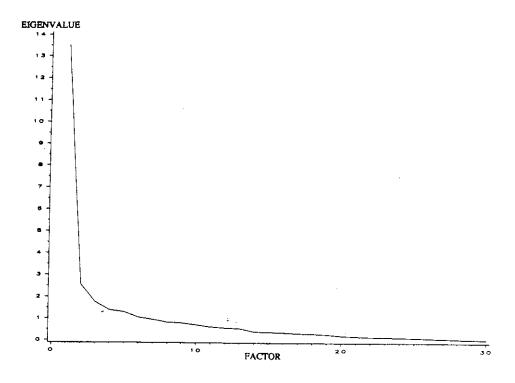


Figure 3. Scree plot of eigenvalues. Sample 2. Subperiod 1.

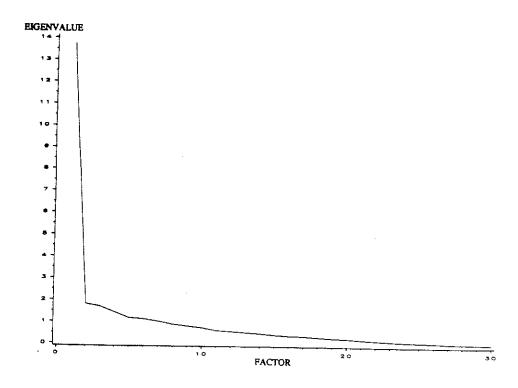


Figure 4. Scree plot of eigenvalues. Sample 2. Subperiod 2

## APPENDIX 4. CATTEL'S SCREE-TESTS

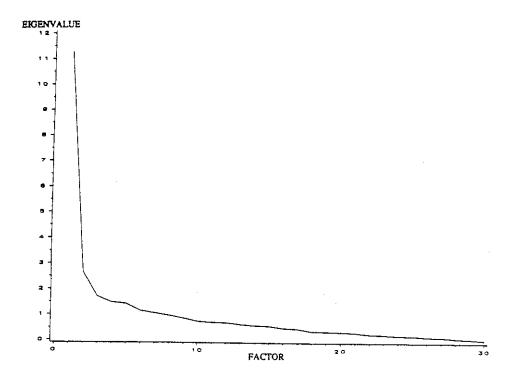


Figure 5. Scree plot of eigenvalues. Sample 3. Subperiod 1.

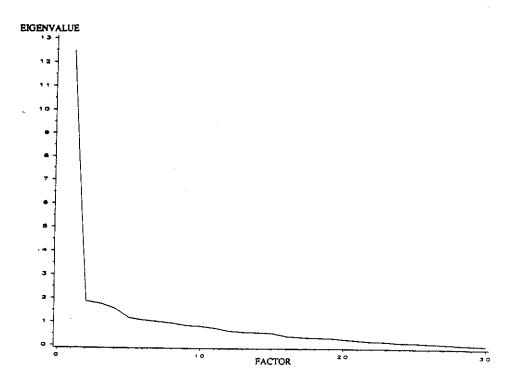


Figure 6. Scree plot of eigenvalues. Sample 3. Subperiod 2.

## APPENDIX 5. TRADING FREQUENCY IN SWEDISH STOCKS

