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ON THE STABILITY OF THE CLASSIFICATION OF FINANCIAL RATIOS

An application of factor and transformation analysis

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ABSTRACT

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The purpose of this study is to give a contribution to the methodological debate in the area of financial ratios. First, existing classifications for financial ratios are surveyed. After that an empirical classification system based on factor analysis is introduced. The stability of the emergent financial patterns during the subperiods of the whole period under study is examined by transformation analysis of the factorial data.

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1. INTRODUCTION

Financial statement analysis is an information-processing system developed to provide relevant data for decision makers. Users of the financial statement analysis are investors, management lenders, labour unions, researchers etc. Traditionally, the information is derived from the published financial statements of the firms. This information can be completed by the nonaccounting data like stock prices and different aggregate economic indicators (Lev 1974: 1-3).

Financial statement analysis includes the following characteristics:

1. the users of information, their objectives and decision-making models
2. the level of standardizing items reported on the financial statement
3. the computational methodology of the financial ratios

The great number of information users, their different objectives and different decision-making models have to some extent meant that the ratios used in financial analysis are numerous and treated as separate figures without an explicit theoretical structure (Horrigan 1968: 294). Financial ratios are classified by the sources of data as follows (Lev 1974: 11-12):

1. balance sheet ratios
2. income statement ratios
3. fund statement ratios
4. mixed ratios

or according to the different economic aspects of the firm's operations:

1. profitability ratios
2. short-term solvency ratios
3. long-term solvency ratios
4. efficiency (turnover) ratios

Many alternative categories of financial ratios and individual ratios have been proposed in the literature (Horrigan 1967 Chapter 6, Foster 1978: 24-37, Courtis 1978: 372-375 and Tamari 1978: 24-44). There is no consensus on each ratio as to what the ratio primarily measures because of the differences in standardizing items reported on the financial statement (Aho 1981: 16-19) and because of the differences in computation of financial ratios (Gibson 1982: 13 and Gombola and Ketz 1983: 105).

Faced with this practically unlimited array of potentially useful ratios - which are often very close to each other - there has arisen a need to develop some acceptable system for classifying financial ratios. Valuable insight into relationships between financial ratios is afforded by the study of Pinches, Mingo and Caruthers (1973). They developed an empirically-based classification system for financial ratios using factor analysis. Principally, the users of ratios can choose from each group such financial ratios as are theoretically acceptable and computationally unambigious in accounting practice and use that small subset of financial ratios to describe the characteristics of the firms. Another interesting feature in the study of Pinches, Mingo and Caruthers was that according to their results the classification patterns of ratios were reasonably stable over time, even when the magnitude of the financial ratios was undergoing change. They analyzed the stability by using correlation coefficients based on all factor loadings in the years 1951, 1957, 1963 and 1969.

Johnson (1978: 207-213) presented an empirically based system for classifying financial ratios and examined the cross-sectional stability of the emergent financial patterns. He had two groups of industrial firms - primary manufacturing versus retail 1972 - and the degree of cross-sectional stability was analyzed by employing congruency coefficients (Harman 1967: 256-259). The results showed that from eight factor patterns six were remarkably stable across industry classification. Two exceptions were capital intensiveness and inventory intensiveness.

Interesting results were recently presented by Gombola and Ketz (1983) and Yli-Olli (1983) independently of each other. They showed that profitability ratios and cash-flow ratios do not measure the same characteristic of firm performance. Accounting profitability measures indicate operating performance whereas cash-flow measures signify solvency and financial flexibility (Gombola and Ketz 1983: 106 and Yli-Olli 1983: 40-50, see also Financial Accounting Standards Board 1980: i. in its Discussion Memorandum on Reporting Funds Flow and Financial Flexibility, "Profitability and funds flow are different"). In previous studies (see e.g. Pinches, Mingo and Caruthers 1973 and Pinches, Eubank, Mingo and Caruthers 1975) the sum of net income plus depreciation is used to serve as a proxy for cash-flow. The proper measure of cash-flow is, however, cash receipts from operations less cash disbursements for operations (see also Aho 1980: 415-417).

Gombola and Ketz found considerable time-series stability of factor patterns. Factor analysis was performed for each year over a 19 years period beginning in 1962 and ending in 1980. In this period they had fifteen years which included the same eight factors. Yli-Olli used the so-called transformation analysis (developed by Ahmavaara 1954 and 1963) to measure the medium-term stability of factor patterns (see Section 3.1. for the accounting principles of variables). Transformation analysis is a more sophisticated method than an analysis based on correlation or congruency coefficients in indicating cross-section or time-series stability of factor patterns (see Section 3.2.).

The general objective of this paper is to corroborate and extend the results of Yli-Olli (1983). The specific purposes of this paper are:

1. to develop empirically-based classification patterns for theoretically acceptable financial ratios where cash-flow ratios are included and cash-flow is defined in a proper way, i.e. cash receipts from operations less cash disbursements for operations,

2. to measure, using transformation analysis, the medium-term stability in these classifications and to show that transformation analysis is a more sophisticated method for measuring and modelling the stability of classification patterns than e.g. correlation or congruency coefficients earlier used in financial ratio analysis.

2. THE SELECTED FINANCIAL RATIOS

The selection of financial ratios is based on the following considerations. The ratios examined should include both so-called traditional financial ratios and cash-flow ratios. The cash-flow analysis and ratios used in Finland were developed by Artto (1968), see e.g. Kettunen-Mäkinen-Neilimo (1979: 96-105), see also Lawson (1980: 11-46). In this approach profit and financing results do not depend on either accounting valuation or periodization (see Artto 1978: 27-29 and Artto 1982: 754-755, and Lawson 1980: 11-46).

Due to the fact that accounting principles and valuations and the level of standardizing items reported on the financial statement differed considerably between firms until the 1970's cash-flow analysis became quite common in Finland.

2.1. The selected traditional financial ratios

The small number of firms in the sample limited the number of ratios examined (see 3.1.1.). Financial ratio analysis is divided into four separate categories according to the needs of users (page 2). Investors are especially interested in profitability ratios, lenders in both short- and long-term solvency ratios and all users are interested in turnover ratios, because they may well be the hallmark of efficient management. This classification is oriented to the needs of most strategic users of information, particularly profitability and solvency ratios. Profitability and solvency are also the most essential theoretical variables in modern finance theories and this classification is quite common (see Lev 1974: 12, Foster 1978: 28, Courtis 1978: 373 and Tamari 1978: 22-44). According to arguments presented above the traditional financial ratios to be chosen in this study are profitability and solvency ratios.

Table 1. The selected ratios.

Ratio	Symbol
Net Income + Interest / Total Assets	x_1
Earnings Before Interest / Total Capital	x_2
Net Income / Equity	x_3
Operating Margin (Fund from Operations) / Sales	x_4
Net Income / Sales	x_5
Total Debt / Shareholders' Equity	x_6
Current Assets / Current Liabilities	x_7
Net Working Capital / Sales	x_8
Financial Assets / Current Liabilities	x_9
Trade Credit / Purchases	x_{10}

Ratios x_1 - x_5 are, a priori, the measures of the firms profitability and they are in common use in ratio analysis (see e.g. Foster 1978: 28-60, Johnson 1979: 1037-1038, and Gombola and Ketz 1983: 108). Ratio x_6 is - a priori - the measure of long-term solvency and ratios x_7 - x_{10} the measures of short-term solvency. Ratio x_{10} - Trade Credit/Purchases - is not common. It is - a priori - supposed that the firm which has limited ability to meet its current obligations wants to use more trade credit than the firm which has good short-term solvency. On the other hand, this ratio is also connected to the theoretical concept profitability, because the use of trade credit is as a rule expensive.

2.2. The selected cash-flow ratios

The first step in the cash-based approach is to compute the cash-flows of the enterprises (cf. Artto 1982: 754-755).

Sales (cash receipts)
- Variable and fixed (short-term) cash disbursements
- Extra expences

Operating cash margin I
+ Extra income

Operating cash margin II
- Profit distribution (interest, taxes and dividends)

Net cash margin
- Net investments

Financing margin
+/- Changes in equity and liabilities
+/- Changes in liquid assets (including cash)

= 0

The different margins presented above are used in the evaluation of profitability ratios (also see cash-flow measures by Lawson 1980, Artto 1982 and Gombola and Ketz 1983). The cash-flow ratios selected for this study are presented in Table 2.

Table 2. The selected cash-flow ratios.

Ratio	Symbol
Operating Cash Margin I / Sales (cash receipts)	x_{11}
Operating Cash Margin II / Sales (cash receipts)	x_{12}
Net Cash Margin / Sales (cash receipts)	x_{13}
Operating Cash Margin II / Total Assets	x_{14}

All presented cash-flow ratios are used to measure the profitability of Finnish firms (see Kettunen, Mäkinen and Neilimo 1979: 96-105 and Aho 1980: 416).

3. DATA AND METHODOLOGY

3.1. Data and empirical variables

The firms included in the study had to satisfy the following conditions:

- they had to be big Finnish industrial firms and quoted on the Helsinki Stock exchange
- the level of standardizing items reported on the financial statements had to be high enough between the firms in the sample and in the same firm during the period examined (1972-1981).

The total number of industrial firms quoted on the Helsinki Stock exchange during the years 1972-1981 was about 30. Rules concerning accounting information of the firms were quite liberal at the beginning of the period to be examined. Therefore the final sample of the firms examined was 19 firms in different industries, see Yli-Olli (1983: 30).

The whole period is divided into two sub-periods: sub-period 1 includes the years 1972-1976 and sub-period 2 the years 1977-1981. During the first sub-period there prevaled a depression and during the latter period a boom in the Finnish economy. Using these periods it is possible to measure the medium-term stability in the structure of financial patterns and to analyze the effect of various cyclical conditions on this structure. So information collected from financial statements of the firms consists of three data matrices, sub-period 1972-1976, sub-period 1977-1981 and the whole period 1972-1981.

The variables in the data matrix of any period (sub-period 1, sub-period 2, the whole period) are cumulative (or weighted) averages of the yearly values of the selected financial ratios over that period. Or more explicitely, let

$$(3.1) \qquad r_t = a_t/b_t\,, \quad t = 1,2,\dots, T$$

be the ratio to be considered (the period consists of the years $1,2,\dots, T$). By the cumulative average $\bar{r}_w$ of the ratio r we mean the expression

(3.2.) $$\bar{r}_w = (\sum_{t=1}^{T} a_t) \; / \; (\sum_{t=1}^{T} b_t) .$$

It is easy to see that expression (3.2) can also be interpreted as a weighted average of the yearly values $r_1, r_2, \ldots, r_T$, i.e. $\bar{r}_w$ can be expressed in the form

(3.3) $$\bar{r}_w = \sum_{t=1}^{T} w_t \, r_t ,$$

where the weights w_t, $t = 1,2,\ldots, T$, satisfy

(3.4) $$0 \leq w_t \leq 1 , \quad \sum_{t=1}^{T} w_t = 1 .$$

To show the consistency of (3.2) and (3.3) we write

(3.5) $$\begin{aligned} \bar{r}_w &= (\sum_{t=1}^{T} a_t) \; / \; (\sum_{t=1}^{T} b_t) \\ &= (\sum_{t=1}^{T} b_t r_t) \; / \; (\sum_{t=1}^{T} b_t) \\ &= \sum_{t=1}^{T} (b_t / \sum_{t=1}^{T} b_t) \, r_t . \end{aligned}$$

The last equation in (3.5) shows that (3.2) really can be written in the form (3.3) if we in (3.3) only set

(3.6) $$w_t = b_t / \sum_{t=1}^{T} b_t , \quad t = 1,2,\ldots, T .$$

This means that the cumulative average $\bar{r}_w$ can also be interpreted as a weighted average of the values of the ratio r, the weights for different years being proportional to denominators of this ratio. As the denominator of any financial ratio is typically a size variable (sales, equity etc.), the result (3.5) implies that in computing the average value of the ratio for a period, the different yearly values of this ratio are weighted according to "the size" of the firm in the respective years. That's why this operation has an eliminating effect on possible extreme yearly values of the ratios. It also smoothens the strong irregular variation in these ratios (see the problem of outliers in ratio analysis e.g. Lev and Sunder 1979: 207-208 and Frecka and Hopwood 1983: 115-128).

3.2. Statistical methods

3.2.1. Factor analysis

The first specific purpose of the study is (cf. Chapter 1) to develop from a set of 14 financial ratios (of the medium-term averages of these ratios in fact) classification patterns for the ratios in much a lower dimension than the original measurements have been made. This is a typical problem to be handled via multivariate factor analysis.

The essential purpose of factor analysis is (Johnson and Wichern 1982: 401) to describe the covariance (or correlation) relationships among many variables in terms of a few underlying, but unobservable random quantities called factors. The factor model may be motivated by the following argument. Suppose variables can be classified by their correlations. That is, all variables within a particular class are highly correlated among themselves but have relatively small correlations with variables in a different class. It is conceivable that each class of variables represents a single underlying construct or latent variable, factor, that is responsible for the observed correlations. For example, high correlations between current ratio, net working capital ratio and quick ratio might suggest an underlying "short-term solvency" factor.

The aim of factor analysis thus is to reduce the space of correlated variables into a factor space of lower dimensionality. This reduction is done in such a way as to retain as much of the original information (the total variance of the original variables) as possible.

Let us assume that we have p original variables $x_1, x_2, \ldots, x_p$ with mean values $\mu_1, \mu_2, \ldots, \mu_p$ and variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_p^2$, respectively. The common factor model postulates, that each x_i is linearly dependent upon a few unobservable variables $f_1, f_2, \ldots, f_r$ $(r < p)$, called common factors, and an additional source of variation u_i, called specific factor. The factor analysis model thus is (see e.g. Johnson and Wichern 1982: 402-407)

$$x_i - \mu_i = l_{i1} f_1 + l_{i2} f_2 + \ldots + l_{ir} f_r + u_i, \quad i = 1,2,\ldots,p \tag{3.7}$$

or, in matrix notation

(3.8) $\mathbf{x} - \boldsymbol{\mu} = \mathbf{L}\mathbf{f} + \mathbf{u}$,

where $\mathbf{x}' = (x_1,x_2,...,x_p)$, $\boldsymbol{\mu}' = (\mu_1,\mu_2,...,\mu_p)$, $\mathbf{L} = (l_{ij})_{pxr}$, $\mathbf{f}' = (f_1,f_2,...,f_r)$ and $\mathbf{u}' = (u_1,u_2,...,u_p)$. The coefficient l_{ij} is called the loading of the ith variable on the jth factor, so the matrix $\mathbf{L}$ is the matrix of factor loadings. Note that the ith specific factor u_i is associated only with the ith response x_i (u_i includes measurement error and quantities that are uniquely associated with the ith individual variable x_i). The p deviations $x_i - \mu_i$, $i = 1,2,..,p$, are expressed in terms of $r+p$ random variables $f_1,f_2,...,f_r$, $u_1,u_2,...,u_p$, all of which are unobservable. This distinguishes the factor model from the multivariate regression model in which the independent variables (whose position is now occupied by the f_i's) can be observed.

Factor analysis contains three main phases: factoring, rotation and interpretation. The first phase, factoring, means the estimation of the factor matrix $\mathbf{L}$, i.e. estimation of the number of factors r and the loadings l_{ij}. With so many unobservable quantities, a direct estimation of the factor model from the observations is hopeless. However, with some additional assumptions about the random vectors $\mathbf{f}$ and $\mathbf{u}$, the model (3.8) implies certain covariance relationships, which can be checked, see e.g. Johnson and Wichern (1982: 403-404). One output of these assumptions is that in the case of orthogonal (uncorrelated) factors the variance of the ith variable x_i can be expressed in the form

(3.9) $\mathrm{var}(x_i) = h_i^2 + \psi_i$,

where

(3.10) $h_i^2 = l_{i1}^2 + l_{i2}^2 + ... + l_{ir}^2$

is the ith communality, that portion of $\mathrm{var}(x_i)$ explained by the r common factors, and ψ_i is the portion of $\mathrm{var}(x_i)$ due to the specific factor (the "unexplained" variance). Further we have

(3.11) $\mathrm{cov}(x_i,f_j) = l_{ij}$, i.e. the loadings l_{ij} give the covariance structure between the variables and factors. In the case of standardized variables, instead of the deviations about the mean used in (3.7), the covariance structure becomes even more simple: the communality and the specific variance in (3.9) add to unity and the loadings l_{ij} give correlation coefficients between the variables and the factors (i.e. all these measures are scaled between 0 and 1 in absolute values).

If the factors f_j are allowed to be correlated, we have the oblique factor model. The oblique model presents some additional estimation difficulties which will not be discussed here. For the oblique factor model see Harman (1967 : ch. 13 and ch. 15).

The main estimation methods in the phase of initial factor extraction are principal component method, principal factor method and maximum likelihood method (see e.g. Johnson and Wichern 1982: 407-420). In this study the initial factor matrix is estimated by the principal component method. In estimating the dimension of the factor space there exists no unambiguous criterion. Several procedures for determining how many common factors to extract have been suggested. In this study the number of factors extracted was in the first hand determined by requiring that the eigenvalues associated with each factor exceed 1. This criterion was replenished by Cattell's scree test and by interpretative aspects.

There is always some inherent ambiquity associated with the factor model (3.8). For, if we have a nonsingular $r \times r$ matrix $\mathbf{T}$, and we denote

(3.12) $$\mathbf{L}^* = \mathbf{LT} \quad \text{and} \quad \mathbf{f}^* = \mathbf{T}^{-1}\mathbf{f},$$

we can write

(3.13) $$\begin{aligned} \mathbf{x} - \boldsymbol{\mu} &= \mathbf{Lf} + \mathbf{u} \\ &= \mathbf{LTT}^{-1}\mathbf{f} + \mathbf{u} \\ &= \mathbf{L}^*\mathbf{f}^* + \mathbf{u}. \end{aligned}$$

From (3.13) we can see that factor loadings $\mathbf{L}$ (and factors $\mathbf{f}$) are determined only up to a nonsingular matrix **T**. Equations (3.12) represent the rotation phase of factor analysis. The initial loading matrix is rotated (multiplied by a nonsingular matrix), where the rotation is determined by some "simple-structure" or "ease-of-interpretation" criterion. The aim of the rotation thus is to provide a clearer resolution of the underlying factors.

If matrix T is orthogonal (i.e. $T^{-1} = T'$) we have an orthogonal rotation. In this case the loadings L and $L^* = LT$ both give the same covariance representation for the original data. The communalities, given by the diagonal elements of $LL' = (L^*)(L^*)'$, are also unaffected by the choice of **T**. The results in this study are mainly based on Kaiser's Varimax rotation which is an orthogonal rotation method. Some additional or interpretatively supporting results are obtained via Promax rotation. This is a nonorthogonal (an oblique) rotation where the resulting factors are allowed to be correlated.

Factor analysis contains several elements which have no unique solution (how many factors to extract?, how to choose the rotation matrix T?, etc.). In applications it is therefore important that these ambiguous quantities are fixed as to produce results which are based on some relevant theory and have meaningful empirical interpretations. Generally speaking, the interpretative phase is a proper part of the entire factoring process.

3.2.2. Transformation analysis

The second specific purpose of this study is (cf. Chapter 1) to measure and model the medium-term stability exhibited by the factor analytical classification patterns of the two sub-periods. The degree of stability (both dynamic and cross-sectional) in factor patterns has been traditionally measured with correlations coefficients (e.g. Pinches, Mingo and Caruthers 1973, Aho 1980) or with congruency coefficients (e.g. Johnson 1978, Gombola and Ketz 1983). Both of these measures give an index for the similarity of two different factor solutions in terms of the pattern of correlations among factor loadings across all variables in the reduced factor space. For the dissimilar part of these factor solutions these indices are, however, unable to describe and explain the reason for the non-invariant part prevailing in these factor solutions.

Recently, Yli-Olli (1983) has introduced the use of transformation analysis for determining the degree and nature of medium-term stability exhibited by the factor patterns of the financial ratios. Transformation analysis is a multivariate statistical method which has been developed to compare factor solutions obtained for diffrent populations or groups. This technique was initiated by Ahmavaara (1954) and further developed by Ahmavaara (1963 and 1966), Ahmavaara and Nordenstreng (1970) and Mustonen (1966). The most applications of transformation analysis exist in the area of Finnish political and sociological research (e.g. Markkanen 1964, Nordenstreng 1968). Originally transformation analysis was developed to compare factor solutions between two (or more) different groups of objects, Yli-Olli (1983) has used the technique to compare two different factor solutions among the same group of objects, the two factor solutions being based on measurements made at different times (sub-period 1, sub-period 2). The latter use of transformation analysis means its use for measuring and modelling the medium-term stability of the financial factor patterns. In the following we sketch out the general idea behind transformation analysis (for a more detailed discussion, see e.g. Ahmavaara 1966, Mustonen 1966).

Let's assume that we have two groups of observations G_1 and G_2 (two different groups of objects or one group measured at two different times) with the same variables, both by number and by content. Let L_1 and L_2 be the factor matrices (cf. equation (3.8)) for G_1 and G_2, respectively. Let's further assume that the factor models used in deriving L_1 and L_2 are both orthogonal and have the same dimension (these assumptions are not, in general, necessary, but the computer program package, HYLPS, containing transformation analysis is restricted to orthogonal factor solutions with the same number of factors). So we may assume that both L_1 and L_2 are pxr-matrices.

If there exists invariance between the two factor structures, there exists a non-singular rxr-matrix T_{12} such that equation

$$(3.14) \qquad L_2 = L_1 T_{12}$$

holds. Matrix T_{12} is called the transformation matrix (between L_1 and L_2, or in direction $G_1 \to G_2$). If equation (3.14) holds exactly, it means that the factor structures in groups G_1 and G_2 are, up to a linear transformation, invariant, all the variables have the same empirical meaning in different groups. Depending on the type of the transformation matrix T_{12}, the formation of the factors from the variables and thereby the interpretation of the factors either is preserved (T_{12} is the identity matrix I) or it changes (T_{12} has also non-zero off-diagonal elements).

In practice, situation (3.14) will not be reached, but, after matrix T_{12} has been estimated, we have $L_2 \neq L_1 T_{12}$. The goodness of fit criterion for the model (3.14) may be based on the residual matrix

$$E_{12} = L_1 T_{12} - L_2 . \tag{3.15}$$

Non-zero elements in E_{12} mean that the empirical meaning of the variables in question has changed. This is called abnormal transformation.

To avoid confusion it is worth to note here that in the case of two factor solutions L and L^* which have been obtained from the same set of observations, we always have an exact solution T for the equation (3.14). In fact, this is the problem of rotation considered in Section 3.2.1. (cf. equations (3.12) and (3.13)). Now the situation is quite different when two separate sets of observations are considered.

The main problem in transformation analysis is the estimation of the matrix T_{12}. The estimation methods are in general based on the minimization of the sum of squares of the residuals e_{ij} (the elements of the residual matrix E_{12}). This is the usual method of least squares. The problem is to minimize

$$\begin{aligned} \|E_{12}\| &= \|L_1 T_{12} - L_2\| \\ &= \text{trace}\,((L_1 T_{12} - L_2)(L_1 T_{12} - L_2)') . \end{aligned} \tag{3.16}$$

Depending on additional constraints set for the matrix T_{12}, we have three different estimation methods (three transformation analysis models).

1. If there are no constraints for T_{12} in minimizing (3.16) we have the naive model. This is the original solution for the transformation

problem presented by Ahmavaara (1954). The naive model has been now superseded by the other models in the applications.

2. If the transformation matrix has to obey the transitivity property $T_{kl}T_{lm} = T_{km}$, we get the relativistic model. The relativistic model has been also developed by Ahmavaara (1963, 1966). The relativistic transformation analysis possesses some general theoretical advantages, wherefore it is prefered by some authors (Ahmavaara 1966, Ahmavaara and Nordenstreng 1970, Markkanen 1964, Nordenstreng 1968).

3. If the transformation matrix T_{12} is required to be orthogonal, i.e. $T_{12}^{-1} = T'_{12}$, we have the symmetric model. In this case we obtain the following symmetry property: $E_{12}E'_{12} = E_{21}E'_{21}$, i.e. the covariance matrix of the residuals is independent of the direction of the transformation. The symmetric model was introduced by Mustonen (1966). The results obtained via this method are in general easy to be interpreted wherefore the symmetric version of transformation analysis has been used in most applications. It is worth to note that the problem of estimating T_{12} in (3.14) under the same assumptions as those used in symmetric transformation analysis has been independently solved by Schönemann (1966). Schönemann has considered the problem, however, purely from the point of view of mathematics, without any explicit connections to factor or transformation analysis.

In this study the symmetric transformation analysis will be used. The results have the following properties (Mustonen 1966: 8):

1. Transformation matrices are orthogonal.
2. Abnormal transformation (measured by residuals) is independent of the direction between the groups.
3. Orthogonal rotations in the original factor spaces have no effect on the results.

Further, in symmetric transformation analysis the abnormal transformation (the total residual) E_{12} may be expressed in form

(3.17) $$\| E_{12} \| = \sum_{i=1}^{p} \sum_{j=1}^{r} e_{ij}^2 = \sum_{i=1}^{p} t_i^2$$

or

$$(3.18) \qquad \|E_{12}\| = \sum_{j=1}^{r}\sum_{i=1}^{p} e_{ij}^2 = \sum_{j=1}^{r} s_j^2$$

where $t_i^2 = \sum_{j=1}^{r} e_{ij}^2$ and $s_j^2 = \sum_{i=1}^{p} e_{ij}^2$ are those portions of abnormal transformation due to the ith variable x_i and jth factor f_j, respectively.

In this study the two groups to be compared via transformation analysis consist of the same objects (the 19 firms), but the groups are formed by observations made at different times. Transformation analysis is thus used as a technique to describe and measure the longitudinal stability existing in the observations. Invariance between the factor patterns of two successive medium-term periods (sub-period 1, sub-period 2) means at least medium-term stability of the underlying factor structure, whereas non-invariance indicates the existence of instable elements in the factor patterns.

Transformation analysis possesses several advantages in analysing the stability of the factor patterns when compared for example with correlation or congruency analysis. With correlation and congruency coefficients one can only measure the degree of similarity of two factor solutions (correlations or congruencies among factor loadings across the variables in the factor space). This is also possible via transformation analysis (coefficients of coincidence on the main diagonal of the transformation matrix). In addition to this we obtain a regression type model for shifting of variables from one factor to another (normal or explained transformation). This is reveait by the non-zero off-diagonal elements in the transformation matrix and indicates interpretatively changes for the factors in question. And at last, large elements in the residual matrix, if any, indicate abnormal or unexplained transformation between the two factor solutions. This means that the empirical content of the corresponding variables has changed. Further, this abnormal transformation can be appointed to separate variables or to separate factors (cf. (3.17) and (3.18)).

4. THE ANALYSIS OF EMPIRICAL FINANCIAL RATIOS

Correlation coefficients between ratios for the first sub-period 1972-1976 are presented in Table 3. The comparable coefficients for the second sub-period 1977-1981 are given in Table 4. The computational method of the variables as medium-term weighted averages was presented in Section 3.1.

Table 3. Correlation matrix for sub-period 1.

Variable	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}
x_1	1.00													
x_2	.92	1.00												
x_3	.89	.95	1.00											
x_4	.02	-.01	.03	1.00										
x_5	.92	.90	.88	-.14	1.00									
x_6	-.56	-.66	-.69	-.21	-.47	1.00								
x_7	.53	.54	.50	-.21	.56	-.45	1.00							
x_8	.26	.26	.41	-.26	.42	-.07	.57	1.00						
x_9	.55	.49	.58	-.42	.63	-.15	.58	.85	1.00					
x_{10}	.38	.52	.46	-.55	.49	-.29	.53	.44	.48	1.00				
x_{11}	-.44	-.38	-.42	.63	-.47	.15	-.33	-.36	-.51	-.53	1.00			
x_{12}	-.43	-.30	-.29	.53	-.45	.13	-.41	-.21	-.33	-.45	.90	1.00		
x_{13}	-.20	-.02	-.06	.30	-.13	.11	-.24	-.16	-.19	-.20	.77	.87	1.00	
x_{14}	-.12	.07	-.10	.14	-.07	.19	-.20	-.34	-.26	-.16	.60	.64	.82	1.00

Table 4. Correlation matrix for sub-period 2.

Variable	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}
x_1	1.00													
x_2	.90	1.00												
x_3	.83	.85	1.00											
x_4	.06	.17	-.05	1.00										
x_5	.96	.92	.91	.01	1.00									
x_6	-.58	-.62	-.82	-.01	-.63	1.00								
x_7	.71	.79	.63	.09	.66	-.59	1.00							
x_8	.40	.44	.49	-.38	.48	-.40	.63	1.00						
x_9	.70	.64	.62	-.23	.70	-.46	.76	.88	1.00					
x_{10}	.67	.56	.60	-.14	.60	-.47	.39	.22	.43	1.00				
x_{11}	-.24	-.11	-.25	.84	-.31	.07	-.09	-.61	-.52	-.18	1.00			
x_{12}	-.36	-.09	-.25	.80	-.35	.08	-.03	-.35	-.41	-.34	.89	1.00		
x_{13}	-.07	.21	.06	.76	-.04	-.13	.15	-.32	-.32	-.08	.87	.90	1.00	
x_{14}	.18	.44	.19	.66	.17	-.05	.28	-.28	-.16	.07	.70	.70	.87	1.00

Tables 3 and 4 show that correlation coefficients among the selected traditional profitability ratios are high with one exception. The correlation coefficients between Operating Margin/Sales (variable x_4) and other traditional profitability ratios are very low. Correlation coefficients among cash-flow variables (x_{11} - x_{14}) are also moderately high but correlations between the traditional profitability ratios and the cash-flow ratios are low, especially during sub-period 2.

An interesting feature is that the correlation coefficients between the ratio Operating Margin/Sales (variable x_4) and cash-flow variables are high in the boom period but moderately low in the depression period. The variables x_4 (Operating Margin/Sales) and x_{11} (Operating Cash Margin I/Sales (cash receipts)) should be very close to each other in the long term. Because the latter is a cash based ratio they differ in the short and medium-term. The changes in trade accounts and notes receivables, in inventories, in prepaid expenses, in trade accounts and notes payable and in accrued liabilities cause differences between the two ratios mentioned above. Changes in those items seem to be larger in the boom period compared to the depression period. The results presented in Tables 3 and 4 indicate that ratio Operating Margin (Funds from Operations)/Sales does not measure the same dimension in the firm's performance during different cyclical conditions.

4.1. Financial ratio patterns for industrial firms

In the previous studies classification patterns of financial ratios have usually been developed via factor analysis by using some ortogonal rotation method. However, it is difficult to find theoretical arguments why the factors to be extracted should be uncorrelated. Therefore the results should at least be verified with some non-orthogonal (oblique) rotation method.

In this study the four factors found by using Kaiser's orthogonal Varimax rotation for sub-period 1 are presented in Table 5 and for sub-period 2 in Table 6. The respective nonorthogonal results obtained via oblique Promax rotation are presented in Tables 7 and 8.

Table 5. Varimax-rotated factor matrix for sub-period 1.

Variable	Factor I	Factor II	Factor III	Factor IV	Communality h_i^2
x_1	**.918**	-.142	.162	.095	.899
x_2	**.967**	.045	.135	.171	.985
x_3	**.922**	-.047	.280	.045	.934
x_4	.146	.198	-.183	**-.901**	.907
x_5	**.856**	-.066	.283	.256	.883
x_6	**-.764**	.155	-.010	.275	.685
x_7	.481	-.171	**.567**	.140	.603
x_8	.110	-.121	**.958**	.106	.957
x_9	.335	-.113	**.819**	.302	.889
x_{10}	.386	-.107	.323	**.628**	.661
x_{11}	-.282	**.714**	-.166	-.554	.925
x_{12}	-.262	**.809**	-.027	-.475	.951
x_{13}	-.027	**.967**	-.023	-.141	.957
x_{14}	.032	**.910**	-.258	.127	.914
Eigenvalue	6.495	2.955	1.607	1.092	
Cumulative proportion of total variance	.464	.675	.790	.868	

Table 5 shows that the four factor solution accounts for 86.8 per cent of the total variance in the original fourteen financial ratios for sub-period 1 (1972-1976). The respective value for sub-period 2 (1977-1981) is 90.6 per cent.

The first factor accounts for 46.4 per cent for sub-period 1 and 46.6 per cent for sub-period 2 of their respective total variances. The interpretation of the first factor is easy and unambiguous. The financial ratios which achieve the highest factor loadings on this factor are all traditional financial ratios of **profitability**, i.e. x_1 (Net Income + Interest/Total Assets), x_2 (Earnings Before Interest/Total Capital), x_3 (Net Income/Equity) and x_5 (Net Income/Sales). The loadings of the variable x_4 (Operating Margin (Funds from Operations)/Sales) on the first factor are very low for both of the sub-periods. This variable does not measure the profitability of the firm like was supposed a priori.

Table 6. Varimax-rotated factor matrix for sub-period 2.

Variable	Factor I	Factor II	Factor III	Factor IV	Communality h_i^2
x_1	**.921**	-.028	-.333	-.018	.960
x_2	**.831**	.211	-.449	-.088	.947
x_3	**.784**	-.010	-.330	-.445	.924
x_4	.049	**.887**	.053	.099	.802
x_5	**.882**	-.049	-.377	-.105	.936
x_6	-.492	-.045	.271	**.809**	.973
x_7	.491	.184	**-.745**	-.154	.855
x_8	.134	-.350	**-.875**	-.183	.941
x_9	.447	-.277	**-.809**	-.038	.934
x_{10}	**.796**	-.153	.061	-.179	.694
x_{11}	-.149	**.905**	.306	-.044	.939
x_{12}	-.337	**.921**	.026	-.079	.969
x_{13}	.008	**.958**	.053	-.153	.944
x_{14}	.285	**.879**	.005	.132	.872
Eigenvalue	6.520	4.371	1.118	.682	
Cumulative proportion of total variance	.466	.778	.858	.906	

The second factor explains 21.1 per cent for sub-period 1 and 31.2 per cent for sub-period 2 of the total variance among the original ratios. This factor can be interpreted as a factor of **cash-flow from operations.** The cash-flow ratios thus load on a separate and distinct factor from the profitability factor in the short (see Gombola and Ketz 1983: 105-114) and also in the medium-term. The variable x_4 (Operating Margin/Sales) has a high loading on the second factor during the latter boom period. The interpretation of this phenomen is that during the boom period variable x_4 (Operating Margin/Sales) and cash-flow variables x_{11} - x_{14} measure the same characteristic of the firm's performance. However, the items mentioned on page 18 (changes in trade accounts and notes etc.) cause remarkable differences between Operating Margin to Sales and cash-flow variables during the recession period. In this case Operating Margin/Sales has a high loading on the fourth factor (together with variable x_{10}). The result suggests that this ratio does not measure the same dimension of the firm's performance during different cyclical conditions, i.e. the stability of this ratio is very weak.

No fault can be found with extraction of the third factor and it is made according to the criteria presented in Section 3.2.1. The interpretation of this factor is also easy. The factor describes the **short-term solvency or liquidity** of the firm (in sub-period 2 the negative of this characteristic).

The extraction of the fourth factor is not unambiguous. The eigenvalue associated with this factor exeeds one for the first sub-period but for the second sub-period its value is less than one. The inclusion of the fourth factor also for sub-period 2 is justified by interpretative aspects. During the second or boom period the fourth factor can be interpreted as **an indicator of long-term solvency** or as **an indicator of debt structure** because Total Debt/Shareholders Equity (variable x_6) has the highest loading on this factor. However, the high loading of this variable shifts to the first factor (profitability) during the recession period (sub-period 1). At the same time this loading gets a negative sign. It indicates that capital structure and profitability are not independent of each other. This also means that at least during the recession period the increased use of debt financing has caused a negative effect on the profitability of Finnish industrial firms. The biggest Finnish industrial firms have had too much debt during the period examined and the results presented earlier (see Yli-Olli 1981: 444) support this conclusion. The results of the oblique factor model will show what the leverage effect was like during the boom period.

The interpretation of the fourth factor in sub-period 1 is cumbersome. The ratios x_4 (Operating Margin/Sales), x_{10} (Trade Credit/Purchases) and x_{11} (Operating Cash Margin I/Sales), which is the same measure as x_4 accounted only on a cash basis, have the highest loadings on this factor. The ratios x_4 and x_{10} indicate the **structure of expenditure** of the firm and this is the name of the fourth factor for sub-period 1.

Tables 7 and 8 show the nonorthogonal results of the four factor solution for sub-periods 1 and 2, respectively, and Tables 9 and 10 present correlations between those oblique factors. These results given in Tables 7-10 support those presented in Tables 5 and 6. However, they show two additional interesting features.

Table 7. Promax-rotated factor matrix (pattern loadings) for sub-period 1.

Variable	Factor I	II	III	IV
x_1	**1.107**	-.328	.489	.292
x_2	**1.144**	-.141	.461	.326
x_3	**1.121**	-.234	.603	.243
x_4	-.081	.344	-.352	**-1.060**
x_5	**1.091**	-.275	.627	.470
x_6	**-.827**	.245	-.207	.163
x_7	.739	-.349	**.834**	.394
x_8	.415	-.283	**1.158**	.394
x_9	.666	-.319	**1.100**	.599
x_{10}	.652	-.307	.607	**.839**
x_{11}	-.560	**.913**	-.467	-.864
x_{12}	-.499	**.984**	-.302	-.766
x_{13}	-.193	**1.069**	-.190	-.414
x_{14}	-.130	**.989**	-.385	-.155

Firstly, the profitability factor and short-term solvency factor seem to be nonorthogonal. During sub-period 1 the correlation between the profitability factor and short-term solvency factor is -.395. During sub-period 2 this correlation is -.452 (the positive value .452 in Table 10 is the correlation between profitability and the negative of short-term solvency). This negative correlation may be interpreted as follows: the maintaining of short-term solvency contains elements which have a reducing effect on the profitability of the firm.

Nonorthogonal results confirm the conclusion that the Finnish firms have had too much debt during the period examined. During the second sub-period, i.e. during the boom period, the correlation between the profitability factor and debt structure is about .30. That indicates a small positive leverage effect. However, during the first sub-period - or depression period - the variable x_6 (Total Debt/Shareholders Equity) has a very high negative loading on the profitability factor. This confirms the negative leverage effect, and the conclusion is that the Finnish firms have been too much indepted during the period examined in this study.

Table 8. Promax-rotated factor matrix (pattern loadings) for sub-period 2.

Variable	Factor I	II	III	IV
x_1	**1.201**	-.057	-.682	-.387
x_2	**1.153**	.184	-.759	-.439
x_3	**1.091**	-.032	-.690	-.788
x_4	.061	**.906**	.197	.146
x_5	**1.181**	-.078	-.736	-.475
x_6	-.770	-.035	.569	**1.058**
x_7	.860	.149	**-1.005**	-.450
x_8	.466	-.397	**-1.129**	-.416
x_9	.799	-.325	**-1.121**	-.351
x_{10}	**.933**	-.164	-.229	-.449
x_{11}	-.241	**.941**	.535	.109
x_{12}	-.362	**.947**	.270	.084
x_{13}	.045	**.982**	.188	-.100
x_{14}	.352	**.892**	.071	.094

Table 9. Correlations between the factors within the oblique factor solution (sub-period 1).

Factor	I	II	III	IV
I	1.000	.144	**-.395**	-.119
II	.144	1.000	.072	.288
III	-.395	.072	1.000	-.249
IV	-.119	.288	-.249	1.000

Table 10. Correlations between the factors within the oblique factor solution (sub-period 2).

Factor	I	II	III	IV
I	1.000	-.116	**.452**	**.298**
II	-.116	1.000	-.199	-.027
III	.452	-.199	1.000	-.154
IV	.298	-.027	-.154	1.000

4.2. The medium-term stability of financial factor patterns

Table 11 presents a transformation matrix between the factors for sub-period 1 and sub-period 2. The transform-analyzed factor matrices were given in Tables 5 and 6. The results show that the three first factors **profitability, cash-flow from operations** and **short-term solvency** are quite stable during different cyclical conditions. This conclusion is based on the coefficients of coincidence on the main diagonal of the transformation matrix (Table 11). The numerical values of the coefficients between those factors are .885, .899 and .992. (The negative value of the coefficient of coincidence for the third factor is only due to the fact that the third, short-term solvency factor is equipped with opposite signs in different solutions; the third factor of sub-period 2 is, in fact, the negative of this solvency.) These relationships are schematically presented in Figure 1 with horizontal lines connecting the respective factors for both sub-periods. These coefficients of coincidence may be compared with other indices of factor similarity. The coefficients of congruency are .957, .907, .905 and .004 for the first, second, third and fourth factor, respectively, and the respective coefficients of correlation are .952, .880, .872 and .005. For the first three factors there is a good agreement between these three measures of factor similarity, whereas for the fourth factor only the coefficient of coincidence (.793) obtained via transformation analysis seems to possess the ability to reveal the stable elements existing in this factor.

Table 11. Transformation matrix between the factors in sub-period 1 (rows) and sub-period 2 (columns).

Factor	I	II	III	IV
I	**.885**	.112	-.107	**.440**
II	.088	**.899**	-.069	**-.423**
III	-.105	-.072	**-.992**	-.012
IV	**.445**	**-.417**	-.007	**-.793**

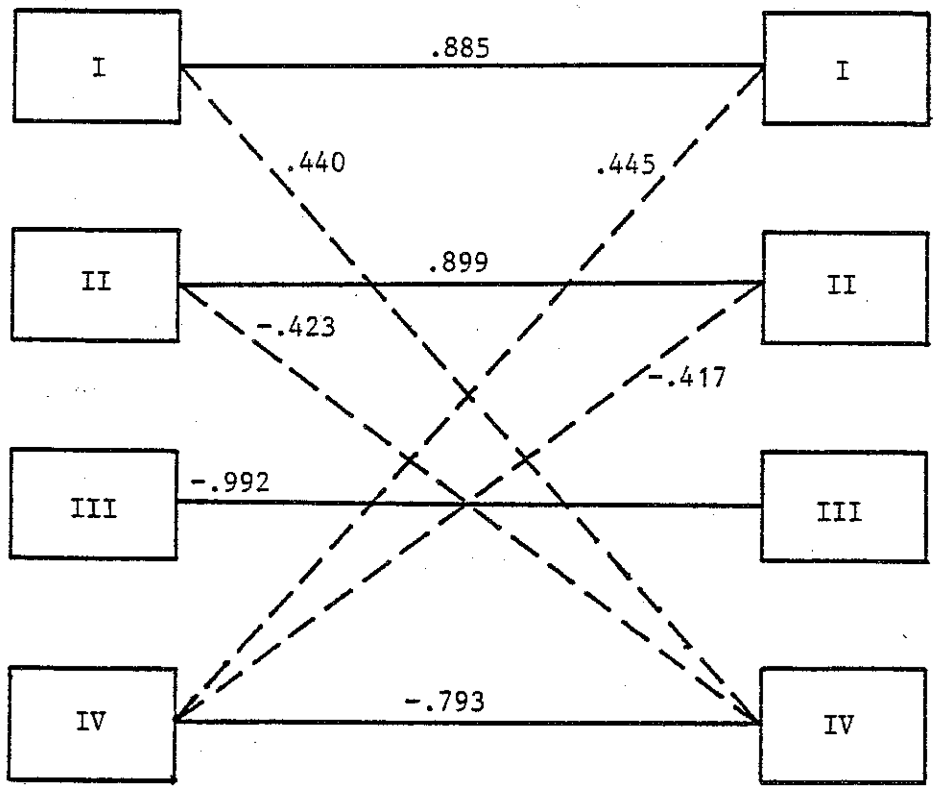

Figure 1. Main transformation connections between the factors in sub-period 1 and sub-period 2.

The transformation matrix in Table 11 indicates the transferance of some variables from the first factor to the fourth factor when the two sub-periods are compared (the value of coefficient of transformation is $t_{14} = .440$). The reason for this change is variable x_6, Total Debt/Shareholders Equity. This variable has a high loading on the profitability factor for sub-period 1 but for sub-period 2 it creates a factor of its own. Another variable that changes the factor is variable x_{10}, Trade Credit/Purchases. It transfers from factor IV to factor I (in sub-period 1 vs. sub-period 2).

In Figure 1 these changes are symbolized by the broken line from the first factor (sub-period 1) to the fourth factor (sub-period 2). Other coefficients of transformation are quite low on the first factor (.112 and -.107). Those coefficients tell that no essential shifting of the variables between the first and second and first and third factor takes place during the two sub-periods examined.

Table 11 further shows that the coefficient of transformation between the second and fourth factor is quite high (-.423). It is caused by the variable x_4 (Operating Margin/Sales) which has a high loading on the fourth factor in sub-period 1 but a high loading on the second factor in sub-period 2.

The third factor is very stable during different cyclical conditions, because all coefficients of transformation between it and other factors are very low.

The fourth factor is very unstable. The transformation coefficient between the fourth and first factor is .455 and between the fourth and second factor -.417. The variables which transfer from one factor to another are x_4 (Operating Margin/Sales), x_6 (Total Debt/Shareholders Equity) and x_{10} (Trade Credit/Shareholders Equity).

Table 12 presents the residual matrix for sub-period 1 (matrix E_{21}). The comparable matrix for sub-period 2 is given in Table 13 (matrix E_{12}). Zero elements in residual matrices show that the variables in question measure the same characteristic of the firms' performance during different cyclical conditions (the changes in the factor matrix between the two sub-periods can for these variables be explained by the linear transformation model, or there exists invariance in the factor structure as far as these variables are concerned).

Table 12. Residual matrix for sub-period 1 (matrix E_{21}).

Variable	Factor I	II	III	IV	Communality h_i^2	Abnormal transformation t_i^2
x_1	-.063	.214	.074	.315	.899	0.155
x_2	-.121	.212	.207	.044	.985	0.104
x_3	.002	-.058	-.040	-.042	.934	0.007
x_4	-.054	**.642**	.062	**.632**	.907	**0.818**
x_5	.006	.082	.001	.077	.883	0.013
x_6	-.061	.083	-.194	.164	.685	0.075
x_7	.121	.368	.106	-.115	.603	0.175
x_8	.144	-.198	-.081	-.040	.957	0.068
x_9	.133	-.057	-.045	-.012	.889	0.023
x_{10}	**.373**	-.040	**-.459**	**-.353**	.661	**0.476**
x_{11}	.238	.047	-.188	.074	.925	0.100
x_{12}	.099	-.046	-.031	-.121	.951	0.027
x_{13}	.203	-.173	-.102	**-.375**	.957	**0.222**
x_{14}	.259	-.039	.161	-.262	.914	0.163
Abnormal transformation s_j^2	.392	.735	.395	**.904**	-	2.426

Non-zero elements in residual matrices, on the other hand, mean that the empirical meaning of the variables in question has changed (the unexplained or non-invariant part of the changes). The residual matrices for sub-period 1 (matrix E_{21}) and for sub-period 2 (matrix E_{12}) show that there are two variables, x_4 (Operating Margin/Sales) and x_{10} (Trade Credit/Purchases) with high abnormal transformation. Also the variable x_{13} (Net Cash Margin/Sales (cash receipts)) has a moderately high abnormal transformation. The abnormal transformation of the variable x_4 (Operating Margin/Sales) can be designated for the most part to factors II and IV. The abnormal transformation of variables x_{10} (Trade Credit/Purchases) and x_{13} (Net Cash Margin/Sales (cash receipts)) can be designated to factors III and IV, and to factor IV, respectively. The abnormal transformation of factor IV (= s_4^2) is very high compared to the respective values of other factors. The big positive numerical value concerning some variable and some factors in residual matrix for sub-period 1 (Table 12) means that the characteristic which the factor in question measures diminishes when we transfer from sub-period 2 to sub-period 1. So the variable x_{10} (Trade Credit/Purchases) is "a better measure" for profitability during sub-period 2 than during sub-period 1. The interpretation of negative values is opposite. So when we are moving from sub-period 2 to sub-period 1 the feature which measures short-term solvency increases in variable x_{10} (Trade Credit/Purchases).

Table 13. Residual matrix for sub-period 2 (matrix E_{12}).

Variable	Factor I	II	III	IV	Communality h_i^2	Abnormal transformation t_i^2
x_1	-.096	-.049	.084	.368	.960	.154
x_2	.090	-.144	.208	.180	.947	.105
x_3	.018	.032	-.043	-.059	.924	.007
x_4	-.284	**-.304**	.105	**.797**	.802	**.819**
x_5	-.047	-.043	.008	.093	.936	.013
x_6	-.047	-.014	-.192	.189	.973	.075
x_7	-.078	-.385	.142	.012	.855	.174
x_8	-.100	.140	-.079	-.179	.941	.068
x_9	-.112	.029	-.034	-.092	.934	.023
x_{10}	-.217	-.185	**-.421**	**-.466**	.694	**.476**
x_{11}	-.267	-.052	-.157	-.029	.939	.100
x_{12}	-.032	-.022	-.024	-.159	.969	.028
x_{13}	-.008	-.031	-.094	**-.461**	.944	**.222**
x_{14}	-.092	-.091	.183	-.336	.872	.163
Abnormal transformation s_j^2	.260	.333	.373	**1.461**	-	2.426

The results of this chapter should prove to be useful to researchers in developing financial ratios. In utilizing the analysis of financial ratios it is valuable to know the theoretical rationships between the classes of ratios under consideration. After choosing a small subset of ratios to be used, it is important to know the empirical behavior of potential ratios. The results of Tables 11, 12 and 13 show that there are different kinds of ratios. Firstly, Tables 12 and 13 show the communalities of the different ratios obtained via factor analysis. If the communality is very close to one, the four-factor solution explains in practice the variation of the ratio in question. This is a good characteristic for the ratio. A high communality does not tell, however, anything about the stability of the ratio which is a very important feature to the decision makers. In the ideal situation where the coefficients of coincidence have all a value of 1 in the transformation matrix, we would have totally stable ratios in different cyclical conditions. However, if the values of the coefficients of transformation differ from zero it is still possible that we have invariance between the factor patterns. In that case, all elements in residual matrices should be zero. The transformation matrix shows the transferance of the variables from certain factors to others. In this case we have much better information about the stability of financial patterns than e.g. by only using correlation or congruence coefficients. Finally, if the elements in the residual matrix differ from zero, the ratios have such a variation that our model cannot explain them. In practice, ratios with low communality and high abnormal transformation are not useful in the analysis of financial ratios. Such ratios in this study are x_4, x_{10} and maybe x_{13}. It is easy to interprete variables which have low abnormal transformation and which hold on to the same factor, or in other words, the ratio remains stable during different cyclical conditions. Further, the variable is ideal if, in addition, the communality is close to one, i.e. the model explains totally the variation of the ratio. From this point of view, the best ratios are x_1, x_2, x_3, x_5, x_8, x_9, x_{11} and x_{12}.

4.3. Some further analysis and implications

According to the results presented in Sections 4.1. and 4.2. we found some unstable ratios. However, we extracted the four factor solution also for the whole period 1972-1981 (Table 14). The ratios are again weighted averages of the yearly values (see equation (3.2.)). This solution shows that it was necessery to divide the whole period into two separate sub-periods. The interpretation of the factors is now more complicated. The variables x_4 (Operating Margin/Sales), x_6 (Total Debt/Shareholders Equity), x_7 (Current Assets/Current Liabilities) and x_{10} (Trade Credit/Purchases) fall troublesamely into two different factors. In addition, the loadings of some other variables are not so clear-cut as in the solutions for the sub-periods.

However, we can find in the solution given in Table 14 the first three factors profitability, cash-flow from operations and short-term solvency or liquidity. Further these results support the conclusions made about the variable x_6 (Total Debt/Shareholders Equity). This variable has a negative loading on the profitability factor and, on the other hand, it also loads on its own factor. This results in a negative leverage effect. The firms with **low profitability** have had too much debt during the period examined.

Table 14. Varimax-rotated factor matrix for the whole period (years 1972-1981).

Variable	Factor				Communality h_i^2
	I	II	III	IV	
x_1	**.934**	-.137	.143	.032	.914
x_2	**.957**	.043	.152	.117	.956
x_3	**.933**	-.059	.227	.028	.927
x_4	-.065	**.647**	-.212	**-.649**	.890
x_5	**.923**	-.100	.196	.149	.925
x_6	**-.741**	-.096	-.027	**.486**	.797
x_7	**.690**	.101	**.498**	-.036	.736
x_8	.182	-.241	**.935**	.062	.971
x_9	**.501**	-.325	**.758**	.091	.942
x_{10}	**.666**	-.094	.048	**.461**	.668
x_{11}	-.227	**.830**	-.274	-.349	.938
x_{12}	-.338	**.861**	.015	-.318	.958
x_{13}	.074	**.959**	-.108	.006	.939
x_{14}	.348	**.823**	-.224	.303	.942
Eigenvalue	6.655	3.800	1.201	.846	
Cumulative proportion of total variance	.475	.747	.833	.893	

5. SUMMARY AND CONCLUSION

The first specific purpose of this study was to develop empirically based classification patterns of financial ratios where cash-flow ratios are included. Cash-flow was defined in a more proper way than in most of the previous studies, i.e. cash-flow was defined as cash receipts from operations less cash disbursements from operations. The period examined (1972-1981) was divided into two five-years sub-periods. Four factors were found for both sub-periods. The interpretation of the first three factors for both sub-periods was very easy. Those three factors were profitability, cash-flow from operations and short-term solvency. The extraction of the fourth factor was not unambiguous and the interpretation of this factor partly changed when we transferred from sub-period 1 to sub-period 2. Classification patterns were developed via factor analysis using orthogonal Varimax rotation. The results were verified with non-orthogonal Promax rotation.

The second specific purpose of this study was to measure and model the medium-term stability of factor patterns between two sub-periods. In this study those factor patterns were compared via transformation analysis. Compared with correlation or congruency analysis used in previous studies we got a more clear-cut picture about the stability and unstability between the factor patterns. We got coefficients of coincidence which measured more accurately the stability of especially the fourth factor than the coefficients of correlation or concruency. Besides this we got the coefficients of transformation which gave a regression type model for shifting of certains variables from one factor to another. Further, we obtained the residual matrices which indicate abnormal or unexplained transformation between the two factor solutions. However, by using residual matricies it was possible to draw conclusions about how much and to what direction the empirical contents of certain variables had changed between the two sub-periods.

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