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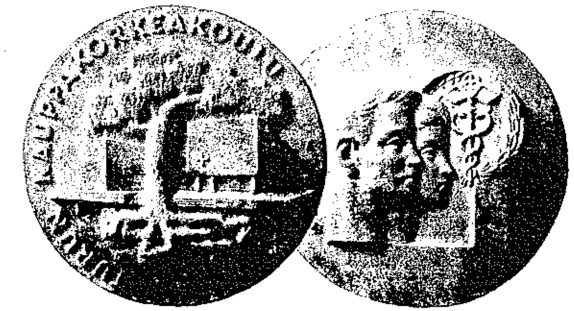

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APPLICATION OF SUPPLEMENTARY VARIABLES AND DISCRETE TRANSFORMS IN AVAILABILITY AND RELIABILITY ANALYSIS OF A PARALLEL REDUNDANT SYSTEM WITH GENERAL FAILURE AND REPAIR TIME DISTRIBUTIONS

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APPLICATION OF SUPPLEMENTARY VARIABLES AND DISCRETE TRANSFORMS IN AVAILABILITY AND RELIABILITY ANALYSIS OF A PARALLEL REDUNDANT SYSTEM WITH GENERAL FAILURE AND REPAIR TIME DISTRIBUTIONS

1. Introduction

In practice, we come across a number of systems containing a strategic component which fails frequently and causes catastrophic damage. In order to achieve a high system reliability the failure of such component should be avoided as much as possible. Often this is possible through a sufficient amount of maintenance. Sometimes, however, it becomes too much expensive or even impossible to reduce the frequency of failures of the strategic component to the desired extent. In such cases, the only alternative way to achieve high reliability is to introduce suitable redundancy in the system.

The following two types of redundancy are usually considered:[1]

(a) Standby redundancy - out of the redundant components only one component operates at a time. The system automatically switches over to the next component when the operating component fails. The system fails when the last standby connected components fails.

(b) Parallel redundancy - all the components connected in parallel start operating together as soon as the system is put into operation and the system fails only when all the components have failed.

Operational behaviour and reliability properties of a redundant system have been earlier studied by several authors.[2] Out of these

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1 See e.g. Pieruschka (1963), p. 76, Barlow and Proschan (1965), p. 162

2 e.g. Kulshrestha (1966,1972), Gnedenko et al. (1969), Das (1972), Srinivasan and Gopalan (1973 I, 1973 II), Govil (1974), Kistner and Subramanian (1974), Kodama and Deguchi (1974), Gopalan (1975)

redundancy models the most general are those given by Kulshrestha. First Kulshrestha studied the effect of standby redundancy[1] on the reliability of a system whereas parallel redundancy was considered in one of his later papers.[2] The assumptions in Kulshrestha's models are quite general: the number of redundant components may be arbitrary (most of the other papers deal with a two-component system) and successive failure times of each individual component as well as the repair times of the system are allowed to be random variables with general distributions (instead of exponential distributions usually assumed). In this paper, the parallel redundancy model given by Kulshrestha will be dealt with, and a lot of new results concerning reliability properties (i.e. reliability, availability, mean time to system failure) of the system will be derived.

In the formulation of the model the supplementary variable technique developed by Keilson and Kooharian[3] will be utilised. The solution of the model is mainly based on the use of Laplace transforms and discrete transforms.[4]

2. Description of the problem

The problem to be considered in this paper is as follows. A simple system consists of N (N > 1) components that are redundantly connected in parallel. The running times of each individual component are identically and independently distributed random variables having a common general distribution. The repair times of the system are distributed according to another independent general distribution. The repair of the system starts when all the N components have failed, after completion of the repair of all the N components the system is put into operation again.

The object of the study is to find out both the transient state and steady state behaviour of the system, and, on the basis of this information to derive the most important reliability characteristics

1 Kulshrestha (1966)
2 Kulshrestha (1972)
3 Keilson and Kooharian (1960)
4 On discrete transforms see e.g. Thiruvengadam and Jaiswal (1964)

(i.e. reliability, availability and mean time to system failure MTSF) for the system. The behaviour of the system is described by state probabilities, a state (at a time t) giving the number of failed components at that time. The system must have a little different state definition for availability analysis than for reliability analysis. This means that there also will be two separate models, one for the availability and another for the reliability and MTSF analysis of the system.

The system to be considered in this paper is the same as that introduced by Kulshrestha.[1] However, the analysis carried out by Kulshrestha has been considerably enlarged and completed:

1. Kulshrestha considered only the availability model, this paper deals also with the reliability model.
2. Kulshrestha limited his considerations to the derivation of the state probabilities, the availability of the system was not found. In this paper all the three reliability characteristics mentioned above are obtained.
3. For expressions of the state probabilities, only an iterative procedure was given in Kulshrestha's paper. Now the expressions for the state probabilities are given in a closed form.

3. The availability model

3.1. Notation

Define[2]

$P_n(x,t)\Delta$ = the joint probability that the system is in operable state at time t and n out of N components are in failed state and the elapsed time since the system was last put into operation lies between x and $x+\Delta$, $n=0,1,\ldots,N-1$.

$P_n(t)$ = the probability that the system at time t is in operable state and n out of N components are in failed state.

1. Kulshrestha (1972)
2. Except a few modifications in subscripts, notation is the same as in Kulshrestha (1972)

$P_N(x,t)\Delta$ = the joint probability that the system is under repair of all the N components at time t and the elapsed repair time lies between x and $x+\Delta$.

$P_N(t)$ = the probability that at time t the system is under repair of all the N components.

$\lambda(x)$ = failure rate of a single component.

$S(x)$ = probability density function of the failure time distribution of a single component.

$\mu(x)$ = repair rate of the system (i.e. of all the N components).

$T(x)$ = probability density function of the repair time distribution of the system.

Now we can easily see that the following relations hold

(1) $$P_n(t) = {}_o\int^\infty P_n(x,t)\,dx, \qquad n=0,1,\ldots,N-1,N$$

(2) $$S(x) = \lambda(x)\, e^{-{}_o\int^x \lambda(x)dx}$$

(3) $$T(x) = \mu(x)\, e^{-{}_o\int^x \mu(x)dx}.$$

The variable x in the expression of $P_n(x,t)$ $(n=0,1,\ldots,N)$ is just the supplementary variable appearing in the name of this paper. With the help of this supplementary variable the originally non-Markovian system becomes semi-Markovian. The probabilities $P_n(t)$ $(n=0,1,\ldots,N)$ are called the state probabilities.

3.2. The model

The model describing the behaviour of the system gets a form of the following set of difference differential equations with variable coefficients:[1]

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1. Except a few modifications in notation, the formulation of the model and the derivation of the solution are presented, up to equation (26), following Kulshrestha (1972)

(4) $$[\partial/\partial x + \partial/\partial t + (N-n)\lambda(x)]P_n(x,t) = (1-\delta_{n,o})(N-n+1)\lambda(x)P_{n-1}(x,t)$$
$$n=0,1,\ldots,N-1,\ x \geq 0,\ t \geq 0$$

(5) $$[\partial/\partial x + \partial/\partial t + \mu(x)]P_N(x,t) = 0, \qquad x \geq 0,\ t \geq 0.$$

Equations (4) and (5) are to be solved under the following boundary conditions:

(6) $$P_o(0,t) = {}_o\int^\infty P_N(x,t)\mu(x)dx, \qquad t > 0$$

(7) $$P_N(0,t) = {}_o\int^\infty P_{N-1}(x,t)\lambda(x)dx, \qquad t > 0$$

(8) $$P_n(0,t) = 0, \quad n=1,2,\ldots,N-1,\ t > 0.$$

If the system initially starts with all the N components new the following initial condition can be set up:

(9) $$P_n(x,0) = \delta_{n,o}\delta(x), \quad n=0,1,\ldots,N,\ x \geq 0.$$

In equations (4) and (9) $\delta_{n,o}$ is the Kronecker delta and $\delta(x)$ is the Dirac delta function.

Let the Laplace transform of a function $f(t)$ be denoted by $\bar{f}(s)$, i.e.

(10) $$\bar{f}(s) = {}_o\int^\infty e^{-st}f(t)dt, \qquad Re(s) > 0.$$

Applying the Laplace transform, the equations (4) to (8) with the initial condition (9) become

(11) $$[\partial/\partial x + s + (N-n)\lambda(x)]\bar{P}_n(x,s) = (1-\delta_{n,o})(N-n+1)\lambda(x)\bar{P}_{n-1}(x,s)$$
$$+ \delta_{n,o}\delta(x),\ n=0,1,\ldots,N-1,\ x \geq 0$$

(12) $$[\partial/\partial x + s + \mu(x)]\,\bar{P}_N(x,s) = 0, \quad x \geq 0$$

(13) $$\bar{P}_o(0,s) = {}_o\int^\infty \bar{P}_N(x,s)\,\mu(x)dx$$

(14) $$\bar{P}_N(0,s) = {}_o\int^\infty \bar{P}_{N-1}(x,s)\lambda(x)dx$$

(15) $$\bar{P}_n(0,s) = 0,\ n=1,2,\ldots,N-1.$$

In order to solve equation (11) the following discrete transforms will be introduced

$$(16)\quad A_k(x,s) = \sum_{n=k}^{N} \bar{P}_{N-n}(x,s)\binom{n}{k},\ k=1,2,\ldots,N.$$

The functions $\bar{P}_n(x,s)$ are then got as inverse transforms

$$(17)\quad \bar{P}_n(x,s) = \sum_{k=0}^{n}(-1)^k\binom{N-n+k}{k}A_{N-n+k}(x,s)$$
$$= \sum_{k=N-n}^{N}(-1)^{k-N+n}\binom{k}{N-n}A_k(x,s),\ n=0,1,\ldots,N-1.$$

Applying the discrete transforms (16) in (11) and simplifying, (11) reduces to

$$(18)\quad [\partial/\partial x + s + k\lambda(x)]A_k(x,s) = \delta(x)\binom{N}{k},\ k=1,2,\ldots,N,\ x \geq 0.$$

Solving equation (18) we obtain

$$(19)\quad A_k(x,s) = \begin{cases} A_k(0,s) & \text{for } x = 0 \\ [A_k(0,s) + \binom{N}{k}]e^{-sx-\int_0^x k\lambda(x)dx} & \text{for } x > 0. \end{cases}$$

From equation (12) we get

$$(20)\quad \bar{P}_N(x,s) = \bar{P}_N(0,s)e^{-sx-\int_0^x \mu(x)dx},\ x \geq 0.$$

Now using (17), (19) and (20) in the boundary conditions (13) and (14), the following expressions for $\bar{P}_o(0,s)$ and $\bar{P}_N(0,s)$ are after some labour obtained:

$$(21)\quad \bar{P}_o(0,s) = \frac{\bar{T}(s)\sum_{k=1}^{N}(-1)^{k-1}\binom{N}{k}\bar{S}_k(s)}{D(s)}$$

$$(22)\quad \bar{P}_N(0,s) = \frac{\sum_{k=1}^{N}(-1)^{k-1}\binom{N}{k}\bar{S}_k(s)}{D(s)},$$

where

$$(23)\quad S_k(x) = k\lambda(x)e^{-\int_0^x k\lambda(x)dx},\ k=1,2,\ldots,N$$

$$(24)\quad D(s) = 1 - \bar{T}(s)\sum_{k=1}^{N}(-1)^{k-1}\binom{N}{k}\bar{S}_k(s)$$

and $\bar{T}(s)$ and $\bar{S}_k(s)$ $(k=1,2,\ldots,N)$ are the Laplace transforms of $T(x)$ and $S_k(x)$ $(k=1,2,\ldots,N)$, respectively.

From equation (1) we get

$$(25)\quad \bar{P}_n(s) = \int_0^\infty \bar{P}_n(x,s)dx,\ n=0,1,\ldots,N.$$

Setting n = N in (25) and using (20) and (22), equation (25) on integration becomes

$$(26)\quad \bar{P}_N(s) = \frac{1-\bar{T}(s)}{s}\,\frac{\sum_{k=1}^{N}(-1)^{k-1}\binom{N}{k}\bar{S}_k(s)}{D(s)}.$$

Equation (26) gives the Laplace transform of the state probability $P_N(t)$ (probability that the system is in the failed state at time t). Next the Laplace transforms of the probabilities of the operable states will be derived. First, setting x = 0 in (16) and using (15) we have

$$(27)\quad A_k(0,s) = \sum_{n=k}^{N}\bar{P}_{N-n}(0,s)\binom{n}{k} = \binom{N}{k}\bar{P}_o(0,s),\ k=1,2,\ldots,N.$$

For x > 0, equation (19) thus becomes

$$(28)\quad A_k(x,s) = \binom{N}{k}[1 + \bar{P}_o(0,s)]e^{-sx-\int_0^x k\lambda(x)dx},\ k=1,2,\ldots,N.$$

Using (17) and (28) in (25) we have

$$(29)\quad \bar{P}_n(s) = \int_0^\infty \sum_{k=N-n}^{N}(-1)^{k-N+n}\binom{k}{N-n}A_k(x,s)$$
$$= \sum_{k=N-n}^{N}(-1)^{k-N+n}\binom{k}{N-n}\binom{N}{k}[1 + \bar{P}_o(0,s)]\int_0^\infty e^{-sx-\int_0^x k\lambda(x)dx}dx$$
$$= \sum_{k=N-n}^{N}(-1)^{k-N+n}\binom{k}{N-n}\binom{N}{k}[1 + \bar{P}_o(0,s)]\frac{1-\bar{S}_k(s)}{s}$$
$$= \binom{N}{n}\frac{1}{sD(s)}\sum_{k=N-n}^{N}(-1)^{k-N+n}\binom{n}{N-k}[1 - \bar{S}_k(s)],\ n=0,1,\ldots,N-1.$$

This can further be written

$$(30)\quad \bar{P}_n(s) = \begin{cases} \dfrac{1-\bar{S}_N(s)}{sD(s)} & \text{for } n=0 \\ \binom{N}{n}\dfrac{1}{sD(s)}\sum_{k=N-n}^{N}(-1)^{k-N+n+1}\binom{n}{N-k}\bar{S}_k(s) & \text{for } n=1,2,\ldots,N-1. \end{cases}$$

Equation (30) contains as particular cases the expressions for $\bar{P}_o(s)$ and $\bar{P}_1(s)$ which Kulshrestha has by an iterative procedure obtained.[1]

The Laplace transforms of the state probabilities are given in (26) and (30). Now for given values of $T(x)$ and $S(x)$ we can invert $\bar{P}_n(s)$ $(n=0,1,\dots,N)$ for obtaining the state probabilities $P_n(t)$.

3.3. Availability of the system

Availability of the system, denoted here $A(t)$, is defined[2] as the probability

(31) $A(t) = P\{\text{the system is operable at time } t\}$.

From the definition of the states it follows that the system is operable in being in one of the states $0,1,\dots,N-1$ and inoperable in state N. The Laplace transform of the availability of the system thus becomes

(32) $$\bar{A}(s) = \frac{1}{sD(s)} \sum_{n=0}^{N-1} \binom{N}{n} \sum_{k=N-n}^{N} (-1)^{k-N+n} \binom{n}{N-k} [1 - \bar{S}_k(s)]$$

or more simply

(33) $$\bar{A}(s) = \frac{1}{s} - \bar{P}_N(s) = \frac{1}{sD(s)}\left\{1 - \sum_{k=1}^{N} (-1)^{k-1} \binom{N}{k} \bar{S}_k(s)\right\}$$

$$= \frac{1}{sD(s)} \sum_{k=1}^{N} (-1)^{k-1} \binom{N}{k} [1 - \bar{S}_k(s)].$$

Again, with given values of $T(x)$ and $S(x)$ equation (33) can be inverted to give the availability $A(t)$.

3.4. Behaviour under steady state

The steady state behaviour of the system can be found out using the well known result in Laplace transforms[3], viz.

1. see Kulshrestha (1972), equations (28) and (29)
2. Gnedenko et al., p. 110
3. the final value theorem, Spiegel (1965), p.20

(34) $$\lim_{t \to \infty} f(t) = \lim_{s \to o} s\bar{f}(s).$$

Applying this relation to the equations (26) and (29) we get the steady state probabilities

(35) $$P_N = \lim_{t \to \infty} P_N(t) = \lim_{s \to o} s\,\bar{P}_N(s)$$

$$= \lim_{s \to o} \left\{ \frac{1 - \bar{T}(s)}{s} \; \frac{\sum_{k=1}^{N} (-1)^{k-1} \binom{N}{k} \bar{S}_k(s)}{D(s)/s} \right\} = \frac{R}{D}$$

(36) $$P_n = \lim_{t \to \infty} P_n(t) = \lim_{s \to o} s\,\bar{P}_n(s)$$

$$= \lim_{s \to o} \left\{ \frac{\binom{N}{n}}{\frac{D(s)}{s}} \sum_{k=N-n}^{N} (-1)^{k-N+n} \binom{n}{N-k} \frac{1 - \bar{S}_k(s)}{s} \right\}$$

$$= \binom{N}{n} \frac{1}{D} \sum_{k=N-n}^{N} (-1)^{k-N+n} \binom{n}{N-k} E_k, \quad n=0,1,\dots,N-1,$$

where

(37) $$R = \int_o^\infty x\mathbf{T}(x)dx = \text{the mean repair time of the system}$$

(38) $$D = \lim_{s \to o} \frac{D(s)}{s} = R + \sum_{k=1}^{N} (-1)^{k-1} \binom{N}{k} E_k$$

(39) $$E_k = \int_o^\infty x S_k(x)dx = \int_o^\infty xk\lambda(x)\exp\{-\textstyle\int_o^x k\lambda(x)dx\}dx, \quad k=1,2,\dots,N.$$

The steady state availability of the system thus is

(40) $$A = 1 - P_N = 1 - \frac{R}{D} = \frac{D - R}{D}$$

$$= \frac{\sum_{k=1}^{N} (-1)^{k-1} \binom{N}{k} E_k}{R + \sum_{k=1}^{N} (-1)^{k-1} \binom{N}{k} E_k}.$$

3.5. Particular case

If both the failure times of the components and the repair times of the system follow exponential distributions with parameters $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ respectively, we have

(41) $R = \mu^{-1}$

(42) $E_k = (k\lambda)^{-1}, \; k=1,2,\ldots,N.$

So the steady state availability of the system becomes

(43)
$$A = \frac{\mu \sum_{k=1}^{N} (-1)^{k-1}\binom{N}{k}k^{-1}}{\lambda + \mu \sum_{k=1}^{N}(-1)^{k-1}\binom{N}{k}k^{-1}} = \frac{\mu \sum_{k=1}^{N}(1/k)}{\lambda + \mu \sum_{k=1}^{N}(1/k)} .$$

4. The reliability model

4.1. Notation

In reliability analysis, unlike in availability analysis, the repair of the system is not considered. The system is only observed until the first failure occurs. So the distribution of the repair times of the system is not needed in the reliability analysis either. The states 0,1,...,N-1 are the same and they have the same meaning as in the availability model. Thus also the functions $P_n(x,t)$ and $P_n(t)$ are for n=0,1,...,N-1 the same as in section 3.1. Only the quantities joining with the state N must be redefined. So define

$P_N(t)$ = the probability that the system is inoperable at time t (the system has totally failed before or at time t).

The state N becomes an absorbing state and so $P_N(x,t)$ is not needed.

4.2. The model

Following similar lines as in the availability model the following set of difference differential equations with variable coefficients can be set up:

(44) $[\partial/\partial x + \partial/\partial t + (N-n)\lambda(x)]P_n(x,t) = (1-\delta_{n,o})(N-n+1)\lambda(x)P_{n-1}(x,t)$

$n=0,1,\ldots,N-1, \; x \geq 0, \; t \geq 0$

(45) $(\partial/\partial t)P_N(t) = \int_o^\infty P_{N-1}(x,t)\lambda(x)dx, \quad x \geq 0, \; t \geq 0.$

Because only the first operation period is considered, the boundary conditions now become

(46) $P_n(0,t) = 0, \quad n=0,1,\ldots,N-1, \; t > 0.$

Assuming all the components as new at time t = 0 we get the following initial conditions

(47) $P_n(x,0) = \delta_{n,o}\delta(x), \quad n=0,1,\ldots,N-1, \; x \geq 0$

(48) $P_N(0) = 0.$

The solution of equation (44) at first proceeds in the same way as the solution of equation (4) in section 3.2: we apply the Laplace transform in (44) to get (11), apply the discrete transforms (16) in (11) to get (18), integrate (18) and obtain (19). Using (16) and the Laplace transform of (46) we obtain

(49) $A_k(0,s) = \sum_{n=k}^{N} \overline{P}_{N-n}(0,s)\binom{n}{k} = 0, \quad k=1,2,\ldots,N.$

Thus, for x > 0 equation (19) now becomes

(50) $A_k(x,s) = \binom{N}{k} e^{-sx-\int_o^x k\lambda(x)dx}, \quad k=1,2,\ldots,N.$

Following the similar lines as in the derivation of equation (29) in section 3.2 we obtain the following expression for $\overline{P}_n(s)$:

(51)
$$\overline{P}_n(s) = \begin{cases} \dfrac{1-\overline{S}_N(s)}{s}, & n=0 \\ \binom{N}{n} \sum_{k=N-n}^{N} (-1)^{k-N+n+1}\binom{n}{N-k}\dfrac{\overline{S}_k(s)}{s}, & n=1,2,\ldots,N-1 \end{cases}$$

From equation (45) we get, taking the Laplace transform on both sides

(52) $s\overline{P}_N(s) = \int_o^\infty \overline{P}_{N-1}(x,s)\lambda(x)dx$

and further, using the inverse of the discrete transform of $\overline{P}_{N-1}(x,s)$

[eq. (17)] and equation (50)

$$(53)\quad s\overline{P}_N(s) = \int_0^\infty \sum_{k=1}^{N} (-1)^{k-1} k A_k(x,s)\lambda(x)dx$$
$$= \sum_{k=1}^{N} (-1)^{k-1}\binom{N}{k}\int_0^\infty k\lambda(x)e^{-sx-\int_0^x k\lambda(x)dx}dx$$
$$= \sum_{k=1}^{N} (-1)^{k-1}\binom{N}{k}\overline{S}_k(s).$$

So we have obtained

$$(54)\quad \overline{P}_N(s) = \frac{1}{s}\sum_{k=1}^{N} (-1)^{k-1}\binom{N}{k}\overline{S}_k(s).$$

Again, with a given value of S(x) equations (51) and (54) can be inverted to give the state probabilities $P_n(t)$ (n=0,1,...,N).

4.3. Reliability and MTSF of the system

Reliability of the system, denoted R(t), is defined[1] as the probability

(55) R(t) = P{the system does not fail during the interval (0,t)}.

Evidently we now have

$$(56)\quad R(t) = \sum_{n=0}^{N-1} P_n(t) = 1 - P_N(t).$$

The Laplace transform of the reliability so becomes

$$(57)\quad \overline{R}(s) = \frac{1}{s} - \overline{P}_N(s)$$
$$= \frac{1}{s}[1 - \sum_{k=1}^{N} (-1)^{k-1}\binom{N}{k}\overline{S}_k(s)]$$
$$= \sum_{k=1}^{N} (-1)^{k-1}\binom{N}{k}\frac{1-\overline{S}_k(s)}{s},$$

which after inversion gives the reliability R(t).

The mean time to system failure (MTSF) has been shown[2] to be

$$(58)\quad MTSF = \lim_{s\to 0} \overline{R}(s).$$

1. Gnedenko et al. (1969), p.79
2. Das (1972), p.69

Thus we have

$$(59)\quad MTSF = \lim_{s\to 0}\sum_{k=1}^{N} (-1)^{k-1}\binom{N}{k}\frac{1-\overline{S}_k(s)}{s} = \sum_{k=1}^{N} (-1)^{k-1}\binom{N}{k}E_k,$$

where E_k is given in equation (39).

4.4. Particular case

If the failure times of the components obey exponential distribution with parameter λ, we have

$$(60)\quad \overline{S}_k(s) = \frac{k\lambda}{s+k\lambda}, \quad k=1,2,\ldots,N$$

and thus

$$(61)\quad \overline{R}(s) = \sum_{k=1}^{N} (-1)^{k-1}\binom{N}{k}(s+k\lambda)^{-1},$$

which after inversion gives the reliability R(t):

$$(62)\quad R(t) = \sum_{k=1}^{N} (-1)^{k-1}\binom{N}{k}e^{-k\lambda t}.$$

MTSF becomes in the exponential case

$$(63)\quad MTSF = \sum_{k=1}^{N} (-1)^{k-1}\binom{N}{k}(k\lambda)^{-1} = \lambda^{-1}\sum_{k=1}^{N}(1/k).$$

We can notice that the result for the exponential case MTSF is the same as earlier obtained e.g. by Gnedenko et al.[1]

1. Gnedenko et al. (1969), p.281

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