

H.S.V. de Snoo &
H.L. Wietsma (Eds.)

Contributions to Mathematics and Statistics

Essays in honor of Seppo Hassi



ACTA WASAENSIA 462



Vaasan yliopisto
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PUBLISHER
University of Vaasa
School of Technology and Innovation
P.O. Box 700
FI-65101 Vaasa
Finland

ISBN
978-952-476-959-4 (print)
978-952-476-960-0 (online)

<http://urn.fi/URN:ISBN:978-952-476-960-0>

ISSN
0355-2667 (Acta Wasaensia 462, print)
2323-9123 (Acta Wasaensia 462, online)



SEPPO HASSI

FOREWORD

With this Festschrift we celebrate the sixtieth birthday of our friend and colleague Professor Seppo Hassi of the University of Vaasa. It consists of papers written by colleagues outside Vaasa, who have been coauthors of Seppo, as well as by colleagues from Vaasa. Although many friends and colleagues have known and worked with Seppo for a long time, quite a few people answered in disbelief "What? Seppo 60?" when first approached about this project.

This collection of essays shows our appreciation of Seppo as a friend and as a colleague. From early on, his main activities have been in the branches of mathematics, known as operator theory and spectral theory, although his interests are much broader. Almost all of the included essays reflect these interests. Unfortunately, due to the consequences of the global pandemic some contributions could not be submitted in time to be part of our collection.

It is our pleasure to thank all the authors, both for contributing their work to this volume and for their readiness to respond to our questions and suggestions. Furthermore, we are grateful to Heinz Langer, Kenneth Nordström, Seppo Pynnönen, and Franek Szafraniec for answering our queries concerning several points about the past and present of the person to whom this volume is dedicated. Finally, our thanks go to the staff at the University of Vaasa, in particular to Riikka Kalmi, for their efficient production of this collection.

Groningen and Vaasa, April 2021

Henk de Snoo and Rudi Wietsma

SEPPO HASSI, 60 YEARS

Seppo Ortamo Hassi was born on July 2, 1961, in Hyvinkää in southern Finland. He received his secondary school education in Pori, located on the western coast of Finland, and in 1980 he went to the University of Helsinki to be a student in mathematics. There Seppo obtained his master's degree in 1984. He would stay at the university and eventually in 1985 became an assistant in the Department of Statistics (which was located at Yliopistonkatu, while the Department of Mathematics was located at Alexanterinkatu, a geographical gap).

The leading people in the statistics department were Hannu Niemi (a student of Louhivaara, whom we will meet below) and Seppo Mustonen. Mustonen somehow awakened Seppo's interest in singular values and canonical representations of operators. This eventually led to the dissertation *A singular value decomposition of matrices in a space with an indefinite scalar product*, with Ilppo Simo Louhivaara (1927 - 2008)[†] as adviser. This thesis in mathematics was approved by the University of Helsinki on January 31, 1991, at the time that Seppo served in the Finnish army (between August 1990 and April 1991). The opponent at the defence was Heinz Langer (originally from Dresden); Langer had first visited Louhivaara in Jyväskylä in 1969 and had been a frequent guest ever since.

Prior to finishing his dissertation, Seppo had been invited to participate in the *Schur Analysis* meeting (October 16 - October 20, 1989) at the Karl Marx Universität in Leipzig, Deutsche Demokratische Republik, organized by Bernd Kirstein and Bernd Fritzsche. This seminar brought together many people from East and West. It took place in the middle of the peaceful protests against the communist regime that had been going on in Leipzig for some time. Loudspeakers in empty streets would advise the public not to follow the protesting crowds: "They are misguided." On November 9, shortly after the conference, the Berlin wall came down. At the beginning of the conference it turned out that Heinz Langer had left the country and at its closing it was announced that the great mathematician Mark Grigorievich Kreĭn (1907-1989) had died. One of the people present from the East was Yury L'vovich Smul'yan (1927-1990), whose work played an important role in Seppo's dissertation and in his later articles.

With the dissertation completed, Seppo started some joint work with his colleague Kenneth Nordström, who was also an assistant in the Department of Statistics. Their interest focussed on antitonicity properties of operators and projections in indefinite inner product spaces. In the meantime Heinz Langer had obtained a professorship at the Technische Universität Wien in 1991. He invited Seppo to spend some weeks in Vienna in 1992 at the same time that Henk de Snoo from Groningen was also visiting. During that period Langer's Dutch and Finnish visitors started to work together, which led to many mutual

[†]Louhivaara had been one of the many students of Rolf Herman Nevanlinna (1895-1980). He was also interested in extension theory and indefinite metrics, like his contemporary fellow students Yrjö Kilpi (1924-2010) and Erkki Pesonen (1930-2006). Louhivaara had been a professor of mathematics at the universities in Helsinki and Jyväskylä, before moving to the Freie Universität Berlin.

visits to Holland and Finland over the years, up till the present day. It was during a number of subsequent conferences in or visits to Vienna, Pula, Timisoara, Warsaw, Krakow, and Budapest that it was possible to meet old and new acquaintances and lay foundations for future work. It is appropriate to mention in this context Michael Kaltenbäck, Harald Woracek, Henrik Winkler, Andreas Fleige, Franek Szafraniec, Zoltán Sebestyén (thanks to Jan Stochel), Jean-Philippe Labrousse, and last, but not least, Yury Arlinskiĭ, Vladimir Derkach, and Mark Malamud. A sabbatical visit to Groningen and Berlin in the academic year 2000-2001 made it possible to meet the group around Karl-Heinz Förster of the Technische Universität Berlin, which consisted of Peter Jonas and Peter's students Carsten Trunk and Jussi Behrndt. Peter Jonas was from East Berlin and had come to the Technische Universität via Ilppo Simo Louhivaara at the Freie Universität. Seppo's visit led to fruitful contacts; also the later December conferences in Berlin were very productive.

Seppo would remain at the Department of Statistics in Helsinki until 2001; in the meantime he had been formally named docent at the Department of Mathematics of the same university. In November-December 2000 there had been a longer visit to Manfred Möller at the University of Witwatersrand in South Africa and it was there that Seppo found out that the University of Vaasa was interested in his person. He obtained a professorship at that university in the spring of 2001. Seppo settled down in Vaasa during the summer and took up the usual teaching and administrative tasks. In the following years the number of coworkers increased with, for instance, Annemarie Luger, Adrian Sandovici, Sergey Belyi, Eduard Tsekanovskii, and Sergii Kužel. As a consequence there has been a regular stream of visitors (all of whom think with a certain melancholy of the old wooden guestrooms of the University of Vaasa). In May 2003 Seppo was the organizer of an Operator Theory Symposium and, a little later, in 2005 he was one of the organizers of the Algorithmic Information Theory Conference, see *Acta Wasaensia* 124, 2005. Moreover, Seppo was one of the organizers of the conferences "Boundary relations" and "Operator realizations of analytic functions" at the Lorentz Center in Leiden in 2009 and 2013, respectively.

The main mathematical interest of Seppo circles around the topics of spectral theory, boundary value problems for differential equations, operator theory and its applications in analysis, mathematical physics, and system theory. This keeps him going with great dedication. In particular, right from the beginning Seppo looked into situations involving indefinite inner product spaces and this interest also led to several doctoral students, Rudi Wietsma, Dmytro Baidiuk, and Lassi Lilleberg, writing a dissertation on this topic under his direction. Being a rather prolific writer himself, he is furthermore an editor for a number of mathematical journals.

When Seppo first arrived in Vaasa he belonged to the Department of Mathematics and Statistics. As the century progresses, so does the university. Seppo now belongs to the School of Technology and Innovations, where he is the leader of the Mathematics and Statistics Research Group. He is also head of the Doctoral Programme in Technical Sciences. Moreover, there are duties beyond Vaasa. For many years Seppo has been involved with the nationwide

entrance exam for Technical Sciences and Architecture studies of the member universities in Finland. And then there is the Academy of Finland: for some years now Seppo has been a member of its Research Council for Natural Sciences and Engineering, and a member of its steering group. All these things coming his way are done with his usual attention to detail.

Those who deal with Seppo, either as colleagues or as students, know that he provides a listening ear and is ready to help whenever needed. And those who are fortunate enough to work with him recognize his quiet determination. Uninterrupted, he can sit behind his desk for hours, like a sphinx – lost in thought (so we assume). But when he returns back to real life, you know that something is going to happen.

On behalf of all his many friends, whether in Vaasa or elsewhere in the world, we congratulate Seppo on reaching his sixtieth birthday and we wish him, together with his wife Merja and their son Leo, good health and happiness. May there be many more productive years to come!

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THE ROLE OF MATHEMATICS AND STATISTICS IN THE UNIVERSITY OF VAASA; THE FIRST FIVE DECADES

Ilkka Virtanen

In dedication to Professor Seppo Hassi on the occasion of his 60th birthday

The tragedy of the world is
that those who are imaginative
have but slight experience,
and those who are experienced
have feeble imaginations.
Fools act on imagination
without experience.
Pedants act on experience
without imagination.
The task of the university is
to weld together imagination and experience.

Alfred North Whitehead
A Cambridge mathematician

1 From a business school towards a university with a clearly defined academic profile

1.1 The beginning

The University of Vaasa is one of the four “new universities” founded in Finland in 1966. Practical activities of the new units started two to five years after the foundation decision was made by the state authorities. Three of the new universities were founded in Eastern Finland whereas the University of Vaasa came to be placed on the west coast of Finland. All these new universities had different academic profiles. Vaasa got a private business school which, however, from the beginning – the first students started their studies in 1968 – got the major part of its financing from the state. After ten years the business school became a state university. Thus the University of Vaasa started as a business school, but it was clear from the beginning that the final target was to develop it into a multidisciplinary university. The University of Lappeenranta was a technical university, the University of Kuopio had disciplines in medicine, biosciences, and social sciences and, finally, the University of Joensuu was based on an earlier teacher training college having also natural sciences, humanities, and social sciences in its discipline palette. The Eastern Finland universities were state universities from the beginning.

The post-war baby boom generation was coming to the university in the 1960s and 1970s, and competition for resources between universities was great. So it turned out that the development of the business school towards the multidisciplinary target would be a hard and long-lasting exercise. Step by step the goal was, however, reached, at least to a decent extent.

Today the University of Vaasa is – on the Finnish scale – a small or medium-sized business-oriented and multidisciplinary science university. The strategic focus areas of the university are management and change, energy and sustainable development, as well as financial and economical decision-making. The university has about 5000 students and about 300 of them are international degree students. The total number of employees is about 510 of whom about 100 are international.

The university has been organized into schools. There are four schools for research and teaching: the School of Management, the School of Accounting and Finance, the School of Marketing and Communication, and the School of Technology and Innovations. The unit of mathematical sciences belongs to the School of Technology and Innovations, although it is responsible for all the teaching of mathematics and mathematics-based quantitative methods in the whole university. The teachers and researchers of this unit are in close cooperation with employees of the other schools.

1.2 Strategy and values of the university today

In its strategy for 2030 the university defines itself to be an internationally competitive, productive and specialized research university with a strong focus on impactful basic scientific research. The core competence of the university consists of high-level expertise in business, technology, management, and communications.

The university is focused on responsible business. Its fundamental purpose is to cultivate new knowledge and nurture civilization as a core value of our society. This is why the focus is on global challenges and opportunities. They provide the university with the core source of motivation for its education and research. The university uses its work as a means to advance positive and sustainable development for individuals, communities, and the world at large.

Based on the strategy for 2030, the university has defined its vision, mission and values as follows.

- *Vision:* The University of Vaasa is regarded internationally as a successful and impactful research university.
- *Mission:* The university carries out impactful research and educates experts that address the needs of society today, and in the future. The university advances competitiveness, innovation and sustainable development in business, technology, and society.
- *Values:* The values of the university are Courage, Community, and Responsibility.

2 The role of mathematics and statistics in the university's palette of sciences

2.1 No own degree programme in master level – a recognized role, however

The role of mathematics and statistics in the University of Vaasa is not typical for universities. According to the strategy applied in the university today and during the past decades, these disciplines have never been offered as the main subjects of bachelor and master level degrees. This has not meant, however, that the disciplines were considered as less important for studies and research than the disciplines with their own degree programmes. On the contrary, the methodological disciplines have for example always been represented by professor chairs. It is noteworthy that even the first chair established in the university – a business school in the beginning – and also having got its first office-holder was the combined chair in economic mathematics and statistics.

The holder of the chair in economic mathematics and statistics was also the first rector of the business school. He started his work already one year earlier than the school was opened and received its first students. That is, he had a central role in planning the first year's curriculum for the school. According to the experiential opinion of the author of this article, the result was that the quantitative sciences got a stronger role in the curriculum for the business students than was the case in the other Finnish business schools at that time.

Nowadays the situation has changed. Other business schools have also become aware that modern economic and business education as well as research needs a good comprehension of the quantitative methods as well as the skill to use them. A large number of students graduated as doctors in business economics in the University of Vaasa who, having been recruited as professors and other academic employees to other business schools, have had a marked effect in this process.

2.2 Mathematical sciences – a close and united academic unit

At the beginning the teaching staff consisted of one professor and one lecturer, both posts being combined posts for business mathematics and statistics. As the number of students increased the unit obtained another professorship and an additional lectureship. This made it possible to focus the duties of all the posts only on business mathematics or statistics, respectively. An assistantship was also an important addition to the staff.

In the beginning of the 1990s the university started to enlarge its branch of activities towards technology. This could progress step by step only. First, technical elements were included in the curriculum of two master programs in economics and business administration and an unofficial label "industrial economist" was given to the graduates. The main subject in these programmes was either industrial management or information technology. In the next step the university started to educate civil engineers in co-operation with the Helsinki Technical University. The University of Vaasa recruited the students who carried out half of their studies in Vaasa and graduated from Helsinki Technical University. In the beginning of the 2000s the University of Vaasa was allowed to educate civil engineers completely as its own activity.

It was clear that the enlargement of the branch of activities into technology presumed additional resources also for the unit of mathematical sciences. The total number of students increased and the amount, type, and level of mathematics needed in technical studies is different from what is needed in economics, business administration, and in the social sciences. The requirements were resolved by providing the unit with a professorship and a lectureship in mathematics.

Today the permanent staff in the unit (department) consists of three professors, three senior lecturers with doctor degrees, one university teacher, and a varying number (3–5) of doctoral students and post-doctoral researchers. Although administratively the unit is a part of the School of Technology and Innovations, the main sphere of responsibility of business mathematics and statistics has been supporting the School of Accounting and Finance and other business oriented schools by teaching quantitative methods both for education and research purposes.

The unit of mathematical sciences is small. Therefore it is important that the mathematically oriented disciplines form also administratively an integrated own unit. Together the unit's academic subjects are stronger, they can flexibly use common teaching resources, and they create and maintain a high-level and international academic atmosphere in research. This administrative solution guarantees that the department can form a close and united academic society. One example of the manifestation of this coherence is that the unit takes care also of maintaining close contacts with its staff in retirement.

Holders of the professor offices in the department have been mainly recruited from other universities. Statistics is an exception. The last two professor appointments in statistics have been candidates who have done their doctoral studies and qualified for professorship when working at the University of Vaasa. All the senior lecturers of the unit have received their doctor degrees while working at the university. As a result, the staff members represents a high quality group of experts in their own fields, but, at the same time, they possess understanding of and positive attitude towards the needs of quantitative methods appearing among the students and researchers of other disciplines in the university.

2.3 The unit's supporting role in undergraduate education, its own intensive postgraduate education, and high-quality international research

As has been mentioned earlier, the unit of mathematical sciences doesn't have and has never had any own master-level education programme in the curriculum of the university. In undergraduate studies the role of mathematical sciences is to contribute as a strong and high-standard supporting discipline offering basic university level knowledge and relevant advanced tools in mathematical and statistical modelling for the students of the other education programmes of the university, especially for students in business and technology. The demand of skills in the use of modern quantitative methods and models is outstandingly high in the postgraduate level of the studies. Besides offering methodological courses the professors of the unit participate as tutors in other subject's doctoral seminars.

Postgraduate studies in mathematics and statistics have been possible in the university from the beginning. Their role has become more and more important during the years. The recruiting of students is challenging due to the non-existing own master level education. The achievements are, however, good and the department is for example a pioneer in the university in recruiting international students for doctoral studies. Today, a majority of the unit's postgraduate students is of foreign origin.

As the mathematical sciences are represented on the professor level in the university, scientific research is, of course, strongly on the department's agenda. The research groups in which the department's researchers participate represent the top quality in the university. The researchers have created and maintain one of the university's research programmes with the theme of mathematical modelling. The projects in the programme are both discipline oriented, especially in mathematics, and more application oriented in statistics and business mathematics. The research groups work actively in co-operation with other researchers inside the university and with researchers in other universities, both in Finland and abroad. Visiting foreign scholars are commonly seen working in the department and the staff members work frequently abroad.

The professors and other staff members participate actively in the general management of the University. Two professors in economic mathematics have served as rectors of the University, several professors as faculty deans and as heads of multidisciplinary departments. Professors and other staff members are members of several managerial working groups.

3 Discussion

The chosen policy for the mathematical sciences in the University of Vaasa – i.e., to operate as an academic education and research unit without any own undergraduate education programme – is challenging. But the work done during the five first decades has shown that the chosen option has been successful. The main measures for reaching success are:

- Careful attention has been paid when recruiting new members to the staff. Besides competence, the new members must also have an understanding of and a positive attitude towards the other disciplines in the university with which they are expected to co-operate both in education and research.
- Active co-operation with other disciplines inside the university has been a necessity for being able to give relevant and up-to-date methodological support to both students and researchers of other disciplines.
- To reach and maintain a high level competence in the area of everyone's own expertise an active communication and co-operation with the representatives of one's own discipline in other universities in Finland and abroad have been strongly on the agenda.
- The absence of own undergraduate education has not meant absence of postgraduate education. On the contrary, active doctoral programmes have guaranteed continuity in research and have helped in recruiting to the unit new employees who possess a relevant orientation towards the operating principle of the unit. Of course, external recruiting has also been important for guaranteeing high levels of competence.
- Special attention has been paid for activating international co-operation in research and post-graduate education.

A small university like the University of Vaasa must carefully focus its activities on areas about which it has successful experience from the past, which are still relevant today and are also or are expected to become crucial in the future, and which form a coherent entity. In education and research, also the needs of the region and the whole society as well as the region's possibilities to

offer co-operation and support must have a strong role in the agenda. The strategy and values of the university meet these requirements well.

Similarly, the unit of mathematical sciences as a small unit inside the university must have a clear and specific high-standard academic culture in its operations. Conclusions from the scrutiny above show that the unit has been successful in creating this culture.

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