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Common Factors in the Arbitrage Pricing Model in Two Scandinavian Countries

P YLI-OLLI

I VIRTANEN

T MARTIKAINEN

University of Vaasa, Finland

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The purpose of this paper is to test the Arbitrage Pricing Theory (APT) using monthly data for Finnish and Swedish stock returns during the 1977–1986 period. The first stage involves estimating the systematic risk components for each asset using factor analysis. The second stage involves testing if the number and structure of factors which influence the security returns remain unchanged across various time periods and across different samples in the two Scandinavian countries. For this purpose, a new method called transformation analysis is introduced. Empirical evidence indicates that two or three common factors in these two neighbouring countries can be found.

Key words—finance, international asset pricing, Arbitrage Pricing Model, stability of risk measures

1. INTRODUCTION

Background

THE CAPITAL ASSET PRICING MODEL (CAPM), developed in [21, 25, 31, 32] is a simple and elegant model for pricing risky assets. The CAPM is an equilibrium model, where the systematic risk of an asset is assumed to be the covariance of the asset return with the market portfolio return divided by the variance of the market portfolio return. The CAPM has been the central topic in the empirical work in finance over the past 20 years. Empirical tests of the CAPM have produced mixed results (see e.g. [7, 18, 19, 24]). The critical point in the estimation of the CAPM is the difficulty of measuring the true market portfolio (more about the importance of a relevant data base in empirical research see [4]). Stock market indices are usually used as a proxy of true market portfolio. Roll [28] reported that the CAPM is extremely sensitive to the choice of a market proxy (see also [3, pp. 110–126]). Thus, he casts serious doubts on the testability of the CAPM itself. The CAPM is not testable unless the exact composition of the true market portfolio is used in the tests.

Several other equilibrium models and extensions of the CAPM have also been presented in literature (e.g. [6, 17, 20]). However, the most frequently tested equilibrium model in recent literature is obviously the model based on the Arbitrage Pricing Theory (APT) formulated by Ross [30]. The APT is based on the similar intuition as the CAPM, but it is more general. The CAPM assumes that a return of any security will be linearly related to a single common factor, to a return of the market portfolio, whereas the APT assumes that a security return is a linear function, not only of one, but of a set of common factors. Typically, in order to test the APT empirically, a two-step procedure is used. First, factor analysis is used to identify the number of factors and the factor loadings from daily, weekly or monthly time series. Second, the estimated factor loadings are used to explain the cross-sectional variation of estimated expected returns (see e.g. [9, 13, 27, 29]).

Unfortunately, there are many problems in testing of the APT. An intensively discussed problem is how to decide the correct number of priced factors. It has been found that the number of significant factors is an increasing function of the size of the groups analyzed [14, 15]. There are also some additional methodological problems with the use of factor analysis [16, pp. 343–354]. First, the decision as how many factors to extract has been made subjectively. Second, there is no guarantee that factors are produced in a particular order. Third, there is no meaning to the signs of the parameters.

In our opinion, however, the most relevant questions in testing the APT is neither the question how to determine the "correct number" of the priced factors in different samples nor the question in what kind of order factors are produced in those samples. For, it is possible to obtain the same number of factors in different samples or in the same sample in different time periods even if the content or empirical interpretation of the factors has not necessarily remained the same in those groups. Therefore, it is more important to find such common factors which are the same across different samples during the same time period (cross-sectional studies) or across different time periods in the same sample (time-series studies) irrespective of what is the number of those factors or in what order the factors are produced. For this purpose, transformation analysis is used in this study. This method offers a versatile methodology with which it is possible to study the stability and invariance existing among different factor structures (for business applications of transformation analysis see [36]).

The purposes of the study

One of the most crucial problems when testing the APT is the question if the contents of the factor structures in different samples during the same time period or the contents of the factors in the same sample in different time periods are the same. In this paper the transformation analysis method is used. This statistical technique makes it possible to analyze the stability of factor structures across different samples in the same time period or across different time periods in the same sample. In addition, using this method it is possible to describe the reason for the non-invariant part prevailing in these structures. In this method, it is not important whether the factors in different samples are produced in a particular order or not. The only limitation is that different samples have the same number of factors. Using transformation analysis it is possible to find such common factors which have the same contents, i.e. the same empirical interpretation for different samples.

The purposes of this study are:

(1) To test the APT using monthly time series data from the Scandinavian firms quoted on the Helsinki and Stockholm Stock Exchanges.

(2) Especially, to test, using transformation analysis, the similarity of the factor structures over time and across samples, i.e. across two Scandinavian countries.

2. THE MODEL

The Arbitrage Pricing Model (APM) based on the theory originally developed by Ross [30], predicts that on the perfectly competitive and frictionless stock markets the stock return is a linear function of a certain number, say k, economic factors. So, the APM starts with the assumption that returns on any stock, $\underline{R}_i$, are generated by a k-factor model of the form (see e.g. [29, pp. 1076–1082]):

$$\underline{R}_i = E(\underline{R}_i) + b_{i1}\underline{\delta}_1 + b_{i2}\underline{\delta}_2 + \dots + b_{ik}\underline{\delta}_k + \underline{\varepsilon}_i, \qquad (1)$$

where a random variable is denoted by underlying the symbol of the variable, and $E(\underline{R}_i)$, $i = 1,2,\dots,n$, is the expected return of the stock i, $\underline{\delta}_j$, $j = 1,2,\dots,k$, are unobserved economic factors, b_{ij} is the sensitivity of the security i to the economic factor j and $\underline{\varepsilon}_i$ are the idiosyncratic risks of the stocks. In addition, we assume that $E(\underline{\delta}_j) = 0$ for $j = 1,2,\dots,k$, $E(\underline{\varepsilon}_i) = 0$ for $i = 1,2,\dots,n$, $E(\underline{\varepsilon}_i\underline{\varepsilon}_h) = 0$ for $i \neq h$, and $E(\underline{\varepsilon}_i^2) = \sigma_i^2 < \infty$.

Ross [30] has shown that if the number of stocks is sufficiently large the following linear risk-return relationship can be written:

$$E(\underline{R}_i) = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + \dots + \lambda_k b_{ik} \qquad (2)$$

where λ_0 is a constant riskless rate of return (the common return on all zero-beta stocks) and λ_j, $j = 1,2,\dots,k$, represents, in equilibrium, the risk premium for the jth factor.

In equation (1) each stock i has a unique sensitivity b_{ij} to each factor $\underline{\delta}_j$ but any factor $\underline{\delta}_j$ has a value that is the same for all stocks. These common factors capture the systematic components of risk in equation (1). Therefore, any $\underline{\delta}_j$ affects necessarily more than one security return. In the other case it would have been compounded in the unsystematic component of the risk, i.e. in the residual term $\underline{\varepsilon}_i$. In order to test the model (1) there are in principle two alternative approaches.

First, one could try to specify *a priori*, on the basis of the theory, the general factors that explain pricing in the stock market. Such macroeconomic variables could be e.g. the spread between long-term and short-term interest rates, expected and unexpected inflation, industrial production and spread between high- and low-grade bonds [10]. In the thin Finnish stock market such variables could be e.g. aggregate future cash-flow of the firms, interest rate of bank deposits or return of the state bonds, the supply of money, and inflation [34] or aggregated accounting numbers [23]. In the case where factors are based on economic theory the estimation procedure should be as follows. In the first stage, time series regressions are run of each series of stocks (portfolios) to estimate each stock's (portfolio's) sensitivities b_{ij} to macroeconomic variables. Then the risk premia λ_j are estimated by running a cross-sectional regression for each time period examined. In every cross-sectional regression the average return of stocks is used as the dependent variable and the sensitivities of the securities as independent variables.

Second, the more general and also much more problematic approach is to estimate the b_{ij} and unknown factors $\underline{\delta}_j$ simultaneously by factor analysis. In that case a theory does not tell, *a priori*, what the exact content or even the number of relevant factors is. Without any theory a decision how many factors to extract from the data has to be made subjectively or by statistical criteria, such as the eigenvalue criterion or the Cattel's scree test. When the systematic components of the risk, b_{ij}, have been obtained, the risk premia λ_j are typically estimated using again cross-sectional regressions.

In the factor analysis approach there are many methodological problems. First, there are no meaning to the signs of the factors produced by factor analysis. Second, the scaling of b_{ij}'s and λ_j's is arbitrary. Third, there is no guarantee that factors are produced in a particular order when analysis is performed on separate samples [16, pp. 336–352]. In addition, it has been difficult to try to decide the correct number of priced factors (see e.g. [15]).

In recent years the international application of the asset pricing models have been discussed widely (see e.g. [12, 22, 33]). Cho *et al.* [12] found the number of international common factors to be about three or four, being dependent on the economic integration between countries. Their analysis was based on inter-battery factor analysis (see [11]). Recently, the main asset pricing models for Finland and Sweden were researched and compared by Östermark [37] who reported that in both countries all ratios were in favour of the APM instead of the traditional CAPM. However, the similarity of the factor patterns over time or across countries was not studied. Doing this is the main object of this paper.

3. DATA AND STATISTICAL METHODS

The data

In Finland, on the Helsinki Stock Exchange, three stock market indices are in common use, Unitas and KOP price indices published by two Finnish commercial banks and a return index developed for research purposes ([5], see also [38]). From the theoretical point of view, the return index is the best measure, and is also selected for our research. This index includes the dividend component wheras Unitas and KOP indices are pure price indices. When no trade has occurred, the "true" price has been proxied by the bid quotations. In Sweden, on the Stockholm Stock Exchange, the returns have been calculated using the same method as in the return index for Finnish data. Using indices from these two closely related economies offers a fruitful starting point for studying the cross-country invariances in the APM since previous research indicates that the stock price behaviour in these two countries is relatively similar to each other (see e.g. [34]).

The empirical verification of the APM and the stability analysis require both a large sample in terms of number of securities, and also a long time period. Monthly values of the selected indices from January 1977 to December 1986 are used in this study. The Finnish sample

(sample 1) consists of the shares of 30 firms. The sample includes shares which have been quoted on the Helsinki Stock Exchange for the entire sample period and have been the most frequently traded stocks in the period 1970–1986. The Swedish data is analyzed in two samples. The first Swedish sample (sample 2) consists of the 30 most frequently traded firms during the research period and the second Swedish sample (sample 3) represents the next 30 stocks ranked by the trading frequency (in both countries only one series for each firm has been selected). The reason why only one sample from Finland has been selected is simply the fact the number of the listed Finnish firms at the end of the period was only 59 and we are not able to select two samples of 30 stocks in Finland. In addition, it should be emphasized that some listed Finnish firms have been traded very infrequently. Using more than 30 firms would have led to serious problems when estimating stock returns. In this context an interesting point is to study how the two Swedish samples are associated with each other. Namely, due to the sample formulation, the thin trading bias in the second Swedish sample, i.e. in sample 3 is expected to be higher than in sample 2. This makes it possible to study the effects of trading frequency on the stability of factor loadings. The trading frequencies of the selected Swedish stocks are plotted in the Appendix 1. The figure shows that the problems of irregular trading are obvious in the third sample. For the stability analysis the whole period is divided into two subperiods: subperiod 1 includes years 1977–1981, and subperiod 2 years 1982–1986.

Statistical methods

The main statistical methods used in the study are factor analysis, regression analysis and transformation analysis. Factor analysis and regression analysis are usual techniques in business applications. Transformation analysis, on the contrary, has been mainly applied only in Finnish research, earlier in sociological research and, in recent years, in business applications also. Therefore, this paper contains a short description of this multivariate method.

The degree of stability in factor patterns has been traditionally measured with correlation or congruence coefficients (the same coefficients are used in measuring the stability of estimated betas in the CAPM; see e.g. [8]). Both of these measures give an index for the similarity of two different factor solutions in terms of the pattern of correlations among factor loadings across all variables in the reduced factor space. For the dissimilar part of these factor solutions these indices are, however, unable to describe and explain the reason for the non-invariant part prevailing in these factor solutions [36].

Originally transformation analysis (initiated by Ahmavaara [1] and further developed by Ahmavaara [2] and Mustonen [26]) was developed to compare factor solutions between two different groups of objects. The technique has also been used to compare two different factor solutions among the same group of objects, the two factor solutions being based on measurements made during two different time periods. The mathematical idea behind transformation analysis is sketched out in Appendix 3 (see also [35, 36]). With correlation and congruence coefficients one can only measure the degree of similarity of two factor solutions (correlations or congruences among factor loadings). This is also possible via transformation analysis (coefficients of coincidence on the main diagonal of the transformation matrix). In addition to this one obtains a regression type model for shifting of variables from one factor to another (normal or explained transformation). This is revealed by non-zero off-diagonal elements in the transformation matrix and indicates interpretative changes for the factors in question. And finally, large elements in the residual matrix indicate abnormal or unexplained transformation between the two factor solutions. This means interpretatively that the empirical content of the corresponding variables has changed. Further, this abnormal transformation can be appointed to separate variables or to separate factors.

4. EMPIRICAL RESULTS

The empirical analysis in this paper is divided in two phases. In the first phase the long-term stability of the factor loadings is investigated by using the three different samples collected from the Helsinki Stock Exchange (sample 1) and the Stockholm Stock Exchange (samples 2 and 3), and introduced in Section 3. In the second phase the cross-sectional invariance of these three samples is considered, i.e. the aim is to find out if the contents in different samples are similar to one another.

Table 1. Cumulative proportions of total variance explained

	Sample 1		Sample 2		Sample 3	
	1977–1981	1982–1986	1977–1981	1982–1986	1977–1981	1982–1986
Factor 1	0.326	0.311	0.438	0.453	0.368	0.419
Factor 2	0.449	0.414	0.558	0.544	0.487	0.525
Factor 3	0.520	0.495	0.629	0.603	0.560	0.580
Factor 4	0.582	0.565	0.693	0.655	0.626	0.631
Factor 5	0.633	0.625	0.735	0.693	0.676	0.669
Factor 6	0.680	0.670	0.770	0.730	0.715	0.705
Factor 7	0.715	0.714	0.801	0.764	0.752	0.738

Long-term stability of factor patterns

The first step in the empirical analysis is to use factor analysis procedure to identify the number of factors affecting equilibrium returns. In earlier studies, this procedure has typically been very problematic because it has been shown that the number of factors discovered depends for example on the size of the groups of securities one deals with (see [14, pp. 345–346]). The estimation of factors can be carried out by different factor analytic methods. In this study the following procedure will be used: the initial factor extraction is carried out by the principal component method based on the covariance matrices of stock returns, and thereafter a varimax rotation is applied.

As stated above, the whole period is divided into two subperiods. In this section it is tried to find such common factors that are stable for different subperiods. For this purpose two-, three-, four-, five-, six- and seven-factor solutions were first derived for each subperiod. Cumulative proportions of total variance explained (of the unrotated factor patterns) are presented in Table 1. These results indicate that the cumulative proportions of the Swedish samples are in each case higher than the proportions of the Finnish sample and the first Swedish sample (sample 2) always outperforms the second (sample 3). Cattell's scree tests showed that 2–5 different factors for each subperiod could be found. However, there is yet no absolute guarantee that the factors extracted have the same interpretation in different subperiods, i.e. one can not be sure that the presented factors are the expected common factors.

Next, the stability of factor patterns over time is measured via transformation analysis. The conclusion about stability is based on the coefficients on the main diagonal of the transformation matrix provided that factors in different samples are produced in the same order. The numerical values of those coefficients are close to one when the factor structure over time is stable. Tables 2–4 present the transformation matrices between the three-factor solutions of subperiods for the three samples. The three-factor solutions are used as a starting point due to the observation carried out by Yli-Olli and Virtanen [35] who found the three-factor solution to be relatively stable in Finnish factor patterns.

The results in Tables 2 and 3 indicate that the stability of the factors in the three-factor solutions has been very high between the two subperiods when the first two samples are considered. This means there are at least three very stable factors in these two samples. Tables 2 and 3 also show that the two successive subperiods for these two samples have produced the first and the second factors in different order. That means the first and second factors have changed their positions in the second subperiod as compared to the first subperiod. However, the three-factor solutions for the third sample indicate that the factors in this sample have not been as stable as in the two first samples. One potential reason for that could be the effect of infrequent trading discussed above. The factor loadings may be biased due to nonsynchronous trading especially in this sample because of the fact that it consists of relatively infrequently traded stocks. Application of transformation analysis

Table 2. Transformation matrix between the factor patterns of returns, three-factor solution. Sample 1

		Subperiod 2		
	Factor	1	2	3
Subperiod 1	1	0.127	0.987	0.094
	2	0.992	−0.127	−0.002
	3	−0.011	−0.094	0.996

Table 3. Transformation matrix between the factor patterns of returns, three-factor solution. Sample 2

		Subperiod 2		
	Factor	1	2	3
Subperiod 1	1	0.365	0.931	0.003
	2	0.929	−0.365	0.059
	3	−0.056	0.019	0.998

Table 4. Transformation matrix between the factor patterns of returns, three-factor solution. Sample 3

		Subperiod 2		
	Factor	1	2	3
Subperiod 1	1	0.597	−0.009	0.802
	2	0.530	0.755	−0.386
	3	−0.602	0.656	0.456

for the two-factor solutions, however, indicated that there has existed two stable factors in the third sample. The large, close-to-one elements in this transformation matrix were 0.998 and 0.998.

As stated above, the stability of the factors in the two first samples was found to be high in the three-factor solutions. Thus, it is naturally interesting to study how the stability is affected when four- and five-factor solutions for these samples are studied. The results gave support to the finding of instability in these factor patterns. In this context, the four and five-factor solutions seemed to be clearly outperformed by the three-factor solutions. This was especially true for the Finnish sample. Thus, the conclusion is that maximum number of common factors in the two first samples is three.

The following step involved examining the effect of factors on equilibrium returns. In cross-sections the dependent variable is the monthly mean return and the independent variables are factor loadings from factor analysis. The risk-free rate of return was not restricted but estimated from the stock returns, assuming the intercept term to represent this return. The OLS regression coefficients are the estimated risk premia. In factor analysis there is no absolute meaning to the signs of the parameters or the scaling of the factors, and then also the signs of regression coefficients are arbitrary. Therefore, only the statistical significance of regression coefficients is relevant instead of their numerical values.

The results of the cross-sectional regression (three-factor solutions) are presented in Table 5. They indicate that in the first subperiod at least two different factors are priced. On the other hand, concerning the first two samples, the transformation analysis showed that we can extract three factors with the same content in different subperiods. The seeming inconsistency of the results rises from the fact that in the first two samples transformation analysis gave the number of factors which have the same content in different subperiods, i.e. it gave the maximum possible number for priced, stable common factors. Regression analysis on the other hand gave the actual number of priced common factors. It is possible that some very stable factors are so firm- or industry-specific that they are not common. This is revealed by low t-statistics in cross-sectional regressions. However, it is very important to remember that transformation analysis is necessary in testing if the contents of factors in different subperiods are the same.

In the second subperiod the rates of determinations in the regression equations are considerably lower than in the first subperiod. An interesting observation is the stability of the results in this sense. The APT seems not to work as well in the second subperiod as it did in the first subperiod in any of the samples. This might

Table 5. Regression analysis estimates

	Coefficients of					
Sample	Constant	Fact 1	Fact 2	Fact 3	R-square	F
			Subperiod 1			
1	0.0179	−0.0162	−0.0036	0.0089	0.395	5.663
	(3.898)	(−2.448)	(−0.584)	(1.431)		
2	0.0145	0.0099	−0.0066	−0.0208	0.392	5.598
	(2.104)	(1.280)	(−0.789)	(−2.617)		
3	0.0076	0.0026	−0.0214	−0.0233	0.613	13.217
	(0.927)	(2.496)	(−2.116)	(−2.637)		
			Subperiod 2			
1	0.0156	0.0150	0.0123	0.0065	0.114	1.075
	(2.179)	(1.574)	(1.371)	(0.629)		
2	0.0144	0.0076	0.0119	0.0037	0.160	1.647
	(2.854)	(1.244)	(2.120)	(0.596)		
3	0.0280	0.0052	−0.0048	−0.0104	0.074	0.695
	(3.034)	(0.525)	(−0.409)	(−1.009)		

Dependent variable: average monthly return for security. Independent variables: factor loadings ($k = 3$).
(t-values in parentheses).

be due to the fact that the price behaviour in the second subperiod was relatively unstable in both countries. In addition, the cross-sectional regression analysis was also performed for the four-factor solutions. The fourth factor had only a bit more explanatory power compared to the three-factor solution (*F*-values were typically lower in the four-factor solution).

Cross-country similarity of factor patterns

As stated above, transformation analysis makes it also possible to compare the factor loadings in cross-sectional samples. In the first phase, the cross-country similarity between the factor patterns of the most frequently traded stocks, i.e. the similarity between factor patterns of samples 1 and 2 was studied. Again, the analysis was started from the three-factor solutions. In the second subperiod, the results gave support to the existence of three common factors across countries. In the first subperiod, however, only two common factors could be found. Therefore, the following analysis is concentrated on the two-factor models only.

The resulting transformation matrices based on the two-factor solutions are presented in Tables 6 and 7. The results clearly suggest that there exist two common factors in Finland and Sweden in both subperiods. An interesting observation is that in the first subperiod these factors have been produced in different order in different countries. Altogether, how these two common factors are produced in different samples in different time periods is presented in Appendix 2.

5. SUMMARY

The main purpose of this study was to test the APT using monthly time series data of Scandinavian firms quoted on the Helsinki Stock Exchange and on the Stockholm Stock Exchange, and as a part of this, especially to test, using transformation analysis, the stability of the factor structure over time and across different samples. That means: via transformation analysis it is possible to discover whether the content of the factors remains the same in different time periods and also in different samples (even across countries) during the same time period.

The empirical verification of the APM involved in the first stage the estimation of the systematic risk components for each asset using factor analysis. The second stage involved testing by transformation analysis if the number and structure of factors remained unchanged or stable across different time periods and between different samples in two Scandinavian stock exchanges. For stability analysis the whole period was divided into three samples (one Finnish, two Swedish) and into two subperiods: 1977–1981, and 1982–1986. The factor and transformation analysis showed that at least three very stable factors could be found for the samples where only the most frequently traded stocks were included. When, on the other hand, the sample consisted of more infrequently traded stocks, only two stable factors could be found. This might well be due to the problems measuring returns and estimating factor loadings for the infrequently traded stocks. Thus, our results indicate that infrequent trading might cause significant problems when testing the Arbitrage Pricing Theory. In addition, the similarity of factor patterns across different samples was also tested. The results suggested that now two stable factors common to different samples could be found.

In addition, the effects of factors on equilibrium returns were also examined. The results from the cross-sectional regressions indicated the relatively high level of similarity between factor patterns in the same subperiods. In the first subperiod, the APM seemed to perform relatively well, but in the second subperiod the results were not as encouraging.

Table 6. Transformation matrix between the factor patterns of returns, two-factor solution. Sample 1 vs Sample 2. Subperiod 1

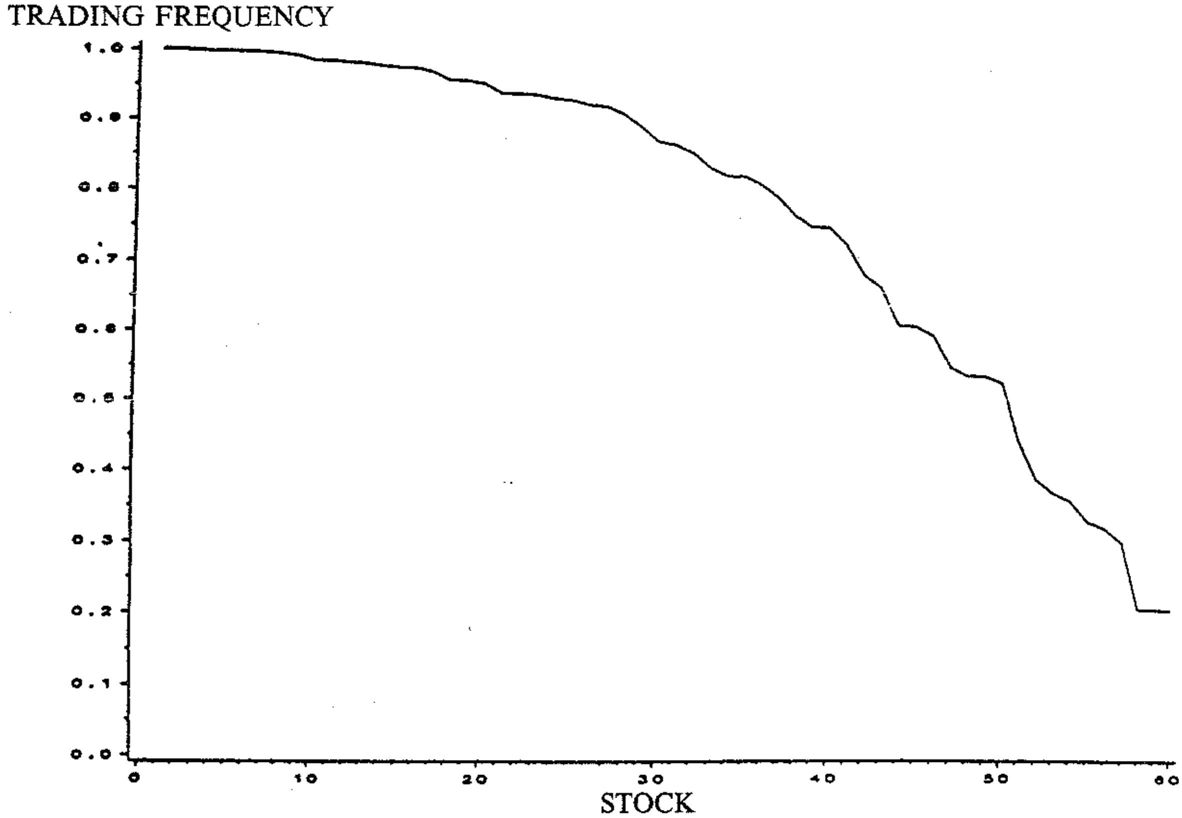
		Sample 2	
	Factor	1	2
Sample 1	1	0.391	0.920
	2	0.920	−0.391

Table 7. Transformation matrix between the factor patterns of returns, two-factor solution. Sample 1 vs Sample 2. Subperiod 2

		Sample 2	
	Factor	1	2
Sample 1	1	0.960	−0.281
	2	0.281	0.960

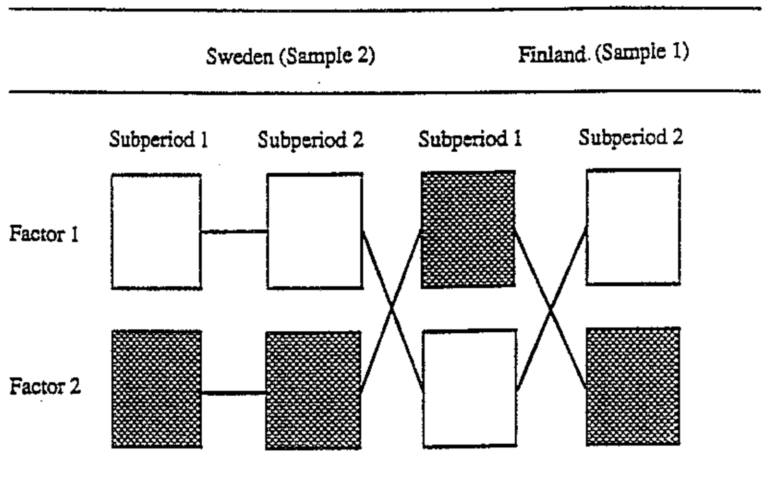
APPENDIX 1

Plot of Trading Frequency in Swedish Stocks



APPENDIX 2

Cross-Country Similarity of Factor Patterns. Two-Factor Solutions



APPENDIX 3

Transformation Analysis

First, it is assumed that there exist two groups of observations G_1 and G_2 with the same variables, both by number and content. Let $\mathbf{L}_1$ and $\mathbf{L}_2$ be the factor matrices for G_1 and G_2, respectively. It is further assumed that the factor models used in deriving $\mathbf{L}_1$ and $\mathbf{L}_2$ are both orthogonal and have the same dimension, $p \times r$, say.

If there exists invariance between the two factor structures, there exists a nonsingular $r \times r$-matrix $\mathbf{T}$ such that equation

$$\mathbf{L}_2 = \mathbf{L}_1\mathbf{T}_{12} \tag{3}$$

holds. Matrix $\mathbf{T}_{12}$ is called the transformation matrix (between $\mathbf{L}_1$ and $\mathbf{L}_2$, or in direction $G_1 \rightarrow G_2$). If equation (3) holds exactly, it means that the factor structures in groups G_1 and G_2 are, up to a linear transformation, invariant, all the variables have the same empirical meaning in different groups. Depending on the type of the transformation matrix $\mathbf{T}_{12}$, the formation of the factors from the variables and thereby the interpretation of the factors either is preserved ($\mathbf{T}_{12}$ is the identity matrix $\mathbf{I}$) or it changes ($\mathbf{T}_{12}$ has also non-zero off-diagonal elements).

In practice, situation (3) will not be reached, but, after matrix $\mathbf{T}_{12}$ has been estimated, one has $\mathbf{L}_2 \neq \mathbf{L}_1\mathbf{T}_{12}$. The goodness of fit criterion for the model (3) may be based on the residual matrix

$$\mathbf{E}_{12} = \mathbf{L}_1\mathbf{T}_{12} - \mathbf{L}_2. \quad (4)$$

Non-zero elements in $\mathbf{E}_{12}$ mean that the empirical meaning of the variables in question has changed. This is called abnormal transformation.

The main problem in transformation analysis is the estimation of the matrix $\mathbf{T}_{12}$. The estimation methods are in general based on the minimization of the sum of squares of the residuals e_{ij} (the elements of the residual matrix $\mathbf{E}_{12}$). This is the common method of least squares. The problem is to minimize

$$\|\mathbf{E}_{12}\| = \|\mathbf{L}_1\mathbf{T}_{12} - \mathbf{L}_2\| = \mathrm{trace}((\mathbf{L}_1\mathbf{T}_{12} - \mathbf{L}_2)(\mathbf{L}_1\mathbf{T}_{12} - \mathbf{L}_2)'). \quad (5)$$

Depending on additional constraints set for the matrix $\mathbf{T}_{12}$, three different estimation methods, i.e. three transformation analysis models are obtained (see e.g. [36]). If there are no constraints for $\mathbf{T}_{12}$ in minimizing (5) the transformation analysis model is called the naive model. The relativistic model is obtained if the transformation analysis model is required to obey the transitivity property, $\mathbf{T}_{kl}\mathbf{T}_{lm} = \mathbf{T}_{km}$. Finally, if the transformation matrix is required to be orthogonal, the symmetric model is obtained. Of these three techniques, the symmetric transformation analysis is the most popular one. It is also applied in this study.

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ADDRESS FOR CORRESPONDENCE: *Teppo Martikainen, University of Vaasa, PO Box 297, SF-65101 Vaasa, Finland.*