

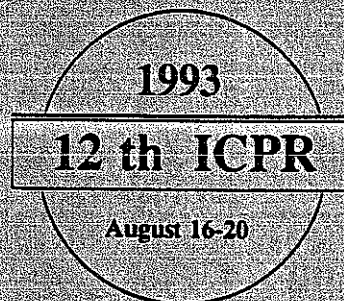
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Economic Production and Order Quantities for Jointly Replenished Items

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Abstract

The problem of determining economic packaging frequency of jointly replenished items has received a lot of attention from production researchers. This problem is usually encountered while packaging the products after completing the manufacture. However, production lot-sizing certainly influences the ordering quantity for jointly replenished items. Considering the importance of the relationship between order quantity for jointly replenished items and economic production quantity, a mathematical model is proposed for determining optimal inventory policies in a multi-stage production-inventory system. An example problem is solved to illustrate the application and behaviour of the model.

1. INTRODUCTION

Packaging is quite important especially for small volume products and batch packaging both for industrial and consumer goods. The problem is usually encountered when an unpackaged product must be packed into a number of packaging sizes immediately after manufacture. The economic advantages of packaging such items jointly is that if items are replenished individually then each item is accountable for full set-up costs involved in undertaking the manufacture of the unpackaged product and for packaging the unpackaged product. For a complete introduction to the problem see Goyal (1973a,b). The aspects of production lot-size and packaging sizes are very much inter-related especially, wherein the products are manufactured in multi-stage production systems. A common characteristics of all the suggested methods is that the manufacturing runs for producing the unpackaged products are undertaken at equal intervals. Goyal (1974) developed an enumerative optimal solution procedure for determining the optimal policy when the manufacturing set-ups are undertaken at equal intervals. In this paper, we develop a model for integrating the decision making of optimal production and packaging policies. The basic criterion considered for determining the economic production and packaging quantities is the minimization of total system cost per unit time.

2. THE MODEL

The production system considered for modelling is a flow-shop serial batch-production system. The following notations are used in the model: m -total number of products; n -total number of production stages; i -product index ($i = 1, 2, \dots, m$), j -stage index ($j = 1, 2, \dots,$

n), D_i -demand for product i per unit time or per year; A_{ij} -setup cost per set-up for product i at stage j ; Q_{ij} -batch size for product i at stage j ; T -Manufacturing cycle time of products on the last processing facility m (decision variable); h_{ij} -value added inventory cost per unit per year, for product i at facility j ; H_i -Inventory cost of finished product per unit per year, for product i at facility j ; S_i -cost of a packaging set-up; k_{ij} -A positive integer; K_i - ratio between frequency of manufacturing set-ups for the unpackaged product and the frequency of packaging set-ups for the item (note that K_i must be an integer and the time interval between packaging set-up is given by $T.K_i$). (decision variables); and Z -total system cost.

The following assumptions are made in developing the model: (i) demand per unit time of a product is deterministic and known, (ii) instantaneous processing of products at different stages of production, (iii) products will be dispatched for packaging processing is completed at the final stage, and (iv) rate of packaging is infinite, that is, the packaging time is negligible. The total system cost consists of the following costs: (i) set-up cost, (ii) inventory cost of manufacturing, (iii) set-up cost for packaging of items, and (iv) inventory cost of packaged items. In this paper, we use condition for the optimal policies proposed by Korgaonker (1979) for multi-item case for evolving the joint replenishment problems considering the appropriate costs factors. The total system cost can be obtained as

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n \frac{A_{ij}}{T \left(\prod_{q=j}^n k_{iq} \right)} + \sum_{i=1}^m \sum_{j=1}^n \frac{\left(\prod_{q=j}^n k_{iq} \right) D_i T h_{ij}}{2} \\ + \frac{1}{T} \sum_{i=1}^m \left\{ \frac{S_i}{K_i} \right\} + \frac{T}{2} \sum_{i=1}^m D_i K_i H_i$$

3. RESULTS AND CONCLUSIONS

A direct pattern search method (Hooke and Jeeves, 1966) has been employed for determining the economic production and packaging quantities by minimizing the total system cost per unit time. The results obtained using this search method (can be obtained from authors on request) illustrate the influence of production lot-sizing policies on the packaging policies of items and potential advantages of integrating the production-packaging decisions. Since the model is based on a number of assumptions and approximations, there are avenues for further investigations.

4. REFERENCES

- Goyal, S.K., 1973a, *Int. J. Prod. Res.*, 11, 195.
- Goyal, S.K., 1973b, *Mgmt. Sci.*, 21, 436.
- Goyal, S.K., 1974, *Mgmt. Sci.*, 20, 232-235.
- Goyal, S. K., 1988, *Int. J. Prod. Res.*, 26, 1237.
- Hooke, R. and Jeeves, J.A., 1966, *J. Assoc. Comp. Mach.*, 8, 212.
- Korgaonker, M.G., 1979, *J. Opl. Res. Soc.*, 30, 355-362.