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Adequacy of depreciation allowances under inflation

1. INTRODUCTION

The present corporate tax law in Finland (= EVL) determines that depreciation on fixed assets is to be based on the original purchase prices. If the general level of prices is rising, the sum of depreciation allowances for tax purposes fall short of the costs of replacing the asset in question. The higher the rate of inflation is, the wider becomes the difference between the purchase price of a new asset and the sum of tax deductible depreciation allowances. Depreciation based on original purchase prices thus increases the taxable profit which is measured in nominal terms.[1]

As the real value of tax deductible depreciation under inflationary conditions is continuously decreasing, it would be beneficial from the firm's point of view to utilize these allowances as soon as possible within the economic life of the asset.[2] Also the present value of depreciation tax shield increases if the depreciation allowances can be made at the earliest possible time. Therefore, accelerated methods of depreciation provide at least a partial hedge against inflation. However, a company must have profits against which to write off the accelerated depreciation allowances.

Riistama [3] has analysed the adequacy of the present EVL tax allowances by comparing the present value of EVL determined depreciation amounts with that derived from using the realization method of depreciation in the light of specific examples. In the case, where investment is equity financed,

[1] Honko (1973), pp. 177—180.

[2] Inflation constitutes another factor, in addition to time preference, which reduces the present value of depreciation. See Davidson (1975), p. 1183.

[3] Riistama (1975), pp. 67—72.

he calculated that rate of interest which equates the present value of depreciation based on the declining balance method, which is the EVL approach to depreciation calculations, with the present value that is derived from the realization method, i.e. one based on the real rate of return on investment, assuming that stable price level prevails. He showed that the former present value would remain unaffected by inflation in real terms, if the discount rate in the declining balance method were adjusted for inflation. This corrected discount rate may be expressed as the sum of the following three items: the real rate of discount, the rate of inflation, and the product of these two terms.[4] If the declining balance method produces a higher present value of depreciation than the realization method, the investment in question may be said to tolerate inflation. This tolerance is the higher the wider this difference in favour of the former method is. For example, it is demonstrated that an equity financed investment in machinery with a yearly rate of return of 10 % and a length of life of 10 years is still viable if the annual rate of inflation is 5 %. Inflation tolerance increases, if debt capital can be used in financing the project.[5]

It has been argued by some commentators of the present corporate tax law that the 30 % depreciation allowance for tax purposes that applies to machinery and equipment is accelerated enough in itself to take inflation into account. However, no systematic analysis of the adequacy of the present EVL allowances in inflationary conditions has been undertaken. Since realization is commonly regarded as the correct basis for determining corporate income, it seems justified to employ the realization method of depreciation as the measuring rod against which the adequacy of EVL depreciation allowances is to be judged. A realization depreciation may be interpreted as the present value which is arrived at, if the annual return on investment is discounted by the project's internal rate of return.[6]

This paper analyses the *concept of inflation tolerance* by using the present values of depreciation in calculating it for equity financed investments. The objective is to analytically determine the inflation rate which makes the inflation adjusted present value based on the declining balance method equal to the present value of realization depreciation under stable prices. Only investment in fixed assets is included in the analysis. In Section 2, the relevant present value expressions for both these methods of depreciation are derived, together with the expression for inflation tolerance. Section 3 examines the relationships between the inflation tolerance on the one hand,

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4 See also Aho (1979), p. 301 and pp. 305—306.

5 Riistama (1975), pp. 70—72.

6 For the notion of income underlying this concept, and for a closer discussion of the method, see Saario (1969), pp. 207—209.

and the profitability of the investment, its economic life and the rate of depreciation on the other, and derives the absolute upper bound for inflation tolerance. This section also contains an analysis which aims at determining those rates of depreciation and service lives which would suffice for protecting the investment against such rate of inflation which on average prevailed in Finland during the 1970's. The discussion ends with a summary and an appraisal of the adequacy of the EVL depreciation allowances.

2. DETERMINING THE INFLATION TOLERANCE OF DEPRECIATION

2.1. Present Values of the Depreciation Series

In the following, a fixed asset investment of size C and with a constant annual return, P_t, is examined using continuous discounting.[7] If the (real) internal rate of return on the investment is denoted by i, P_t can be expressed as

$$(2.1)\quad P_t = \bar{c}_{n\,|\,i}\, C = \frac{C}{\bar{a}_{n\,|\,i}}$$

where $\bar{c}_{n\,|\,i} = \dfrac{e^{i} - 1}{1 - e^{-ni}}$ = the annuity factor for discrete payments made at the end of each period, and

$\bar{a}_{n\,|\,i} = \dfrac{1 - e^{-ni}}{e^{i} - 1}$ = the present value factor for discrete payments made at the end of each period.

The realization depreciation in year t, D^R_t, equals the present value of P_t discounted by the internal rate of return on the investment, i.e.

$$(2.2)\quad D^R_t = P_t e^{-it}.$$

The present value for D^R_t, NPV (D^R_t), equals

$$(2.3)\quad \text{NPV}\,(D^R_t) = D^R_t e^{-it} = P_t e^{-2it},$$

and the sum of all realization depreciation amounts:[8]

$$(2.4)\quad \text{NPV}\,(D^R) = \sum_{t=1}^{n} \text{NPV}\,(D^R_t) = \frac{C}{\bar{a}_{n\,|\,i}} \sum_{t=1}^{n} e^{-2it} = C\,\frac{\bar{a}_{n\,|\,2i}}{\bar{a}_{n\,|\,i}}.$$

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7 For the justification of using continuous method of discounting, see Aho — Virtanen (1981), pp. 4—5.

8 For the derivation of (2.4), see Aho — Virtanen (1981), p. 19.

According to EVL, depreciation is based on the declining balance method. In year t, the declining balance based depreciation, D_t^{DB}, is

(2.5) $D_t^{DB} = j(1 - j)^{t-1}C, \qquad t = 1, 2, \ldots, n - 1.$

In the last year of the length of life for the investment, year n, (2.5) must be supplemented with an additional depreciation of $(1 - j)^n C$ so as to completely write off the original purchase price. Therefore:

(2.6) $NPV\ (D_n^{DB}) = j(1 - j)^{n-1}Ce^{-in} + (1 - j)^n Ce^{-in},$

and from (2.5) and (2.6).

$$(2.7)\quad NPV\ (D^{DB}) = \sum_{t=1}^{n} j(1 - j)^{t-1}Ce^{-it} + (1 - j)^n Ce^{-in} = \frac{j + (e^i - 1)e^{-in}(1 - j)^n}{e^i - (1 - j)}\ C.$$

Inflation reduces the present value of depreciation amounts based on the declining balance method. This is taken into account by adjusting the discount rate, i, used in calculations by the rate of inflation, s. Therefore, the inflation adjusted rate of discount becomes the sum i + s.[9] The real present value of depreciation under the declining balance option is then:

$$(2.8)\quad NPV\ (D^{DB}) = \frac{j + (e^{i+s} - 1)e^{-n(i+s)}\,(1 - j)^n}{e^{i+s} - (1 - j)}\ C.$$

The present value expression (2.8) can be approximated by the present value based on infinite horizon, i.e.[10]

$$(2.9)\quad NPV\ (D^{DB}) = \frac{j}{e^{i+s} - (1 - j)}\ C.$$

2.2. Determining the inflation tolerance of depreciation based on the declining balance method

In the case of an equity financed investment, the inflation tolerance may be found by solving for that rate of inflation, s, from expression (2.8) or

[9] The product term does not appear in the formula, because continuous discounting is used. See Aho — Virtanen (1981), pp. 7—8.

[10] Cf. Kettunen (1976), p. 202.

(2.9), which results in NPV (D^R) = NPV (D^{DB}).[11] First, the expression for NPV (D^R), (2.4), is examined. If the interest factor e^i is denoted by R, it follows that [12]

$$(2.10)\quad NPV\,(D^R) = C\,\frac{\bar{a}_{n\,|\,2i}}{\bar{a}_{n\,|\,i}} = C\,\frac{1-e^{-2ni}}{e^{2i}-1}\cdot\frac{e^i-1}{1-e^{-ni}}$$
$$= C\,\frac{1-R^{-2n}}{R^2-1}\quad\frac{R-1}{1-R^{-n}}$$
$$= C\,\frac{(1-R^{-n})\,(1+R^{-n})}{(R-1)\,(R+1)}\quad\frac{R-1}{1-R^{-n}}$$
$$= C\,\frac{1+R^{-n}}{R+1}\,.$$

Correspondingly, the present value NPV (D^{DB}) becomes

$$(2.11)\quad NPV\,(D^{DB}) = C\,\frac{j+(RS-1)\,(RS)^{-n}(1-j)^n}{RS-(1-j)}$$
$$= C\,\frac{j+(RS-1)\,(\frac{1-j}{RS})^{n}}{RS-(1-j)}\,,$$

where S is used to denote e^s, the inflation factor.

Inflation tolerance [13] is determined by the following expression, derived from (2.10) and (2.11):

$$(2.12)\quad \frac{1+R^{-n}}{1+R} = \frac{j+(RS-1)\,(\frac{1-j}{RS})^n}{RS-(1-j)}\,.$$

It is not possible to solve for the inflation factor S, and consequently the rate of inflation, s, analytically. However, for given values of R, j and n a numerical solution is always possible.

From the point of view of further analysis, the functional dependency of s, inflation tolerance, on i, j, and n must be determined. This may be accomplished by employing the approximation formula (2.9) for the present value in the declining balance method. In this case

$$(2.13)\quad \frac{1+R^{-n}}{1+R} = \frac{j}{RS-(1-j)}$$

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11 See Riistama (1975), p. 70.

12 e^i corresponds to the term $1+i$ in discrete discounting.

13 Measured first in terms of S, the inflation factor, and consequently in terms of $s = \ln S$.

which first results in

$$(2.14)\quad RS - (1 - j) = j\,\frac{1+R}{1+R^{-n}}$$

from which S can be solved for as:

$$(2.15)\quad S = \frac{1}{R}\,[1 - j + j\,\frac{1+R}{1+R^{-n}}]$$
$$= \frac{1}{R}\,[1 + j\,\frac{R - R^{-n}}{1+R^{-n}}]$$
$$= \frac{1}{R}\,[1 + j\,\frac{R^{n+1} - 1}{R^{n}+1}]$$

Taking into account the definitions $e^s = S$ and $e^i = R$, it is possible to write

$$(2.16)\quad e^s = e^{-i}\,[1 + j\,\frac{e^{(n+1)i} - 1}{e^{ni}+1}],$$

which in its logarithmic form produces the following formula for inflation tolerance

$$(2.17)\quad \boxed{s = -i + \ln[1 + j\,\frac{e^{(n+1)i} - 1}{e^{ni}+1}].}$$

It may be seen from (2.17) that inflation tolerance depends on three variables: the profitability of investment (the discount rate), its length of life and the rate of depreciation, i.e. $s = s(i, n, j)$. In practice, the most important partial dependencies are $s = s(i)$ and $s = s(n)$, since j may be regarded as relatively fixed by corporate tax laws.

Expression (2.17) gives inflation tolerance as an approximation since the underlying present value formula was based on the assumption of infinite horizon. The error in this approximation becomes greater, as the life of the investment under investigation shortens. Table 2.1. gives examples of the magnitude of this error under different combinations of economic life, rate of discount and rate of depreciation. If the length of life is 10 years, the approximation error is 0.1 %, while with economic life of 20 years or over, no error arises. In the case where the life is 7 years, the approximation error equals 0.6 %. The approximativeness of the above formula must, therefore, be born in mind, when conclusions in connection with very short lengths of actual economic life are made.[14]

[14] It may be noted that the present value formula based on infinite length of life has been applied to finite cases by e.g. Riistama (1975), p. 68, Kettunen (1976), pp. 202—203 and Yli-Räisänen (1977), pp. 341—342.

Table 2.1. Examples on the Magnitude of the Approximation Error.

n	i	j	$s_{appr.}$	s_{exact}	Error
7	0.10	0.30	0.015	0.021	
7	0.20	0.30	0.011	0.017	— 0.006
7	0.30	0.30	— 0.016	— 0.010	
10	0.10	0.30	0.050	0.051	
10	0.20	0.30	0.052	0.053	— 0.001
10	0.30	0.30	0.016	0.017	
20	0.10	0.30	0.128	0.128	
20	0.20	0.30	0.103	0.103	0.000
20	0.30	0.30	0.039	0.039	
40	0.10	0.10	0.001	0.001	
40	0.20	0.10	— 0.085	— 0.085	0.000
40	0.30	0.10	— 0.173	— 0.173	
60	0.10	0.10	0.004	0.004	
60	0.20	0.10	— 0.085	— 0.085	0.000
60	0.30	0.10	0.173	— 0.173	

Next section analyses the dependency of inflation on the profitability of the investment, its length of life and the rate of depreciation, as shown by expression (2.17), in greater detail.

3. ANALYSING THE INFLATION TOLERANCE OF DEPRECIATION

3.1. Inflation tolerance as a function of the profitability of the investment

In what follows, the length of life, n, and the rate of depreciation, j, are assumed to be fixed. The investment under investigation is a marginal project financed from equity sources, whose internal rate of return equals the rate of discount. The functional relationship $s = s(i)$ is analysed by letting i vary within the limits $(0, \infty)$. It may be noted from (2.17) that if i equals zero, s(0) equals zero as well. Thus, the function $s = s(i)$ passes through the origin. Correspondingly, as i approaches infinity, the limit for inflation tolerance can be found as follows:

$$(3.1)\quad \lim_{i\to\infty} s(i) = \lim_{i\to\infty}\{-i + \ln[1 + j\,\frac{e^{(n+1)i} - 1}{e^{ni} + 1}]\}$$
$$= \lim_{i\to\infty} \ln\{e^{-i}[1 + j\,\frac{e^{(n+1)i} - 1}{e^{ni} + 1}]\}$$
$$= \ln\{\lim_{i\to\infty}\ [e^{-i} + j\,\frac{1 - e^{-(n+1)i}}{1 + e^{-ni}}]\}$$
$$= \ln j.$$

Thus, the limit of inflation tolerance, as i approaches infinity, is given by the natural logarithm of the rate of depreciation. As $0 < j < 1$, it follows that $\lim_{i\to\infty} s(i) < 0$. Within the interval $(0, \infty)$ the function s(i) may be analysed using the partial derivative $\frac{\partial s}{\partial i}$. It equals

$$(3.2)\quad \frac{\partial s}{\partial i} = -1 + j\,\frac{(n+1)e^{(n+1)i}[e^{ni} + 1] - ne^{ni}[e^{(n+1)i} - 1]}{[1 + j\,\frac{e^{(n+1)i} - 1}{e^{ni} + 1}]\,[e^{ni} + 1]^2}$$
$$= -1 + j\,\frac{e^{ni}[e^{(n+1)i} + (n+1)e^{i} + n]}{(e^{ni} + 1)[(e^{ni} + 1) + j(e^{(n+1)i} - 1)]}$$
$$= \frac{-e^{2ni} + jne^{(n+1)i} + [j(n+1) - 2]e^{ni} - (1 - j)}{(e^{ni} + 1)\,[(e^{ni} + 1) + j(e^{(n+1)i} - 1)]}.$$

At the origin, (3.2) equals

$$(3.3)\quad [\frac{\partial s}{\partial i}]_{i=0} = -1 + \frac{j(n+1)}{2}.$$

Therefore, this expression is positive, if

$$(3.4)\quad -1 + \frac{j(n+1)}{2} > 0,$$

i.e. one obtains the condition

$$(3.5)\quad j > \frac{2}{n+1} \qquad\qquad ([\frac{\partial s}{\partial i}]_{i=0} > 0).$$

This implies that as the profitability of the investment becomes positive, *depreciation based on the method of declining balance tolerates inflation (at least during the initial phases), if the rate of depreciation j exceeds the ratio 2/ (length of life + 1)*. Alternatively, this condition may be presented as $n > 2/j - 1$.

The value of the partial derivative at the origin equals zero, if

$$(3.6)\quad j = \frac{2}{n+1} \qquad ([\frac{\partial s}{\partial i}]_{i=0} = 0),$$

or if $n = 2/j - 1$. And at last, the derivative at the origin is negative, if

$$(3.7)\quad j < \frac{2}{n+1} \qquad ([\frac{\partial s}{\partial i}]_{i=0} < 0).$$

The order of magnitude of the quantities j and $2/(n + 1)$ is therefore decisive for the behaviour of the function $s = s(i)$ at the origin. The following discussion demonstrates that behaviour at the origin determines the qualitative behaviour of the function over the entire interval $(0, \infty)$.

First, *the case* $j > 2/(n + 1)$ or $n > 2/j - 1$, is examined. At the origin $s(0) = 0$ and $[\frac{\partial s}{\partial i}]_{i=0} > 0$. Inflation tolerence becomes positive as the profitability of investment increases (from zero). On the other hand, if i is high enough, it follows that $\frac{\partial s}{\partial i} < 0$. The denominator of (3.2) is always positive, wherefore the sign of the whole expression is given by the sign of the numerator. Under high values of i, this, in turn, is determined by the highest power of e^i, which in this case equals $(e^i)^{2n} = e^{2ni}$. Its coefficient is negative, which implies that under high i-values the sign of the partial derivative is negative as well, i.e. inflation tolerance is decreasing. Taking into account the earlier result: $\lim_{i \to \infty} s(i) < 0$, a typical pattern for the function $s = s(i)$ may be depicted as in Diagram 3.1. If the internal rate of return on the investment equals zero, depreciation does not tolerate inflation, i.e. $s(0) = 0$. As the profitability of the investment increases, inflation tolerance becomes positive $(\frac{\partial s}{\partial i} > 0)$, and increases up to a certain value of profitability, i_m, at which point it achieves its maximum value (s_m). After that internal rate of return, i_m, inflation tolerance is decreasing $(\frac{\partial s}{\partial i} < 0)$, until at i_0 it completely disappears. From i_0 onwards, s takes increasingly larger negative values as i increases, approaching its mathematical limit ln j.

The profitability of investment, i_m, which corresponds to the maximal inflation tolerance (s_m) can be solved for from the condition $[\frac{\partial s}{\partial i}]_{i=i_m} = 0$, i.e. by solving the following equation with respect to i:

$$(3.8)\quad -e^{2ni} + jn\ e^{(n+1)i} + [j(n+1) - 2]e^{ni} - (1 - j) = 0.$$

Denoting e^i again by R, (3.8) becomes

$$(3.9)\quad -R^{2n} + jn\ R^{n+1} + [j(n+1) - 2]R^n - (1 - j) = 0.$$

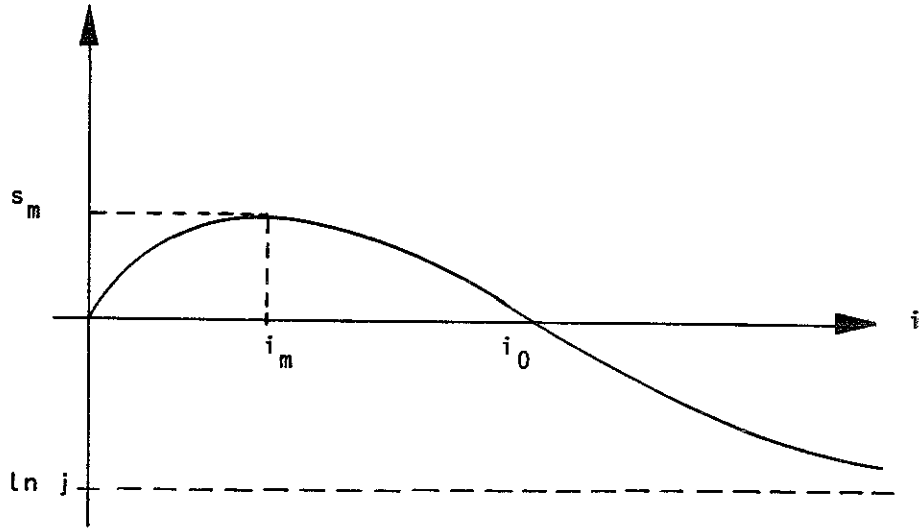

Diagram 3.1 Inflation Tolerance as a Function of the Profitability of Investment When $j > 2/(n + 1)$.

The left-hand side of the above equation is a polynomial of order 2n in terms of R, the roots of which comprise the solutions for (3.9). Only the solution which is realized when $R \geq 1$ (or $i \geq 0$) is of interest here, i.e. negative internal rates of return are not considered. It is not possible to solve for R explicitly as a function of j and n. Numerical solutions based on given combinations of j and n are, however, easy to arrive at. After locating the point R_m, the profitability, i_m, which corresponds to the maximal inflation tolerance is found as the logarithm of R_m:

$$(3.10) \quad i_m = \ln e^{i_m} = \ln R_m$$

and the maximal inflation tolerance from the expression:

$$(3.11) \quad s_m = -i_m + \ln[1 + j \frac{e^{(n+1)i_m} - 1}{e^{ni_m} + 1}].$$

That profitability i_0, which results in the disappearance of inflation tolerance, can be solved for from the condition

$$(3.12) \quad s(i_0) = 0,$$

i.e. i_0 is the solution for

$$(3.13) \quad -i + \ln[1 + j \frac{e^{(n+1)i} - 1}{e^{ni} + 1}] = 0.$$

Once again, only solutions where $i > 0$ are of interest.[15] (3.13) can also be written as

(3.14) $(1 - j)R^{n+1} - R^n + R - (1 - j) = 0,$

where R is used to denote e^i.

The largest root R_0 of the polynomial above, which is of order $n + 1$, may be found numerically once n and j have been fixed. The corresponding profitability is obtained from

(3.15) $i_0 = \ln\, e^{i_0} = \ln\, R_0.$

Of course, i_0 may also be numerically solved directly from equation (3.13).

Next, the function $s = s(i)$ is examined *in the case where* $j = 2/(n + 1)$. This analysis, like the earlier one, employs partial derivatives. Furthermore, the general results $s(0) = 0$ and $\lim_{i \to \infty} s(i) = \ln j < 0$ are available. The partial derivative (3.2) may now be expressed in the condensed form:

$$(3.16) \quad \frac{\partial s}{\partial i} = \frac{- e^{2ni} + (2 - j)\, e^{(n+1)i} - (1 - j)}{(e^{ni} + 1)\,[(e^{ni} + 1) + j(e^{(n+1)i} - 1)]}$$

Since the denominator of (3.16) is always positive, the sign of the whole expression may be determined by analysing the sign of the numerator. Once again, the interest factor e^i is denoted by R, which enables the presentation of the numerator as the following polynomial:

(3.17) $P(R) = - R^{2n} + (2 - j)\, R^{n+1} - (1 - j).$

The requirement that the investment possesses a nonnegative internal rate of return ($i \geq 0$) implies that the polynomial is analysed only for values of R which fulfill the condition $R \geq 1$. Clearly,

(3.18) $P(1) = 0,$

i.e. the derivative $\frac{\partial s}{\partial i}$ equals zero at the origin ($[\frac{\partial s}{\partial i}]_{i=0} = 0$).

It is demonstrated in the following that $\frac{\partial s}{\partial i} < 0$ as $i > 0$. Differentiation of the polynomial P(R) produces:

(3.19) $P'(R) = - 2nR^{2n-1} + (2 - j)\,(n + 1)R^n.$

When $R = 1$ (or $i = 0$), (3.19) becomes

$$(3.20) \quad \begin{aligned} P'(1) &= - 2n + (2 - j)\,(n + 1) \\ &= - 2n + 2n + 2 - j(n + 1) = 0.^{16} \end{aligned}$$

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[15] $i = 0$ constitutes one of the solutions as is evident from the preceding discussion.

[16] $j\,(n + 1) = 2.$

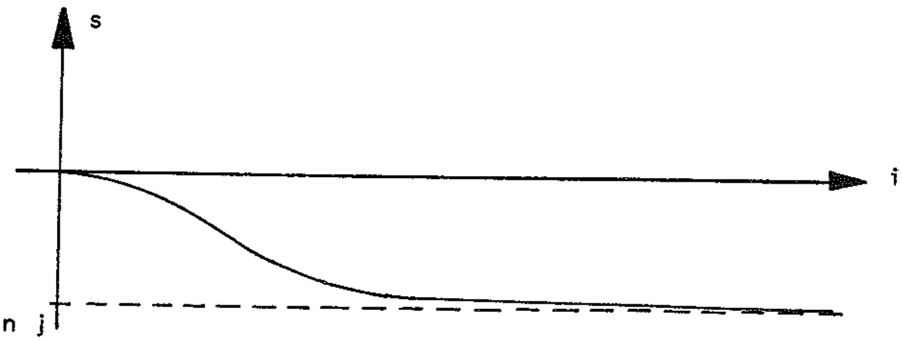

Diagram 3.2 Inflation Tolerance as a Function of the Profitability of the Investment When $j \leq 2/(n+1)$.

When $R > 1$ (or $i > 0$), (3.19) can be written as

$$(3.21)\quad \begin{aligned} P'(R) &= -2nR^{2n-1} + (2-j)(n+1)R^n \\ &= R^n[(2-j)(n+1) - 2nR^{n-1}] \\ &= R^n 2n[1 - R^{n-1}] < 0 \qquad \text{(if } n > 1). \end{aligned}$$

The results derived so far can be summarized as follows: $P(1) = 0$, $P'(1) = 0$, $P'(R) < 0$ when $R > 1$. Therefore, $P(R) < 0$ when $R > 1$. The numerator of $\frac{\partial s}{\partial i}$ is negative, and consequently the whole expression negative as well. By combining this result with the earlier ones: $s(0) = 0$ and $\lim_{i \to \infty} s(i) = \ln j$, it is possible to sketch out the general pattern of the function $s = s(i)$ in the case where $j = 2/(n+1)$, as shown in Diagram 3.2. *In this case inflation tolerance is never positive,*[17] *and increasing profitability merely serves to increase the negativity of s.*

Finally, *the case $j < 2/(n+1)$ is examined.* The general pattern of the function $s = s(i)$ resembles that derived in the preceding analysis. At the origin $s(0)$ equals 0, s is decreasing (and therefore under positive values of i always negative) and approaches ln j at the limit. The function s is even more strongly negative than in the case $j = 2/(n+1)$. This may be inferred directly from the expression for s:

$$(3.22)\quad s = -i + \ln\left[1 + j\,\frac{e^{(n+1)i} - 1}{e^{ni} + 1}\right].$$

It was shown in the preceding discussion that if $j = 2/(n+1)$ holds, the relation $s < 0$ is fulfilled for all positive values of i and all values of n ($n >$

[17] Cf. Riistama (1978), pp. 132—133.

1). Since s is monotonically increasing with respect to j,[18] moving away from the situation $j = 2/(n + 1)$ and into the situation $j < 2/(n + 1)$ must imply a reduction in s, i.e. increased negativity of s (whatever the values of n and i as long as $i > 0$ and $n > 1$).

It can be concluded from the two analyses above that if $j \leq 2/(n + 1)$ holds, positive inflation tolerance is not achieved at any level of profitability. *Therefore, the relation between the rate of depreciation and the length of life is extremely important.* If the necessary (but not sufficient) condition for positive inflation tolerance, $j > 2/(n + 1)$, is not fulfilled, depreciation based on the declining balance method does not tolerate inflation whatever the profitability of the investment in question.

Diagram 3.3 illustrates the function $s = s(i)$ under different combinations of length of life and rate of depreciation. Graph s^{I} describes the behaviour of this function when $n = 5$ years and $j = 0.30$.[19] This might, for example, be a suitable combination for n and j where the investment is an equity financed purchase of computer equipment.[20] As j falls short of $2/(n + 1)$, i.e. $0.3 < 0.33$, a project of this type has no positive inflation tolerance. The profitability of investment affects s, the inflation tolerance, by increasing its negativity with increasing i.

Diagram 3.3 Inflation Tolerance as a Function of the Profitability of the Investment Using Specific Combinations of the Length of Life, n, and the Rate of Depreciation, j.

Graph s^{II} describes the function $s = s(i)$ for a typical case of investment in machinery. The graph initially increases and reaches the maximal inflation tolerance at the value i_m of i which here equals 0.153 (15.3 %). If the internal rate of return on the investment is 15.3 %, the tolerance value is 5.7 %. This inflation tolerance may be examined in relation to, say, average annual inflation in Finland during the 1970's, 11.5 %,[21] which reveals that the 30 % depreciation allowance for tax purposes is not accelerated enough to hedge investment projects of this type against inflation.[22] If the internal rate of return on investment is high enough (i.e. $i \geq 33$ %) positive inflation tolerance ceases to exist.

Graph s^{III} applies to an investment in buildings with an economic life of 40 years and an annual depreciation of 10 %. This is calculated using the declining balance method in accordance with EVL 34:2:1. Here as well, the

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18 This, although self-evident, will later be demonstrated in analysing the effects of j on s.

19 EVL 30: 3.

20 Yritystutkimusneuvottelukunta recommends the economic life for machinery and equipment in many industries to be set at five to six years. See Yritystutkimusneuvottelukunta (1979), Appendix 2.

21 The average annual rise in the wholesale index corresponds to a rate of inflation of 10.9 % in the case of continuous discounting.

22 This will be more closely examined in section 3.5.

function initially increases with the profitability of the investment. The maximal inflation tolerance amounts to mere 2.8 %, which is reached when the rate of return equals 4.5 %. With higher rates of return, inflation tolerance begins to decrease. When $i_0 = 0.102$, inflation tolerance disappears and higher rates of return produce negative values of s. Graph s^{IV} ($n = 60$ years, $j = 0.09$) reaches the maximal inflation tolerance even sooner, i.e. at the internal rate of return of 3.8 %. This implies that the investment in question tolerates a yearly rate of inflation of 3.7 %. Inflation tolerance disappears when the internal rate of return equals 9.3 %.

On the basis of the preceding examples, it may be concluded that the present depreciation allowances stipulated by the corporate tax laws are not even nearly adequate as hedges against the harmful effects of inflation from the firm's point of view. The situation is most unsatisfactory in the case of such investments, where the condition $j > 2/(n + 1)$ cannot be fulfilled.

3.2. Inflation Tolerance as a Function of the Economic Life of the Investment

Without loss of generality, i and j are assumed to be fixed in the following analysis of $s = s(n)$. Although in practice a narrower range is possible, (depending on, say, the value of j), the analysis is accomplished over the entire interpretatively meaningful range of n, i.e. $1 \leq n \leq \infty$.

Expression (2.17) is first utilized in determining the values of s for the extreme values of n. One gets

$$\text{(3.23)} \quad \underline{s} = s(1) = -i + \ln[1 + j\frac{e^{2i} - 1}{e^{i} + 1}] = -i + \ln[1 + j(e^{i} - 1)] = \ln[e^{-i} + j(1 - e^{-i})]$$

and

$$\text{(3.24)} \quad \overline{s} = \lim_{n \to \infty} s(n) = \lim_{n \to \infty} \{-i + \ln[1 + j\frac{e^{(n+1)i} - 1}{e^{ni} + 1}]\} = \lim_{n \to \infty} \ln\{e^{-i} + j\,\frac{e^{ni} - e^{-i}}{e^{ni} + 1}\} = \ln \lim_{n \to \infty} \{e^{-i} + j\,\frac{1 - e^{-(n+1)i}}{1 + e^{-ni}}\} = \ln[e^{-i} + j].$$

Next, it will be shown that s(n) is monotonically increasing, i.e. as n grows from one to infinity, s grows from its lower bound to its upper bound. By differentiating s with respect to n, one gets

$$(3.25) \qquad \frac{\partial s}{\partial n} = \frac{j\{ie^{(n+1)i}[e^{ni}+1] - ie^{ni}[e^{(n+1)i}-1]\}}{[1 + j\,\dfrac{e^{(n+1)i}-1}{e^{ni}+1}]\,[e^{ni}+1]^2} = \frac{ije^{ni}\,(e^{i}+1)}{(e^{ni}+1)\,[(e^{ni}+1) + j(e^{(n+1)i}-1)]}$$

It is clear from the above that $\frac{\partial s}{\partial n} > 0$, i.e. s(n) increases monotonically from the value $\underline{s}$ (at $n = 1$) to the value $\overline{s}$ (as $n \to \infty$). A closer analysis of the two values $\underline{s}$ and $\overline{s}$ follows.

The lower bound of the inflation tolerance, $\underline{s}$, is always negative, since

$$(3.26) \qquad \begin{aligned} \underline{s} &= -i + \ln[1 + j(e^{i}-1)] \\ &= \ln[e^{-i} + j(1-e^{-i})] \\ &\lesseqgtr \ln[e^{-i} + 1 - e^{-i}] = \ln 1 = 0. \end{aligned}$$

The inequality in (3.26) results from excluding j-values larger than one from the analysis. When j equals one, $\underline{s}$ equals zero. Otherwise, $\underline{s}$ is genuinely negative.

The sign of $\overline{s}$, the upper bound of the inflation tolerance, depends on the relation between the two fixed parameters, i and j, since the positivity condition of $\overline{s}$:

$$(3.27) \qquad \overline{s} = \ln[e^{-i} + j] > 0$$

results in the condition

$$(3.28) \qquad e^{-i} + j > 1,$$

and further,

$$(3.29) \qquad j > 1 - e^{-i}.$$

Thus, increasing the length of life produces positive inflation tolerance only if condition (3.29) is fulfilled, ceteris paribus. Since $e^{-i} \simeq 1 - i$, the above result implies that positive inflation tolerance is possible only if [23]

$$(3.30) \qquad j > i.$$

[23] And only if the values of n are sufficiently large.

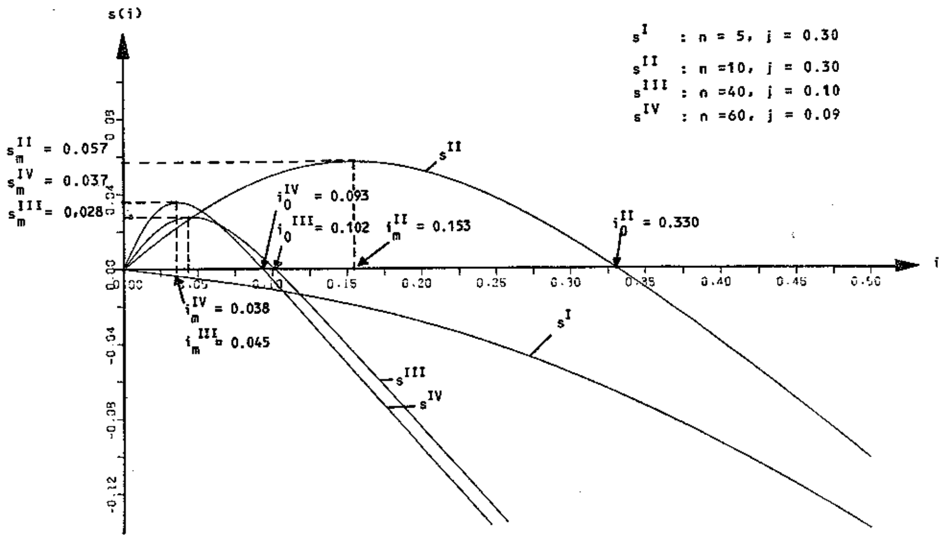

Diagram 3.3. Inflation Tolerance as a Function of the Profitability of the Investment Using Specific Combinations of the Length of Life, n, and the Rate of Depreciation, j.

In other words, *the rate of depreciation must exceed the (real) internal rate of return on the investment.* In the case[24] $j \leq 1 - e^{-i}$ inflation tolerance is negative whatever the value of n. If $j > 1 - e^{-i}$, there exists such a value n_0 for the length of life, n, that when the actual value of n is higher than this, inflation tolerance is positive (since s is monotonically increasing and $\bar{s} > 0$). This value, n_0, is the smallest value of n which satisfies the inequality:

$$(3.31)\quad s(n) = -i + \ln[1 + j\frac{e^{(n+1)i} - 1}{e^{ni} + 1}] \geq 0,$$

or further,

$$(3.32)\quad 1 + j\frac{e^{(n+1)i} - 1}{e^{ni} + 1} \geq e^{i},$$

and, using the interest factor R,

$$(3.33)\quad 1 + j\frac{R^{n+1} - 1}{R^{n} + 1} \geqq R,$$

which, after simplification, becomes

$$(3.34)\quad (1 - j)R^{n+1} - R^{n} + R - (1 - j) \leqq 0.$$

[24] Or as an approximation: $j \leq i$.

The first n that satisfies the inequality (3.34), using given values of i, R and j, is the value n_0. The solution is easy to arrive at when numerical methods are employed. For example, given the values j = 0.30 and i = 0.10 (R = 1.1052), n_0 equals six years. An investment of this type must therefore have an economic life of at least six years in order to tolerate inflation.

Diagram 3.4 graphs the function s(n) using two value combinations of i and j. These are for graph s_1 i = 0.08 and j = 0.10, and for s_2 i = 0.20 and j = 0.10. The extreme values of s_1 are:

$$\underline{s}_1 = \ln[e^{-0.08} + 0.10(1 - e^{-0.08})] = -0.072$$

$$\bar{s}_1 = \ln(e^{-0.08} + 0{\cdot}10) = 0.023$$

and of s_2:

$$\underline{s}_2 = \ln[e^{-0.20} + 0.10(1 - e^{-0.20})] = -0.178$$

$$\bar{s}_2 = \ln(e^{-0.20} + 0.10) = 0.085.$$

Along the graph s_1, inflation tolerance becomes positive when the length of life increases, n_0 = 25 years. Positive inflation tolerance is made possible by the fact that the positivity condition (3.29) is saitsfied in this case, since $j > 1 - e^{-i}$ (0.1 > 0.077). *Although the second graph, s_2, describes an investment with a higher rate of return (20 %), it does not produce positive inflation tolerance whatever the length of life. This is because $j < 1 - e^{-i}$ (0.1 < 0.181).*

On the basis of the above discussion it may be concluded that in order to hedge against inflation, the firm should invest in projects with as long economic lives as possible,[25] provided that the rate of depreciation exceeds the internal rate of return on the investment in question.

3.3. Inflation Tolerance as a Function of the Rate of Depreciation

In analysing the effects of the rate of depreciation it is natural to consider only those j-values that fall in the interval $0 \leq j \leq 1$. From the basic formula for inflation tolerance, expression (2.17), s may be solved for the extreme values of j to give

$$(3{\cdot}35) \quad s(0) = -i$$

25 Using very different methods, Poensgen and Straub arrive at the same general recommendation. See Poensgen-Straub (1976), p. 21.

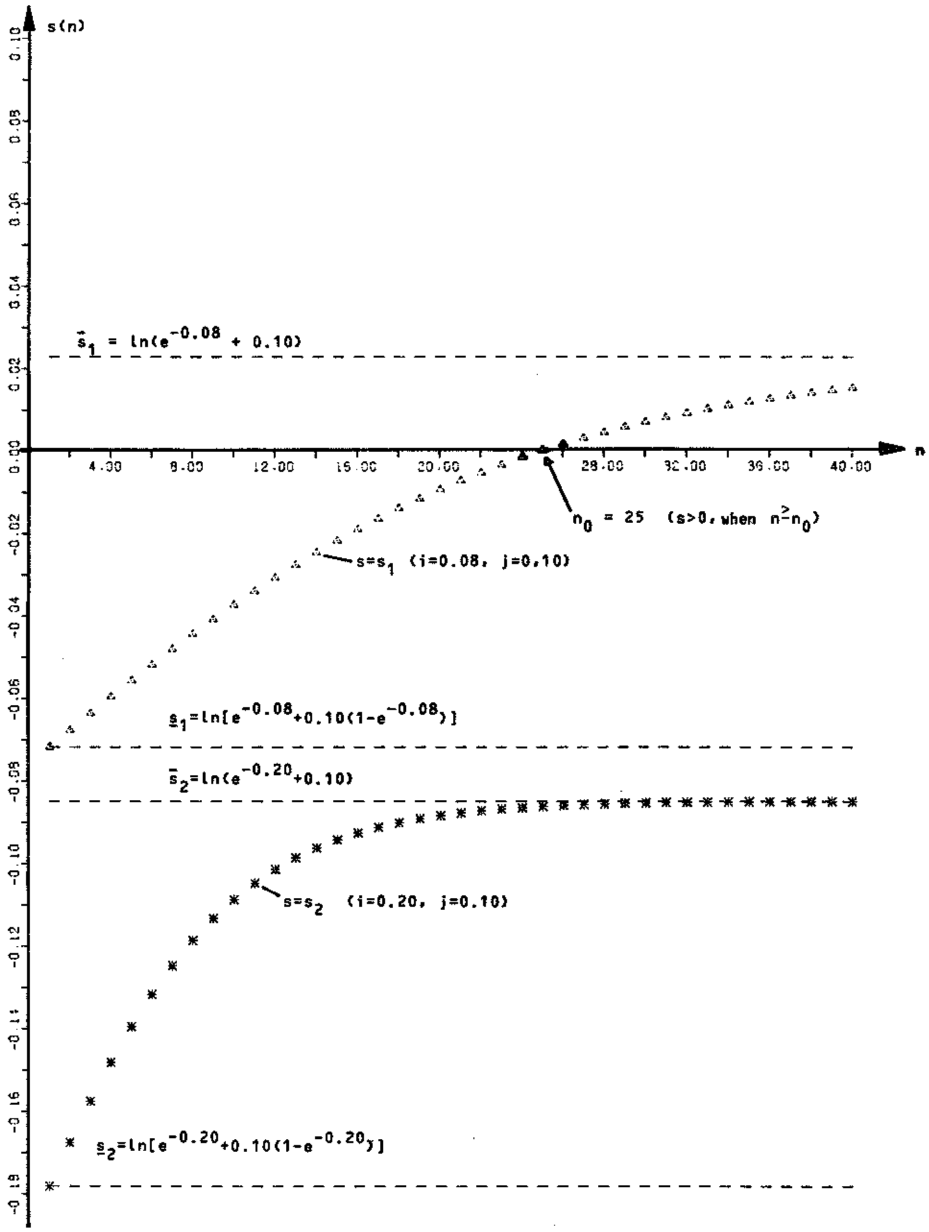

Diagram 3.4. Inflation Tolerance, s, as a Function of the Length of Life, n. Cases: $s_1(i < j)$ and $s_2(i > j)$.

$$(3.36)\quad s(1) = -i + \ln[1 + \frac{e^{(n+1)i} - 1}{e^{ni} + 1}]$$
$$= \ln \frac{e^{(n+1)i} + e^{ni}}{e^{(n+1)i} + e^{i}} .$$

For positive values[26] of i, therefore, $s(1) > 0$.

On the basis of (3.35) it may be noted that an investment which cannot be written off against income (for example land property) has a negative value of s, equal to the internal rate of return in absolute terms, whatever its economic length of life. On the other hand, the unlimited depreciation allowances that have been applied during the recent years, if they are fully exploited during the first year, result in inflation tolerance that always remains positive (see Table 3.1). For example, the inflation tolerance of an investment with a length of life of 10 years and a rate of return of 10 % is, from expression (3.36), 0.33 (33 %). A similar investment but with a length of life of 40 years tolerates inflation up to the value of 62.6 %. Thus, unlimited depreciation allowances provide an efficient hedge against inflation. It must be kept in mind, however, that in order to write off the whole purchase price in one period, the firm must have adequate income also from other sources than the investment under investigation from which this depreciation may be deducted.

Table 3.1. Inflation Tolerance of Lump Sum Depreciation (j = 1).

i \ n	5	10	40	60
0.05	0.093	0.194	0.542	0.620
0.10	0.170	0.331	0.626	0.642
0.20	0.285	0.471	0.598	0.598

The function s(j) is monotonically increasing. This is evident from the expression for s(j) itself, and may also be ascertained by examining the partial derivative

$$(3.31)\quad \frac{\partial s}{\partial j} = \frac{\dfrac{e^{(n+1)i} - 1}{e^{ni} + 1}}{1 + j\,\dfrac{e^{(n+1)i} - 1}{e^{ni} + 1}} = \frac{e^{(n+1)i} - 1}{e^{ni} + 1 + j(e^{(n+1)i} - 1)} ,$$

26 When $i = 0$, $s(j) \equiv 0$.

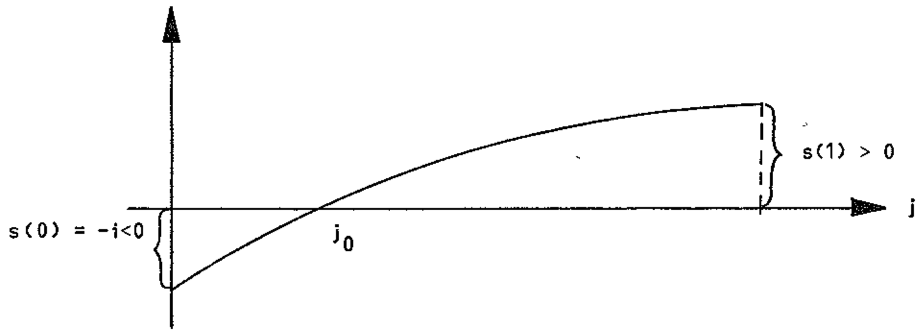

Diagram 3.5 Inflation Tolerance as a Function of the Rate of Depreciation, $i > 0$.

which is clearly positive (when $i > 0$). Typically, therefore, s(j) can be described as in Diagram 3.5. It begins to increase from the value $s(0) = -i$, reaches the value zero at a point j_0 $(0 < j_0 < 1)$, and from j_0 onwards becomes positive, being always monotonically increasing. The graph demonstrates the importance of j with respect to the size of the inflation tolerance. As long as the internal rate of return is positive, all combinations of i and n enable the achievement of positive inflation tolerance, if the rate of depreciation j can be made high enough $(j > j_0)$.

The critical rate of depreciation, j_0, can be solved for from equation

$$(3.38) \quad s(j) = -i + \ln[1 + j \frac{e^{(n+1)i} - 1}{e^{ni} + 1}] = 0,$$

i.e. one gets

$$(3.39) \quad j_0 = \frac{(e^{ni} + 1)(e^{i} - 1)}{e^{(n+1)i} - 1} = \frac{e^{ni} + 1}{e^{ni} + e^{(n-1)i} + \ldots + e^{i} + 1} = \frac{e^{ni} + 1}{\bar{s}_{n+1|i}}$$

where $\bar{s}_{n|i} = \sum_{t=1}^{n} R^{t-1} = \sum_{t=1}^{n} e^{(t-1)i}$ is the prolongation factor for discrete payments made at the end of each n periods in the case of continuous discounting.

3.4. The Absolute Upper Bound for Inflation Tolerance

It is evident from the preceding partial analyses that the function $s = s(i, n, j)$ is bounded from above, i.e. it possesses an absolute upper bound which cannot be exceeded whatever the parameter values employed.[27]

In what follows, this upper bound is derived. The function s, being considered as a function of three variables:

$$(3.40) \quad s = s(i, n, j) = -i + \ln[1 + j \frac{e^{(n+1)i} - 1}{e^{ni} + 1}]$$

is increasing with respect to j. Since $j \leq 1$, it holds for all values of i and n that:

$$(3.41) \quad s \leqq s(i, n, 1) = \ln[e^{-i} + \frac{e^{(n+1)i} - 1}{e^{(n+1)i} + e^{i}}]$$
$$= \ln \frac{e^{(n+1)i} + e^{ni}}{e^{(n+1)i} + e^{i}}.$$

If $s(i, n, 1)$ is denoted by $\tilde{s}(i, n)$, (3.41) becomes

$$(3.42) \quad s \leqq \tilde{s}(i, n) = \ln \frac{e^{(n+1)i} + e^{ni}}{e^{(n+1)i} + e^{i}}.$$

The function $\tilde{s}(i, n)$ increases with n. Thus, for all values of i:

$$(3.43) \quad s \leqq \tilde{s}(i, n) \leq \lim_{n \to \infty} \tilde{s}(i, n) = \lim_{n \to \infty} \ln \frac{e^{(n+1)i} + e^{ni}}{e^{(n+1)i} + e^{i}}$$
$$= \ln \lim_{n \to \infty} \frac{1 + e^{-i}}{1 + e^{-ni}} = \ln (1 + e^{-i}).$$

If $\lim_{n \to \infty} \tilde{s}(i, n)$ is denoted by $\tilde{\tilde{s}}(i)$, it holds that

$$(3.44) \quad s \leqq \tilde{\tilde{s}}(i) = \ln (1 + e^{-i}).$$

Function $\tilde{\tilde{s}}(i)$ is, in turn, a decreasing function of i. Therefore, its maximum value, when i is limited to its nonnegative values, equals $\tilde{\tilde{s}}(0) = \ln 2$. Thus, it always holds that

$$s \leqq \ln 2 \sim 0.693,$$

i.e. inflation tolerance cannot exceed 69.3 %, whatever the parameter values of i, j and n (where $i \geq 0$, $0 \leq j \leq 1$, $n \geq 1$).

---

27 The absolute maximum value of s with respect to i is s_m. The extreme values of s under varying values of n are $\underline{s}$ and $\overline{s}$, and the extreme values with respect to j, s(0) and s(1).

3.5. Adequacy of Present Tax Depreciation Allowances in the Light of Recent Inflation Experience

Let the actual or predicted rate of inflation equal $\hat{s}$. It is possible to examine, how large the rate of depreciation, j, should be so that the investment in question tolerates the inflation rate $\hat{s}$. If this crucial rate of depreciation is denoted by $\hat{j}$, it follows that $s(\hat{j}) = \hat{s}$ and $s(j) > \hat{s}$ when $j > \hat{j}$.[28] Since $\lim_{j \to \infty} s(j) = \infty$ this value $\hat{j}$ always exists. Of course, it is interpretatively meaningful only if $\hat{j} \leq 1$.

This depreciation rate, $\hat{j}$, can be solved for from equation:[29]

$$(3.45) \quad s(j) = \ln[1 + j \frac{e^{(n+1)i} - 1}{e^{ni} + 1}] - i = \hat{s}.$$

i.e. one gets

$$(3.46) \quad \hat{j} = \frac{(e^{ni} + 1)(e^{i+\hat{s}} - 1)}{e^{(n+1)i} - 1}.$$

Using the wholesale index, the average rate of inflation during the 1970's equalled 11,5 % p.a. The corresponding rate of inflation in continuous discounting is therefore ln 1.115 = 0.109 (10,9 %). Table 3.2 presents the rates of depreciation, $\hat{j}$, which are calculated from (3.46) using the value of 0.109 for $\hat{s}$, and employing several combinations of assumed values for the parameters i and n. If the lengths of life are five years and ten years, the EVL approved rate of depreciation equals 0.30. For example, if the investment produces a rate of return of 10 %, and its economic life is ten years, the crucial $\hat{j}$-value equals 0.43, i.e. in order to have protected the investment against the harmful effects of the average inflation during the 1970's ($\hat{s}$ = 0.109), the declining balance method should have allowed a depreciation rate of 0.43. *This implies that the actual allowed rate of depreciation, 0.30, was not sufficiently accelerated so as to provide a hedge against inflation.*

The discrepancy between the actual rate of depreciation allowance and the $\hat{j}$-value is the larger, the shorter the life of the relevant investment is. The EVL approved rate of depreciation for buildings varies (depending on the building material and the use purpose of the building) between five and ten percent. Table 3.2 demonstrates that the crucial $\hat{j}$-values corresponding to the lengths of life of 40 and 60 years vary from 0.18 to 0.30. Consequently,

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[28] Cf. with the monotonically increasing behaviour of s = s(j).

[29] See also Diagram 3.6.

Table 3.2. The Rate of Depreciation that Guarantees Tolerance of Inflation of 10,9 % p.a. under Selected Combinations of i and n.

i \ n	5	10	40	60
0.05	(1.13)	0.62	0.21	0.18
0.10	0.75	0.43	0.22	0.21
0.20	0.58	0.38	0.30	0.30

the EVL allowances vere totally inadequate in the case of buildings as well, if they were to protect the investment against the effects of inflation. In view of the above $\hat{j}$-values for buildings, the proposed increase[30] in the relevant EVL allowances to 15 % is still insufficient from the inflation protection point of view. The maximum depreciation allowance should equal at least 20 %.

3.6. The Minimum Required Economic Length of Life as a Protection Factor against Inflation

It was noted earlier that inflation tolerance increases monotonically with the length of life of the investment, n. Positive inflation tolerance was found to be possible when the length of life increased, if

(3.47) $\bar{s} = \lim_{n \to \infty} s(n) = \ln[e^{-i} + j] > 0,$

i.e. if

(3.48) $j > 1 - e^{-i}.$

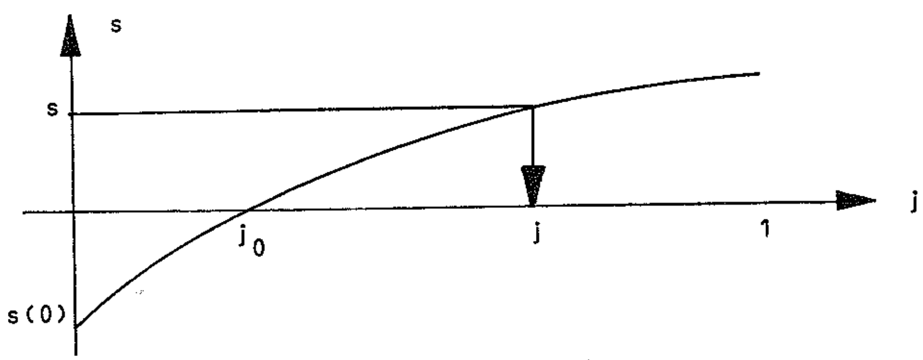

Diagram 3.6. Determining the Rate of Depreciation, $\hat{j}$, which Guarantees Tolerance of Inflation of Size $\hat{s}$.

30 See Yritysverotustoimikunnan mietintö (1980), p. 59.

It may be asked, whether it is possible for an investment which satisfies (3.48) to tolerate the inflation rate $\hat{s}$, i.e. whether it holds that $\bar{s} \geq \hat{s}$, and if so, what is the minimum length of life, $\hat{n}$, which enables the achievement of inflation tolerance of this magnitude.

The first part of the above question may be answered in a straightforward manner. An inflation tolerance of size $\hat{s}$ requires that the following condition is fulfilled:

(3.49) $\bar{s} = \ln(e^{-i} + j) \geqq \hat{s},$

i.e.

(3.50) $e^{-i} + j \geqq e^{\hat{s}}$

or

(3.51) $j \geqq e^{\hat{s}} - e^{-i}$

and, as an approximation [31]

(3.52) $j > i + \hat{s}.$

The rate of depreciation must therefore at least equal the nominal internal rate of return on the investment.[32]

If the condition $j \geq e^{\hat{s}} - e^{-i}$ is fulfilled, $\hat{n}$, the minimum required length of life, may be determined from the inequality:[33]

(3.53) $s(n) = \ln[1 + j \dfrac{e^{(n+1)i} - 1}{e^{ni} + 1}] - i \geqq \hat{s}.$

This can further be expressed as

(3.54) $1 + j \dfrac{e^{(n+1)i} - 1}{e^{ni} + 1} \geqq e^{i+\hat{s}},$

which becomes, using R to denote e^{i} and $\hat{S}$ to denote $e^{\hat{s}}$:

(3.55) $1 + j \dfrac{R^{n+1} - 1}{R^{n} + 1} \geqq R\hat{S},$

and finally,

(3.56) $(\hat{S} - j)R^{n+1} - R^{n} + \hat{S}R - (1 - j) \leqq 0.$

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[31] $e^{x} \approx 1 + x.$

[32] The condition for positive inflation tolerance was that the rate of depreciation had to equal or exceed the real rate of return on the investment.

[33] Cf. section 3.2. where n_0 is determined.

Table 3.3. The Upper Bound of Inflation Tolerance, $\overline{s}$, and the Minimum Required Length of Life, $\hat{n}$, Under Selected Combinations of i and j.

i \ j	0.05	0.06	0.09	0.10	0.30	
0.05	0.001	0.011	0.040	0.050	0.224	$\overline{s}$
	—	—	—	—	24	$\hat{n}$
0.10	— 0.046	— 0.036	— 0.005	0.005	0.186	$\overline{s}$
	—	—	—	—	17	$\hat{n}$
0.20	— 0.141	— 0.129	— 0.096	— 0.085	0.112	$\overline{s}$
	—	—	—	—	26	$\hat{n}$

The first n that for given values of i, (R), j and $\hat{s}$, $(\hat{S})$ satisfies the above inequality, is the required minimum economic life, n. This value must be solved for numerically.

Table 3.3 presents the upper bound values for s, $\overline{s}$, and (if $\overline{s} \geq \hat{s}$) the required minimum lengths of life which correspond to the average rate of inflation in the 1970's, 10.9 %, using selected combinations of i- and j-values. Increasing the length of life is a feasible method of hedging against inflation only, if the investment in question is eligible for the 30 % depreciation allowance. This minimum economic life is 24 years, if the internal rate of return on the investment is 5 %. The corresponding minimum lives are 17 years and 26 years, if the internal rates of return are 10 % and 20 % respectively. In practice, the economic life of machinery and equipment rarely reaches those levels which produce protection against inflation.[34]

4. SUMMARY AND CONCLUSIONS

This paper has examined the concept of inflation tolerance by using present values of the depreciation amounts in analysing all equity-financed capital projects. The investment under consideration must be regarded as marginal, since the internal rate of return was assumed to equal the rate of discount.

It has been emphasized in discussions concerning the adequacy of tax deductible depreciation allowances that only profitable firms find it feasible to protect against the harmful effects of inflation.[35] This study has demon-

34 See Yritystutkimusneuvottelukunta (1979), Appendix 2.

35 For example, Riistama (1980), p. 9.

strated that the relation between the length of life of the investment and the EVL approved rate of depreciation is of crucial significance in determining the conditions for positive inflation tolerance. In the case $j \leq 2/(n+1)$ positive inflation tolerance is impossible, whatever the profitability of the investment under consideration. Instead, increasing profitability merely accentuates the negativity of the inflation tolerance, s. This arises from the fundamental characteristics of the realization depreciation. It is very sensitive to changes in both the length of economic life and the rate of discount because of the double discounting procedure employed in its calculation. However, realization is the commonly acknowledged principle to be observed in determining corporate income and as the basis for corporate taxation.

The necessary, but not sufficient, condition for positive inflation tolerance was found to be $j > 2/(n+1)$. In order to satisfy this condition, it is profitable for the firm to invest in assets with long estimated economic lives provided that the EVL determined rate of depreciation applicable to these is higher than the internal rate of return on the relevant asset. Once the condition $j > 2/(n+1)$ is satisfied, increasing profitability raises inflation tolerance until the latter achieves its maximum value at the level of the internal rate of return i_m. After this point, higher profitability is associated with decreasing inflation tolerance.

Inflation tolerance of depreciation is the higher, the longer the economic life of the asset in question. Inflation tolerance was also shown to increase with the rate of depreciation.

The adequacy of EVL depreciation allowances was examined in the light of the average rate of inflation in Finland during the 1970's (10.9 %). It was shown that the present allowances do not enable protecting the investment against a rate of inflation of this magnitude. For example, an investment with a length of life of ten years and an internal rate of return of ten percent should have required a rate of depreciation of 43 % in order tolerate inflation of 10.9 % p.a, while the actual EVL allowance equalled 30 %. The EVL accepted rates of depreciation applicable to buildings were shown to be equally inadequate. They vary from five to ten percent, whereas at least a 20 % depreciation based on the declining balance method would have been necessary for producing protection against the average rate of inflation during the 1970's.

The determination of inflation tolerance in this paper does not take into account the option of using debt capital as a form of finance. If debt capital can be utilized, the inflation tolerance of depreciation increases.[36] The strength of this effect is being analysed in the ongoing research based on the present paper.

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[36] Riistama (1975), p. 71.

Inflation tolerance has here been analysed with respect to separate investment projects. The tolerance of inflation for the whole firm may be defined as consisting of the sum of all these separate tolerance values, which implies that the growth of the firm is one of the factors to be taken into account in determining the firm-specific inflation tolerance. This remains to be accomplished in future research.

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Appendix. List of Symbols

Symbol	Interpretation	Units
$\bar{a}_{n\rceil i}$	present value factor for discrete payments made at the end of each period (n periods, continuous discounting using the rate i)	—
C	purchase price of the investment	FIM
$\bar{c}_{n\rceil i}$	annuity factor (n periods, continuous discounting using the rate i)	—
D_t^{DB}	depreciation in year t based on the declining balance method	FIM
D_t^R	realization depreciation in year t	FIM
i	real internal rate of return = rate of discount	1/year
i_m	internal rate of return associated with maximal inflation tolerance (when $s = s(i)$)	1/year
i_0	critical internal rate of return: $s(i) < 0$ as $i > i_0$	1/year
j	rate of depreciation in the declining balance method	—
j_0	critical rate of depreciation, $s(j) > 0$ as $j > j_0$	—
$\hat{j}$	minimum rate of depreciation associated with inflation tolerance of the actual/predicted rate of inflation, $\hat{s}$	—
n	economic life of the investment	years
n_0	critical n; $s(n) > 0$ as $n \geq n_0$	years
$\hat{n}$	minimum n associated with inflation tolerance of the actual/predicted rate of inflation $\hat{s}$	years
$NPV(D_t^{DB})$	present value of D_t^{DB} (see above)	FIM
$NPV(D^{DB})$	present value of the sum of all D_t^{DB}'s	FIM
$NPV(D_t^R)$	present value of D_t^R (see above)	FIM
$NPV(D^R)$	present value of the sum of all D_t^R's	FIM
P_t	(constant) annual return on investment in year t	FIM
R	interest factor: $R = e^i$	—
s	rate of inflation; inflation tolerance	1/year
s_m	maximum inflation tolerance	1/year
$\underline{s}$	lower bound for inflation tolerance when $s = s(n)$; $\underline{s} = s(1)$	1/year
$\bar{s}$	upper bound for inflation tolerance when $s = s(n)$; $\bar{s} = \lim_{n \to \infty} s(n)$	1/year

Symbol	Interpretation	Unit
$\hat{s}$	actual/expected rate of inflation	1/year
$\bar{s}_{n\rceil i}$	prolongation factor for discrete payments made at the end of each period (n periods, continuous discounting using the rate i)	—
S	inflation factor; $S = e^s$	—
t	subindex denoting the number of the year	
t	length of the discounting period	years