TEEMU AHO, D.Sc. (Econ.)
ILKKA VIRTANEN, D.Sc.
Lappeenranta University of Technology
Finland

Analysis of lease financing under inflation

1. INTRODUCTION

11. Principles of Leasing in Finland

Leasing as a method of finance is becoming increasingly common. In particular, this applies to investment in machinery. In Finland, leasing implies that the firm rents the capital asset to be acquired from the lessor, who then charges the firm with equal monthly lease payments.

From the firm's point of view, leasing has several advantages as the chosen method of finance:

- No capital needs to be tied in acquiring the asset.
- Lease payments are fixed in advance. Thus, they are protected from the effects of inflation and, under inflationary conditions, decrease in real value. Only if the general level of interest rates changes, it is possible to adjust the payments.
- Lease payments are fully deductible for tax purposes. This implies that (in profitable years at least) the costs of leasing are reduced in proportion to the firm's tax rate.
- After the termination of the initial lease period, the contract can be renegotiated on very favourable terms; the new annual payments being only one twelfth of the original lease payments.

12. Review of Prior Studies

Investment decisions which involve the possibility of lease finance can be seen as consisting of the following two stages:[1]

[1] For similar views, see Bierman-Smidt (1975), p. 269 and Roenfeldt-Osteryoung (1973), p. 78.

1. Is the investment project under consideration in itself worth undertaking? This is the basic investment decision, and is usually analysed using investment profitability calculations.
2. If the proposal is accepted, is leasing the best method of finance? In normal circumstances, the alternative is to purchase the piece of equipment using income, debt and/or equity in financing this purchase.

It is justified to assume that the income flows and operating costs arising from the investment are not dependent on the method of finance selected. *Thus, the actual decision to invest (1) can be seen as separate from the finance decision (2).* It has been common in the literature to combine these, in which case the profitability of leasing cannot be determined without *analysing the overall profitability of the investment.*[2]

Comparisons of the profitability of leasing with other modes of finance are based on the respective costs of finance, i.e. minimising these costs constitutes the criterion for choice between different methods. The prevalent technique in the literature is based on the present value of the costs of finance.[3] In this, the present values of the costs of leasing and purchasing are compared, and the alternative with the smallest present value chosen. Tax deductibility of lease payments is taken into account in calculating the former. Likewise, tax deductibility of interest on debt capital and depreciation allowances for tax purposes are taken into account in determining the present value of interest payments and amortization. This, together with that part of the original purchase price that is financed from income and/or equity, constitute the present value of the costs of finance in the purchase alternative.[4] The present value of the costs of equity and/or income finance is equal to the above mentioned part of the purchase price, if this present value is calculated using the costs of equity/income finance as the rate of discount. Differences in defining and applying the discount rate are probably the most important reason for the fact that different variants of the present value method result in differing profitability rankings in the lease-vs.-buy comparison.[5]

Beechy's method of defining the costs of leasing is based on the implicit internal rate of return (IRR) on leasing.[6] In this method, that IRR is solved

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[2] See e.g. Harwood—Hermanson (1976), pp. 83—87, Johnson—Lewellen (1972), pp. 819—822, Levy—Sarnat (1979), pp. 48—53 and Lewellen—Long—McConnell (1976), pp. 787—798.

[3] For example Mao (1969), pp. 323—325 and Bower (1976), pp. 265—273.

[4] See Merrett—Sykes (1974), pp. 260—263.

[5] Bower (1976), pp. 265—269, presents a good summary of these.

[6] Beechy (1969), pp. 375—381. For a critique of this method, see Mitchell (1970), pp. 308—309.

for which equates the present value of the net cash flows caused by the leased equipment with the purchase price of the equipment. Leasing is the rational choice, if this IRR is lower than the interest rate on debt capital. Beechy assumes that the purchase option would be completely debt financed, which in practice is rarely feasible. His method is based on a before tax analysis, while Reilly for example takes the tax deductibility of interest into account in determining the costs of leasing.[7] In this case the relevant comparison rate for the IRR is the interest on debt capital after tax.

The after tax analysis has traditionally assumed that the tax deductibility can be fully exploited. This implicitly presupposes that the firm's profitability allows this to take place throughout the relevant planning horizon, which, in Finland at least,[8] is unrealistic. This assumption may be seen as defining an upper limit for the utilisation of tax allowances rather than as describing the actual situation.[9] For this reason, reliance on this method necessitates an additional analysis of the effects of taxes and their variation limits on the profitability ordering.

Another serious shortcoming in the previous models on the profitability of leasing is that in general the possibility of inflation is ignored.[10] As has been noted before, from the firm's point of view lease payments are protected against inflation. Since the purchase option, either partly or completely debt financed, also contains cash flow components with varying degrees of inflation protection, the analysis of inflation offers in the present context an important and interesting topic for research.

13. Objectives and Structure of the Paper

The present study attempts to analyse the effects of inflation on the economic consequences and profitability ordering of the purchase and lease alternatives in financing investment in equipment. It is assumed that the potential purchase is either totally or partly debt financed. The effect on the profitability ranking that the structure of this finance may have becomes thus one of the main questions. The method to be used in the comparisons is based on present values after tax.

The present values are calculated using discrete flows and continuous discounting. The former usage can be justified, in addition to its prevalence in the literature, also by the fact that most components of the model

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7 Reilly (1980), p. 15.

8 The present Finnish corporate tax system is characterized by a relatively high income tax rate and generous opportunities for tax deductions.

9 See also Honko (1973), pp. 162—163.

10 For an exception, see e.g. Merrett—Sykes (1974), pp. 260—263.

to be presented in the next section actually are discrete in character. The justification of continuous discounting may be seen in reasons of mathematical simplicity; for example the discount rate under inflationary circumstances can be presented as a sum of the relevant terms ignoring the product term of inflation and interest factors.[11]

In Section 2, present value formulae for the costs of finance under the two alternatives are constructed with special reference to inflation. In the case of the purchase option, a basic formula is first derived, which then is further analysed using different assumptions about the method of depreciation and the type of loan selected. The following depreciation methods are examined: declining balance method, straight line method and realization method. The types of loan selected for analysis are serial and annuity loans. Inflation is taken into account in that all flows in the model are treated in nominal prices, and the discount rate is defined on similar basis.[12] The present values are expressed as the value of money at the time of decision making.

Section 3 contains a partial analysis of the model. In particular, the tax rate and the structure of finance in the purchase alternative are of special interest as to their effects on the profitability ranking of the lease and buy alternatives under changing inflation. The analysis attempts to establish those conditions which cause a reversal in the ordering of these alternatives.

Finally, Section 4 concludes the paper with a discussion and evalution of the model.

2. FORMULATION OF THE MODEL

21. *The Present Value of Lease Payments*

It is usual to express the monthly lease payments as a percentage of the purchase price. These payments are due at the beginning of the relevant month. In what follows, C denotes the purchase price of the capital equipment and k' the magnitude of one lease payment expressed as a fraction of the purchase price. This lease payment coefficient is a function of both C and n, the length of the lease period, i.e. $k' = k'(C,n)$. Table 1 below presents the possible values of k'.

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[11] See Aho (1979), p. 301 for the discrete case.

[12] The alternative method of incorporating inflation would be to use fixed prices in the calculations. However, since e.g. taxation is based on nominal prices, the option chosen in the text seems more justified.

TABLE 1. MONTHLY LEASE PAYMENT COEFFICIENTS (= MONTHLY LEASE PAYMENTS AS PERCENTAGES OF THE ORIGINAL PURCHASE PRICE).

Length of lease period (years)	Purchase price C (1000 FIM)		
	$C \geqq 50$	$50 > C \geqq 30$	$30 > C \geqq 14$
3	3.449	3.491	3.491
4	2.758	2.800	—
5	2.345	—	—
6	2.071	—	—

The single lease payment at the beginning of each month is simply the product $k' \cdot C$.

When the flows are all expressed in discrete terms, the normal practice is to assume that these take place at the end of each year t ($t = 1, 2, \ldots, n$). In order to conform with this, the monthly payments are transformed into annual payments, $L_t = k \cdot C$, by prolongating the monthly payments within each year to the end of the year using the rate of interest, i. This means that the lease payment coefficient for the yearly payments is a function of, not only C and n, but of the rate of interest as well, i.e. $k = k(C, n, i)$. The coefficient k can be obtained from

$$(2.1) \quad k = (12 + \frac{13}{2} \cdot i)k'.$$

First, the present value for lease payments (= NPV(L)) is determined assuming that the price level is stable. As lease payments are fully tax deductible, the lease payment after tax in year t becomes $(1 - f)L_t$ where fL_t is the tax deductible proportion of the payment and where this deduction is assumed to be feasible in its full extent.[13] Thus, NPV(L) in this case can be written as

$$(2.2) \quad NPV(L) = \sum_{t=1}^{n} (1 - f)L_t e^{-it} = (1 - f) \cdot k \cdot C \sum_{t=1}^{n} e^{-it}.$$

Using the expression for the sum of geometric series, (2.2) becomes

$$(2.3) \quad NPV(L) = (1 - f) \cdot k \cdot C \; \frac{1 - e^{-ni}}{e^{i} - 1}.$$

[13] All after tax models in the previous literature treat the effect of taxes in this schematic way. See for example Johnson—Lewellen (1972), p. 820.

The term $(1 - e^{-ni})/(e^{i} - 1)$ is the discount or present value factor [14] based on continuous discounting and discrete flows that applies to payments taking place at the end of the relevant period. It is commonly abbreviated by the symbol $a_{n \mid i}$. In the following, the discount factor based on continuous interest is denoted by $\bar{a}_{n \mid i}$. Thus, (2.3) can be rewritten as

$$(2.4) \quad NPV(L) = (1 - f) \cdot k \cdot C \cdot \bar{a}_{n \mid i}.$$

The above derivations are based on the assumption that tax allowances can in fact be fully exploited. This will be retained in the following derivations as well. However, Section 3 contains a closer analysis of its implications on the lease-vs.-buy decision.

When inflation is allowed for, NPV(L) becomes somewhat different, since lease payment coefficients are fixed and therefore reduce the real value [15] of the payments in inflationary conditions. If s is the rate of inflation, the lease payment in year t, L_t, equals $L_t e^{-st}$ in the money of the time of decision-making. The higher the expected rate of inflation, the lower is the real value of lease payments. The real NPV(L) of all lease payments can be written as

$$(2.5) \quad NPV(L) = \sum_{t=1}^{n} (1 - f) L_t e^{-it} e^{-st}$$

$$= (1 - f) k \cdot C \sum_{t=1}^{n} e^{-(i+s)t}.$$

The above can be expressed as a geometric series, and thus simplified to:

$$(2.6) \quad NPV(L) = (1 - f)\, k\, C \, \frac{1 - e^{-n(i+s)}}{e^{i+s} - 1} = (1 - f) k\, C\, \bar{a}_{n \mid i+s},$$

where $i + s$ is the discount rate adjusted for inflation.

Thus, (2.6) indicates that inflation can be taken into account in calculating the NPV(L) by raising the after tax rate of discount (i) by the rate of inflation (s) in determining the discount factor for discrete payments.[16] This, of course, is the smaller the higher the inflation adjusted discount rate one uses.

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[14] For a more detailed discussion of calculating present values on the basis of continuous discounting and discrete flows, see Beenhakker (1976), pp. 23—26.

[15] In prices prevailing at the time of the decision.

[16] Cf. Poensgen—Straub (1976), p. 14.

Denoting the inflation adjusted rate of discount by i_s, NPV(L) can be expressed as

$$(2.7)\quad NPV(L) = (1 - f)\,k\,C\,\frac{1 - e^{-ni_s}}{e^{i_s} - 1} = (1 - f)\,k\,C\bar{a}_{n\,|\,i_s}$$

which is the form to be used in the rest of this paper.

22. *The Present Value of the Costs of Finance under the Purchase Alternative*

221. The Basic Formula and its Components

First, the present value of the costs of finance under the buy alternative (NPV(B)) is examined assuming that stable prices prevail. The analysis is based on after tax values. As a rule, depreciation on fixed assets (D_t) and interest on debt capital (I_t) are fully tax deductible, which implies that these deductions are determined by the relevant tax rate, if other circumstances allow the full exploitation of tax allowances.[17] The earlier the depreciations are deducted, the larger will the value of their tax shield be. Thus, accelerated depreciation reduces the net costs of finance. The costs of finance under the purchase alternative consist of the costs of debt service and of that part of the purchase price that is financed from income/equity. In year t, the costs of debt service after tax can be expressed as $I_t + K_t - f(D_t + I_t)$, where I_t = interest on debt capital and K_t = amortization. If d is used to denote the fraction of purchase price financed by equity/income, NPV(B) can be written as [18]

$$(2.8)\quad NPV(B) = d\,C + \sum_{t=1}^{n} [I_t + K_t - f(D_t + I_t)]e^{-it}$$

$$= d\,C + \sum_{t=1}^{n} [K_t + (1 - f)I_t - fD_t]e^{-it}.$$

The term dC, $0 \leqq d \leqq 1$, gives the absolute share of equity/income finance in the purchase. It is not distributed in the form of dividends and repayments

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17 Saario examines the tax deductibility and related assumptions in his article (II, 1969), pp. 183—194.

18 Mao arrives at an identical formula excepting the fact that his derivation is based on discrete discounting. See Mao (1969), p. 324.

of principal over the length of life of the equipment, since the present value of these exactly equals dC when i is the relevant discount rate. This implies that i must be interpreted as the required rate of return on equity after tax.[19]

Under inflationary conditions, real NPV(B) is determined somewhat differently. Interest on debt capital is calculated in nominal money, which is also the case with amortization. Therefore, the costs of debt service remain the same in nominal terms as in the previous analysis. However, the real costs of debt service are reduced by inflation, which serves to decrease the NPV(B). The before tax costs of debt service in year t equal $(I_t + K_t)e^{-st}$ in the money of the time of decision-making. Depreciation and interest on debt capital are also calculated in nominal prices, which implies that the real value of their tax shields decreases. The earlier the depreciation allowances for tax purposes can be utilized, the weaker the impact of inflation in reducing their value.

The present value of the costs of equity/income finance does not change due to inflation, if protecting the real present value of the corresponding service payments against the effects of inflation is regarded as one of the firm's objectives.

On the basis of the above discussion, real NPV(B) can be written as

$$\text{(2.9)}\quad NPV(B) = dC + \sum_{t=1}^{n} [K_t + (1-f)I_t - fD_t]e^{-it}e^{-st}$$

$$= dC + \sum_{t=1}^{n} [K_t + (1-f)I_t - fD_t]e^{-(i+s)t},$$

or, using the inflation adjusted discount rate, i_s,

$$\text{(2.10)}\quad NPV(B) = dC + \sum_{t=1}^{n} [K_t + (1-f)I_t - fD_t]e^{-i_s t}.$$

Once again, raising the rate of discount by s, the rate of inflation, is sufficient for incorporating inflation effects into the NPV formula.

Equation (2.10) can be divided into the following three components:

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19 For other methods of defining the discount rate, see Honko—Virtanen (1975), p. 45.

$$(2.11)\quad NPV(K) = dC + \sum_{t=1}^{n} K_t e^{-i_s t},$$

$$(2.12)\quad NPV(I) = (1 - f) \sum_{t=1}^{n} I_t e^{-i_s t},$$

$$(2.13)\quad NPV(D) = f \sum_{t=1}^{n} D_t e^{-i_s t}.$$

The first component, NPV(K), defines the present value of equity and amortizations. NPV(I) gives the present value of interest on debt capital after tax, and NPV(D) the present value of the depreciation tax shield. Therefore

$$(2.14)\quad NPV(B) = NPV(K) + NPV(I) - NPV(D).$$

In the following discussion, the components of (2.14) are examined in greater detail.

222. Amortization

The two types of loan that are selected for analysis are serial loan and annuity loan. In the former, the annual amortization is a constant, i.e. in year t it equals

$$(2.15)\quad K_t = \frac{(1 - d)C}{n}.$$

The present value NPV(K) can in this case be written as

$$(2.16)\quad NPV(K) = dC + \sum_{t=1}^{n} \frac{(1 - d)C}{n} e^{-i_s t}$$

or, using the present value factor,

$$(2.17)\quad NPV(K) = dC + \frac{(1 - d)C}{n} \bar{a}_{n | i_s}$$

In view of the partial analysis to be presented in Section 3, it is useful to rearrange (2.17) into

$$(2.18) \quad NPV(K) = C\frac{\bar{a}_{n\,|\,i_s}}{n} + dC\,[1 - \frac{\bar{a}_{n\,|\,i_s}}{n}].$$

The first term on the right-hand side of (2.18) expresses the real present value of the amortization of a serial loan whose magnitude is C. This may be denoted by K*. K* also equals the component $(C\bar{a}_{n\,|\,i_s})/n$ in the second term on the right-hand side. Therefore, NPV(K) can be rewritten as

$$(2.19) \quad NPV(K) = K^* + d(C - K^*).$$

In the case of an annuity loan, the annual costs of debt service, $\bar{A}$, remain constant over time. When the loan is of magnitude C and the nominal interest rate on debt r [20], the annuity $\bar{A}$ equals

$$(2.20) \quad \bar{A} = \frac{1}{\bar{a}_{n\,|\,r}} C = \frac{e^r - 1}{1 - e^{-nr}} \quad C = \bar{c}_{n\,|\,r}\,C,$$

where $\bar{c}_{n\,|\,r}$ is the amortization or annuity factor based on continuous interest.

The yearly amortization payments of an annuity loan can be treated as an increasing geometric series, and in general form expressed as [21]

$$(2.21) \quad K_t = \bar{c}_{n\,|\,r} \;\; C\,e^{-(n+1-t)r}.$$

Using (2.21), and noting that the magnitude of the loan is $(1 - d)C$, $0 \leq d \leq 1$, NPV(K) in the case of an annuity loan equals

$$(2.22) \quad NPV(K) = dC + \sum_{t=1}^{n} (1 - d)\bar{c}_{n\,|\,r} \;\; C\,e^{-(n+1-t)r}\,e^{-ti_s}$$

or, after simplification,

$$(2.23) \quad NPV(K) = dC + (1 - d)\bar{c}_{n\,|\,r}C\,e^{-(n+1)r} \sum_{t=1}^{n} e^{-t(i_s - r)}.$$

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[20] See Aho—Virtanen (1981), pp. 12—13.

[21] For the derivation of (2.21), see Aho—Virtanen (1981), pp. 13—14.

Once again, the term $\sum_{t=1}^{n} e^{-t(i_s-r)}$ defines a geometric series, the sum of which and the present value factor can be used to give:

$$(2.24) \quad NPV(K) = dC + (1-d)C \; e^{-(n+1)r} \frac{\bar{a}_{n|i_s-r}}{\bar{a}_{n|r}}.$$

In the partial analysis of Section 3, (2.24) is utilized in the following rearranged form

$$(2.25) \quad NPV(K) = C\, e^{-(n+1)r} \frac{\bar{a}_{n|i_s-r}}{\bar{a}_{n|r}} + dC\,[1 - e^{-(n+1)r} \frac{\bar{a}_{n|i_s-r}}{\bar{a}_{n|r}}],$$

which, analogously with (2.19), results in

$$(2.26) \quad NPV(K) = K^* + d(C - K^*),$$

where $K^* = C\, e^{-(n+1)r} \bar{a}_{n|i_s-r}/\bar{a}_{n|r}$. In the present context K^* must be interpreted as the real present value of the amortizations of an annuity loan whose magnitude is C.

223. Interest on Debt Capital

The outstanding balance of a serial loan at the end of each year may be described as a declining arithmetic series. This implies that the series of interest payments is also arithmetically declining. In general, the interest payments in year t equal [22]

$$(2.27) \quad I_t = (e^r - 1)(1-d)\, C\, (1 - \frac{t-1}{n}).$$

The real present value of interest payments after tax is thus, using equations (2.12) and (2.27)

$$(2.28) \quad NPV(I) = \sum_{t=1}^{n} (1-f)(e^r - 1)(1-d)\, C\, (1 - \frac{t-1}{n})\, e^{-ti_s}.$$

The term $\sum_{t=1}^{n} (1 - \frac{t-1}{n})\, e^{-ti_s}$ can be disaggregated into n geometric series, the sums of which in turn form a geometric series. Therefore, the final form of (2.28) is [23]

[22] Aho—Virtanen (1981), p. 15.

[23] Aho—Virtanen (1981), p. 16.

$$(2.29)\quad NPV(I) = (1-f)(1-d)\,C\,\frac{e^{r}-1}{e^{i_s}-1}\left(1-\frac{\bar{a}_{n\,|\,i_s}}{n}\right)$$
$$= (1-d)I^*$$

where $I^* = (1-f)\,C\,[(e^{r}-1)/(e^{i_s}-1)]\;[1-\bar{a}_{n\,|\,i_s}/n]$, and gives the real present value of interest payments after tax on a serial loan whose magnitude is C.

In the case of an annuity loan, interest payments in year t are given by [24]

$$(2.30)\quad I_t = (1-d)\,\bar{c}_{n\,|\,r}\,C(1-e^{-(n+1-t)r}).$$

When this is substituted into (2.12), one gets

$$(2.31)\quad NPV(I) = \sum_{t=1}^{n}(1-f)(1-d)\,\bar{c}_{n\,|\,r}\,C(1-e^{-(n+1-t)r})e^{-ti_s},$$

which, after simplification and using the present value factor, further gives

$$(2.32)\quad NPV(I) = (1-f)(1-d)C\left[\frac{\bar{a}_{n\,|\,i_s}}{\bar{a}_{n\,|\,r}} - e^{-(n+1)r}\,\frac{\bar{a}_{n\,|\,i_s-r}}{\bar{a}_{n\,|\,r}}\right].$$

If I^* is used to denote $(1-f)\,C\,[\bar{a}_{n\,|\,i_s}/\bar{a}_{n\,|\,r} - e^{-(n+1)r}\,\bar{a}_{n\,|\,i_s-r}/\bar{a}_{n\,|\,r}]$, (2.32) can, analogously with (2.29), be presented in the following form

$$(2.33)\quad NPV(I) = (1-d)I^*,$$

where I^* now stands for the real present value of interest payments after tax on an annuity loan of the magnitude C.

224. Depreciation

In what follows, the third present value component in equation (2.13), NPV(D), is closer analysed using different assumptions about the depreciation method chosen. Under straight line depreciation, depreciation in year t is simply

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[24] Aho—Virtanen (1981), p. 14.

$$(2.34)\quad D_t = \frac{C}{n}\,.$$

When this is substituted into (2.13), it becomes

$$(2.35)\quad NPV(D) = \sum_{t=1}^{n} f\frac{C}{n} e^{-ti_s} = fC\frac{\bar{a}_{n\,|\,i_s}}{n}.$$

In the declining balance method, which is in accordance with the present Finnish corporate tax laws, depreciation in year t can be expressed as [25]

$$(2.36)\quad D_t = j(1-j)^{t-1}C;\ t = 1, 2, 3, \ldots, n-1$$

where j ($0 \leqq j \leqq 1$) is the rate of depreciation.

In the last year of the planning horizon, n, the above regular depreciation must be complemented with an additional depreciation, $(1-j)^nC$, so that the total purchase price is written off. This quarantees comparability with other depreciation methods. Thus, the total depreciation in year n is

$$(2.37)\quad D_n = j(1-j)^{n-1}C + (1-j)^nC = (1-j)^{n-1}C.$$

Using (2.36) and (2.37), the real present value of the depreciation tax shield becomes

$$(2.38)\quad NPV(D) = \sum_{t=1}^{n} fjC\,(1-j)^{t-1}e^{-ti_s} + fC\,(1-j)^n e^{-ni_s},$$

which, by an application of the sum of the geometric series, and after simplification, gives [26]

$$(2.39)\quad NPV(D) = fC\,\frac{j + (e^{i_s}-1)\,e^{-ni_s}\,(1-j)^n}{e^{i_s}-(1-j)}\,.$$

In the realization method, depreciation is seen as the present value of the annual return on investment discounted by its IRR.[27] This depreciation

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25 See Aho (1979), p. 302.

26 For detailed derivation of (2.39), see Aho—Virtanen (1981), p. 18. Cf. also Aho (1979), p. 302.

27 For closer discussion of this method, see Saario (I, 1969), pp. 207—209.

concept is based on a notion of income, according to which the costs that are sacrificed 'transform into' income at the rate of the internal return on the investment in question. Thus, the relevant, 'realized', depreciation is the present value of the income to be earned from the investment. In what follows, it is assumed that the IRR on the piece of equipment to be acquired equals the discount rate. Further, it is assumed that the decision to invest has already been made (see subsection 12 earlier in this paper).

The realization depreciation in year t is [28]

$$(2.40) \quad D_t = \bar{c}_{n \mid i_s} C e^{-ti_s}.$$

When (2.40) is substituted into (2.13), the real present value of the depreciation tax shield becomes

$$(2.41) \quad NPV(D) = \sum_{t=1}^{n} f \bar{c}_{n \mid i_s} C e^{-ti_s} e^{-ti_s}$$

$$= f \bar{c}_{n \mid i_s} C \sum_{t=1}^{n} e^{-2ti_s},$$

which gives, using present value factors:[29]

$$(2.42) \quad NPV(D) = fC \frac{\bar{a}_{n \mid 2i_s}}{\bar{a}_{n \mid i_s}} .$$

23. *Summary of the Model*

Subsections 21 and 22 above present the derivations of present values for the costs of finance in both lease and purchase alternatives. The final form for the former is given by equation (2.7). The general form for the latter is expressed by equation (2.10), and its components by equations (2.11) — (2.14). As both the general formula for the present value and the component formulae under the purchase alternative are crucially dependent on the assumptions regarding the depreciation method and loan form chosen, it is worthwhile to summarize these present value equations for further reference.

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28 Cf. Aho (1981).

29 Cf. Saario (1969), p. 214.

The after tax present value of the costs of finance under the purchase alternative is, according to (2.14):

$$\text{(2.43)} \quad NPV(B) = NPV(K) + NPV(I) - NPV(D)$$

where NPV(K) is solved for from (2.17) in the case of a serial loan (SL) or from (2.24) in the case of an annuity loan (AL):

$$\text{(2.44)} \quad NPV(K) = \begin{cases} dC + (1-d)\, C \dfrac{\bar{a}_{n|i_s}}{n} & \text{(SL)} \\ dC + (1-d)\, C\, e^{-(n+1)r} \dfrac{\bar{a}_{n|i_s-r}}{\bar{a}_{n|r}} & \text{(AL)} \end{cases}$$

and NPV(I) from equation (2.29) (SL) or equation (2.32) (AL):

$$\text{(2.45)} \quad NPV(I) = \begin{cases} (1-f)(1-d)\, C \dfrac{e^r - 1}{e^{i_s} - 1} \left(1 - \dfrac{\bar{a}_{n|i_s}}{n}\right) & \text{(SL)} \\ (1-f)(1-d)\, C \dfrac{\bar{a}_{n|i_s}}{\bar{a}_{n|r}} - e^{-(n+1)r} \dfrac{\bar{a}_{n|i_s-r}}{\bar{a}_{n|r}} & \text{(AL)} \end{cases}$$

Finally, NPV(D) can be found from equation (2.35) in the case of straight line depreciation (I), from (2.39) in the case of declining balance method (II) and from (2.42) in the case of realization method (III):

$$\text{(2.46)} \quad NPV(D) = \begin{cases} fC \dfrac{\bar{a}_{n|i_s}}{n} & \text{(I)} \\ fC \dfrac{j + (e^{i_s} - 1)\, e^{-ni_s} (1-j)^n}{e^{i_s} - (1-j)} & \text{(II)} \\ fC \dfrac{\bar{a}_{n|2i_s}}{\bar{a}_{n|i_s}} & \text{(III)} \end{cases}$$

In view of further analysis in this paper, the present value formulae for amortization and interest payments can be summarized as follows:

$$\text{(2.47)} \quad NPV(K) = K^* + d\,(C - K^*)$$

$$\text{(2.48)} \quad NPV(I) = (1-d)\, I^* = I^* - dI^*,$$

where

$$(2.49)\quad K^* = \begin{cases} C\,\dfrac{\bar{a}_{n\,|\,i_s}}{n} & \text{(SL)} \\[2ex] C\,e^{-(n+1)r}\dfrac{\bar{a}_{n\,|\,i_s-r}}{\bar{a}_{n\,|\,r}} & \text{(AL)} \end{cases}$$

and

$$(2.50)\quad I^* = \begin{cases} (1-f)\,C\,\dfrac{e^r-1}{e^{i_s}-1}\left(1-\dfrac{\bar{a}_{n\,|\,i_s}}{n}\right) & \text{(SL)} \\[2ex] (1-f)\,C\left(\dfrac{\bar{a}_{n\,|\,i_s}}{\bar{a}_{n\,|\,r}} - e^{-(n+1)r}\dfrac{\bar{a}_{n\,|\,i_s-r}}{\bar{a}_{n\,|\,r}}\right) & \text{(AL)} \end{cases}$$

3. ANALYSING THE MODEL

31. *Selecting Parameters for Analysis*

In order to analyse the constructed model in greater detail, it is necessary to fix most of its numerous parameters in advance. In what follows, the following four parameters are considered as fixed:

— the lease payments (L_t)
— the real discount rate (i)
— the nominal rate of interest on debt capital (r), and
— the length of the lease period (n).

The type of loan and method of depreciation are allowed to vary in analysing the parameters that are of main interest, viz.
— the tax rate (f) and
— the structure of finance in the purchase alternative (d).

Analysing the implications of varying the tax rate and relaxing the assumption of full utilisation of tax deductibility is necessary in view of the different actual circumstances in which the firm may find itself. The model takes into account the tax deductibility of lease payments, of interest on debt capital and of depreciation charges. To assume that these all can be fully utilized, defines the upper limit for the effects of taxation, while the lower limit would be represented by a situation where no positive tax liability has arisen. By analysing the tax parameter, it is possible to determine whether changes in the tax rate or in the firm's profit position change the profitability ordering of the lease and purchase alternatives. A further question to be examined is how the effects of taxation depend on inflation under different combinations of loan types and methods of depreciation.

The second parameter to be analysed is the structure of finance under the purchase alternative. It has been common in the earlier literature on leasing to assume that the purchase would be totally debt financed,[30] in order to enable the comparison of alternatives with similar financial risks. However, exclusive reliance on debt capital seems to be possible very rarely.[31] Thus, financing the purchase from debt capital only is in the following discussion regarded as the upper bound for potential debt finance rather than a factual finance option. Correspondingly, the upper bound for equity/income finance is 100 % of the purchase price, which may be a feasible option in the case of replacement investment in machinery. Therefore, in analysing the effects of the structure of finance and their dependence on inflation, d is allowed to vary between the values 0 and 1. It is considered important in this context as well, to determine the critical value of d that makes the lease and buy alternatives equally profitable.

32. *The Partial Analysis*

321. Effects of Taxation

In order to establish the effects of taxation, the difference, G, between the present values of finance costs under the lease and purchase alternatives is considered:[32]

(3.1) $G = NPV(L) - NPV(B)$,

which gives the following choice criterion:

$G > 0$,	select the buy alternative
$G = 0$,	choice is indifferent
$G < 0$,	select the lease alternative

[30] For example Bower—Herringer—Williamson (1966), pp. 257—265.

[31] For more realistic calculations about the profitability ordering of leasing vs. purchasing see Merrett—Sykes (1974), pp. 260—263.

[32] An alternative method would be to define a ratio of the present values, i.e. $G' = NPV(L)/NPV(B)$. In this case, the value of the choice criterion that results in indifference would be 1 instead of 0. The difference approach has been adopted, because of its analytical and interpretative simplicity, and will be utilized in the following analyses as well.

Taking into account the discussion in Section 2, G can also be written as

$$(3.2)\quad G = NPV(L) - NPV(K) - NPV(I) + NPV(D)$$

$$= \sum_{t=1}^{n} (1-f)L_t e^{-ti_s} - \sum_{t=1}^{n} K_t e^{-ti_s} - dC - \sum_{t=1}^{n} (1-f)I_t e^{-ti_s}$$

$$+ \sum_{t=1}^{n} fD_t e^{-ti_s}$$

$$= f \sum_{t=1}^{n} (I_t + D_t - L_t) e^{-ti_s} + \sum_{t=1}^{n} (L_t - I_t - K_t) e^{-ti_s} - dC.$$

Thus, the difference between the present values is linearly dependent on the tax rate f, and it is possible to denote

$$(3.3)\quad G(f) = \left[\sum_{t=1}^{n} (I_t + D_t - L_t) e^{-ti_s} \right] f + \sum_{t=1}^{n} (L_t - I_t - K_t) e^{-ti_s} - dC.$$

Graphically, G(f) can be represented by a straight line, the slope of which equals the multiplicative term preceding f in (3.3). The slope of G(f) can also be expressed as the partial derivative [33]

$$(3.4)\quad \frac{\partial G}{\partial f} = \sum_{t=1}^{n} (I_t + D_t - L_t) e^{-ti_s},$$

i.e. the partial derivative of G with respect to f equals the difference between the sum of the real present values of interest on debt capital and depreciation and the present value of lease payments. The sign of the quantity (3.4) can not be unambiguously deducted. However, *in the case $\partial G/\partial f < 0$, leasing becomes more profitable with increasing tax rate, while in the case $\partial G/\partial f > 0$ the purchase option would be more profitable the higher the tax rate.* It is not, of course, possible to infer the sign of G itself from (3.4), i.e. which alternative should be preferred.

In order to analyse the implications of varying capabilities for actually

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[33] Cf. Mao (1969), p. 325.

exploiting the maximum tax allowances, G is defined for the two cases: $f = 0$ and $f = 1$ (100 %). If f equals zero, equation (3.3) becomes

$$(3.5)\quad G(0) = \sum_{t=1}^{n} (L_t - I_t - K_t)e^{-ti_s} - dC.$$

The first term on the right-hand side gives the difference between the present values of lease payments and the costs of debt service. The second term expresses the fraction of the purchase price that is equity/income financed. Depreciation does not appear in equation (3.5), since its tax deductibility in the taxless case equals zero.

If f equals 1, equation (3.3) becomes

$$(3.6)\quad G(1) = \sum_{t=1}^{n} (D_t - K_t)e^{-ti_s} - dC.$$

In this case the tax shields of the interest and lease payments equal the corresponding payments, and can be ignored. G(1) equals the difference between the (real) present value of depreciation charges and amortization minus the equity/income financed fraction of the purchase price.

In addition to the above cases, it is of interest to determine the tax rate, denoted by f_0, that makes the alternatives equally profitable, i.e. $G = 0$. This may be called the critical tax rate, and can be solved for from (3.3) by setting it equal to zero:

$$(3.7)\quad f_0 = \frac{\sum_{t=1}^{n} (L_t - I_t - K_t)e^{-ti_s} - dC}{\sum_{t=1}^{n} (L_t - I_t - D_t)e^{-ti_s}} .$$

The numerator of (3.7) expresses the difference between the (real) present values of lease payments and the costs of finance under the purchase alternative on a before tax basis. The denominator gives the difference between the (real) present values of lease payments on the one hand, and of the sum of interest and depreciation charges on the other. In order to guarantee that the critical tax rate f_0 in equation (3.7) makes sense both conceptually and interpretatively, i.e. $0 \leqq f_0 \leqq 1$, the following conditions must hold:

$$(3.8)\quad \begin{cases} \sum_{t=1}^{n} (L_t - I_t - K_t)e^{-ti_s} - dC \geqq 0 \text{ and} \\ \sum_{t=1}^{n} (L_t - I_t - D_t)e^{-ti_s} > 0, \\ \text{or} \\ \sum_{t=1}^{n} (L_t - I_t - K_t)e^{-ti_s} - dC \leqq 0 \text{ and} \\ \sum_{t=1}^{n} (L_t - I_t - D_t)e^{-ti_s} < 0, \end{cases}$$

which defines the sign condition for f_0, and

$$(3.9)\quad \left| \sum_{t=1}^{n} (L_t - I_t - K_t)e^{-ti_s} - dC \right| \leqq \left| \sum_{t=1}^{n} ((L_t - I_t - D_t)e^{-ti_s} \right|$$

or, after simplification

$$(3.10)\quad \sum_{t=1}^{n} K_t e^{-ti_s} + dC \begin{cases} \geqq \sum_{t=1}^{n} D_t e^{-ti_s} & \text{if the first alternative of (3.8) is relevant} \\ \leqq \sum_{t=1}^{n} D_t e^{-ti_s} & \text{if the second alternative of (3.8) is relevant} \end{cases}$$

which gives the magnitude condition for f_0.

The sign condition (3.8) requires that the numerator and denominator of expression (3.7) must both be either positive or negative. The inequalities (3.10) in turn determine the following magnitude condition: If the first sign condition holds, the real present value of depreciation charges must not exceed the sum of the real present value of amortization and the equity/income financed fraction of the purchase price. If the second sign condition holds, it must not fall short of this sum.

322. Effects of the Structure of Finance in the Buy Alternative

To analyse the effects of the structure of finance, the NPV(I) and NPV(K) expressions from equation (2.48) and (2.47), respectively, are substituted into the first form of equation (3.2) presented. This gives

$$(3.11)\quad G = NPV(L) + NPV(D) - [(1 - d)I^* + K^* + d(C - K^*)],$$

where I^* and K^* are obtained from equations (2.50) and (2.49). The difference between present values, G, can the be expressed as a function of d:

$$(3.12)\quad G(d) = [NPV(L) + NPV(D) - (I^* + K^*)] + (I^* + K^* - C)d.$$

Graphically, this function is represented by a straight line, the slope of which is

$$(3.13)\quad \frac{\partial G}{\partial d} = I^* + K^* - C,$$

i.e. the difference between the (real) present value of the costs of debt service, (where the magnitude of the loan is C and interest is calculated on an after tax basis), and the purchase price of the equipment. If the nominal rate of interest on debt capital is lower than the inflation adjusted discount rate, the (real) present value of costs of debt service will always be less than the purchase price.[34] Thus, the slope $\partial G/\partial d$ is negative, which implies that leasing becomes more profitable as the equity/income financed fraction of purchase price increases. In practice, the discount rate used is always higher than the interest on debt capital. Therefore, the above conclusion about the sign of (3.13) can be said to hold in general. It may also be noted that as inflation increases, $I^* + K^*$ in (3.13) decreases, and consequently the absolute value of $I^* + K^* - C$ becomes greater. *Thus, when the expected rate of inflation is high, the value of G is more sensitive to variations in d than under conditions of stable prices or low inflation.*

The straight line defined by (3.12) intersects the G-axis as shown by:

$$(3.14)\quad G(0) = NPV(L) + NPV(D) - (I^* + K^*).$$

G(0) gives the difference between present values in the case where the purchase is totally debt financed, i.e. $d = 0$. The other extreme case, where the purchase is completely financed from equity/income sources, gives the following value for the choice criterion:

$$(3.15)\quad \begin{aligned} G(1) &= NPV(L) + NPV(D) - C \\ &= NPV(L) - [C - NPV(D)], \end{aligned}$$

i.e. the difference between real present values equals the real present value of lease payments minus the purchase price of the equipment adjusted for the real present value of the depreciation tax shield.

34 See Aho—Virtanen (1981), pp. 28—29.

Figure 3.1. presents the three possible functional relationships between G and d. In the first alternative, A, purchasing is always preferable to leasing, as $G(0) > 0$, $G(1) > 0$. In the second case, B, the profitability ordering changes at d_0; for larger values of d lease alternative becomes preferable, while for smaller values purchase should be chosen. Finally, in case C, leasing is always more profitable, as $G(0) < 0$, $G(1) < 0$.

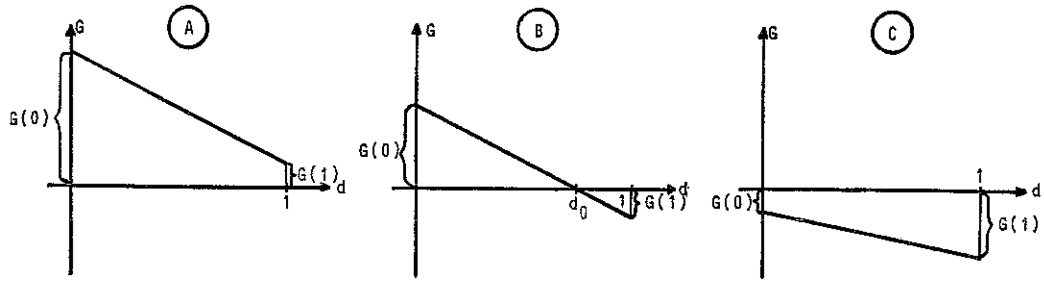

Figure 3.1. The Dependence of G on d.

Next, the critical structure of finance, d_0, is determined, which gives $G(d_0) = 0$. From equation (3.12):

$$(3.16) \quad d_0 = \frac{NPV(L) + NPV(D) - (I^* + K^*)}{C - (I^* + K^*)}.$$

Using the given values for parameters, the existence of a meaningful critical structure of finance requires that d_0 in (3.16) lies between 0 and 1. As the denominator of (3.16) is always positive,[35] becomes the positivity condition for d_0:

$$(3.17) \quad I^* + K^* \leqq NPV(L) + NPV(D).$$

Comparisons of (3.17) with (3.14) reveal that the former in fact is the positivity requirement for G(0), i.e. condition (3.17) excludes the alternative C in Figure 3.1 (leasing is always more profitable).

In order to satisfy the requirement $0 \leqq d_0 \leqq 1$, the magnitude condition for d_0 must also be fulfilled, i.e. it must be the case that

$$(3.18) \quad NPV(L) + NPV(D) \leqq C.$$

In terms of the choice criterion G, condition (3.18) implies that $G(1) < 0$ (compare with (3.15)). Thus, (3.18) excludes the first alternative, A, in Figure

[35] The denominator of (3.16) is the negative of the slope of G.

3.1 (purchasing is always more profitable). Combining the conditions (3.17) and (3.18) gives the following existence condition for the critical structure of finance, d_0:[36]

(3.19) $I^* + K^* \leqq NPV(L) + NPV(D) \leqq C.$

As it always holds that $I^* + K^* \leqq C$, the existence of d_0 is guaranteed, if the sum of NPV(L) and NPV(D) lies between the two end-point values.

33. *Numerical Analysis*

331. Effects of Taxation

In the analysis of taxation (and, later, of the structure of finance) the following fixed values of parameters are utilized (see Table 3.1). These can be regarded as typical.

Table 3.1. Assumed Values for Fixed Parameters

Parameter	Value
Purchase Price, C	100.000 FIM
Length of Lease Period = Loan Period, n	5 years
Nominal Interest on Debt Capital, r	0.10/year
Real Discount Rate, i	0.12/year
Rate of Depreciation under Declining Balance Method	0.30
Monthly lease payment coefficient, k'	0.02345
Equity/Income Financed Fraction of Purchase Price, d	0, 0.35, 1
Tax Rate, f	0.55
Rate of Inflation, s	$0 \leqq s \leqq 1$

First, it is assumed that the purchase is completely debt financed (d = 0). Tables 1—6 of Appendix 2 present the values of the partial derivative of the present value difference with respect to the tax rate ($\partial G/\partial f$), of the present value differences in the two cases f = 0 and f = 1, and of the critical tax rate (f_0) under varying rates of inflation. It is evident from these that *the purchase alternative is always preferable,* and, therefore, no critical tax rate exists, save in the extreme case where the critical tax rate is found at the level of 100 %, when straight line depreciation and serial loan are utilised. In this case, because the yearly depreciation equals amortization,

[36] See case B in Figure 3.1.

the present values of lease costs and finance costs under the purchase alternative are equalized, i.e. $G = 0$. It may also be noted that *raising the tax rate reduces the profitability of the purchase alternative in alsolute terms, i.e.* $\partial G/\partial f < 0$. An exception to this is provided by the case where the rate of inflation, s, fulfills the condition $s \geqq 0.30$, and realization method is utilised. Rising f will then increase the absolute profitability of the purchase option.

When $d = 0$, the strength of the effect of tax rate is dependent on the rate of inflation. Under straight line method of depreciation, this effect ($= |G(0) - G(1)| = |\partial G/\partial f|$) is strongest at a rate of inflation, which in Finland may be considered still tolerable.[37] However, increasing inflation changes the size of this effect but slowly. Under declining balance method of depreciation, this effect becomes slowly weaker when the rate of inflation rises. Under realization method of depreciation, the effect of tax rate is strongly dependent on inflation; $\partial G/\partial f$ takes large negative values at low rates of inflation, becomes zero (at $s \sim 0.3$) and then strongly positive when inflation increases, before it finally returns to its limiting value, zero, which is the same as in other alternatives. The real present value of depreciation and thus, of the depreciation tax shield, decreases faster with rising inflation when the realization method of depreciation is used, as compared with the previous cases, because the relevant discount rate in calculating the present value factor for discrete flows is $2i_s$, and i_s equals $i + s$.

As a summary of the case $d = 0$, it may be noted that the purchase alternative should always be chosen.[38]

In the following discussion, the purchase is assumed to be either partly or completely financed from equity/income. An example of this case is given by Tables 7—12 in Appendix 2, where $d = 0.35$.[39] If there is no inflation ($s = 0$), and also the tax rate equals zero, the buy alternative is preferable, more so if d is small than if it takes higher values. As the tax rate increases (while s still equals zero), leasing improves in profitability the faster the higher d is. The critical tax rate fulfilling the condition $0 \leqq f_0 \leqq 1$ can also be found, (largely irrespectively of the method of depreciation and type of loan chosen). For example, if d equals 0.35, the critical tax rate is about 0.5. *If the tax rate is higher than the critical value ($f > f_0$), leasing is the preferable financing alternative under stable prices.*

When low or moderate inflation is allowed for, the above conclusions

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37 12 % in the case of a serial loan, 18 % in the case of an annuity loan. It may be noted here that in the 1970's the cost of living index in Finland showed an average annual increase of 11.3 %, and the wholesale price index of 11.5 %.

38 If the values of fixed parameters are as depicted in Table 3.1.

39 For other values of d, see Aho—Virtanen (1981), Appendix 4.

pertaining to the case of no inflation must be complemented with the following: The effect of taxation ($|\partial G/\partial f|$) becomes weaker. The purchase alternative ceases to be the preferred one in the case of no taxes ($f = 0$) when inflation increases, i.e. $G(0) \rightarrow 0$ as $s \rightarrow s_0$. The higher the value of d, the sooner this critical rate of inflation, s_0, is met. For example, if d equals 0.35, the corresponding s_0 is 15 %. If the expected rate of inflation (s) is higher than this critical value, s_0, leasing becomes the preferable choice. It may be also noted that as s approaches s_0, the critical tax rate, f_0, approaches zero.

In the case of very high rates of inflation, leasing is always preferable to buying. Then $G(0) < 0$, $G(1) < 0$ and $f_0 < 0$.

Once again, under the realization method of depreciation, the changes in profitability with changing inflation differ from those under the other depreciation methods. In general, as inflation increases, G approaches its mathematical limit, $-dC$. In this case the present value of lease payments becomes zero, and that of the costs of finance in the purchase option dC, which implies that dC gives the absolute difference in the respective present values in favour of leasing.

Figure 3.2 gives a summary of the dependence of f_0, the critical tax rate, on the rate of inflation, using all combinations of loan types and methods of depreciation.[40] If the actual tax rate f lies below the graph of f_0 ($f < f_0$), buying is preferable to leasing. The profitability difference in terms of present values is the greater the further below the actual tax rate lies from the f_0-graph.

Correspondingly, if $f > f_0$, leasing is preferable to buying. Figure 3.2 can be used in assessing the importance of taxes in each lease-vs.-buy comparison. If the profit position of the firm and its effective tax rate (i.e. one adjusted for the tax deductibility of dividends) make it desirable to use a higher tax rate than f_0 in profitability comparisons, leasing is chosen as the method of finance. In the opposite case buying is the preferred choice.

Figure 3.2 can be utilized in examining other aspects of the model as well. If the expected rate of inflation is assumed to be fixed, the figure gives the effects of the method of depreciation (when the type of loan is fixed) and the type of loan (when the method of depreciation is fixed) on the critical tax rate. For example, if the expected yearly inflation rate is 10 %, the combination serial loan-declining balance depreciation gives a critical tax rate of 0.159 (case a). If the loan were changed into an annuity loan, f_0 would increase to 0.225 (case b). On the other hand, if serial loan were combined with straight line depreciation, the critical tax rate would

40 Figure 3.2 is based on the numerical results in Tables 7—12 of Appendix 2. Thus, $d = 0.35$.

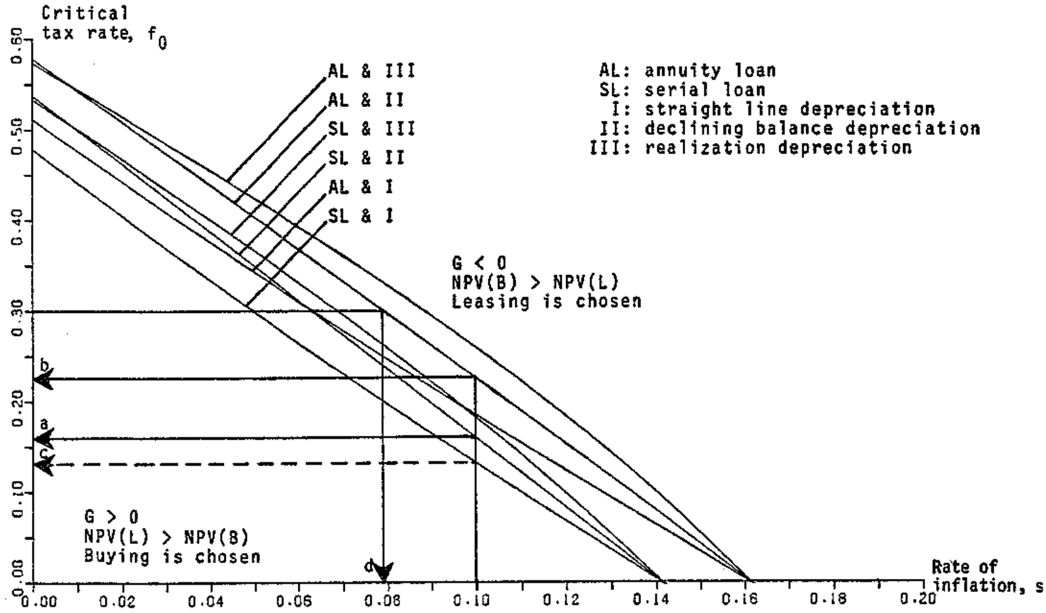

Figure 3.2. The Critical Tax Rate f_0 as a Function of the Rate of Inflation, s (d = 0.35)

be lowered to 0.131 (case c). If all other parameters are assumed to be fixed except for the rate of inflation, Figure 3.2 can be used to examine its effects on the profitability ordering. When, for example, tax rate equals 0.30, and annuity loan and declining balance depreciation are chosen, buying becomes preferable to leasing, if the expected annual rate of inflation is less than 7.9 % (case d). If inflation is expected to be higher than that, leasing is the best method of finance.

332. The Effects of the Structure of Finance under the Purchase Alternative

In order to analyse the effects of the structure of finance, the partial derivatives of the present value differences with respect to the equity/income financed fraction of the purchase price ($\partial G/\partial d$) were calculated, together with the present value differences for the two cases: $d = 0$ and $d = 1$, and the critical d-values (d_0) for different rates of inflation. The tax rate was fixed to the value of 55 % ($f = 0.55$).[41]

First, it is assumed that the purchase is completely debt financed, i.e. the relevant present value difference is G(0). In this case purchase is always the preferred choice ($G(0) > 0$), as was pointed out in the analysis of taxation. The method of depreciation affects the magnitude of G(0): under straight

[41] For numerical results, see Aho—Virtanen (1981), Appendix 5. The cases: $f = 0$ and $f = 1$ were analysed earlier in this paper.

line depreciation the difference in favour of buying is smallest, and under realization method largest. Exceptionally, in the case of stable prices or very low inflation ($s = 0$ to 0.02), the realization and declining balance methods result in G(0)-values of roughly similar magnitude. The type of loan chosen affects the present value difference G(0) in that the latter is larger in the case of an annuity loan than when serial loan is used.

Leasing becomes more profitable when d, the equity/income financed fraction of the purchase price is increased. Using the assumed values for fixed parameters, a critical d-value can always be found. As the rate of inflation increases, the rate of change $|\partial G/\partial d|$ also increases, which implies that the critical value, d_0, approaches zero. Under straight line depreciation, the resulting d_0-value is clearly smallest. In the case of stable prices, for example, d_0 equals approximately 0.27 (if serial loan is chosen) or 0.31 (if annuity loan is chosen), whereas the other two methods of depreciation produce d_0-values of 0.33 and 0.37, respectively. When inflation is allowed for, the above relationship remains unchanged except for the fact that now realization and declining balance methods result in differing d_0-values. The complete ordering is then: $d_0(I) < d_0(II) < d_0(III)$. When serial loan is chosen, the resulting d_0-values are always smaller than in the case of an annuity loan, irrespectively of the method of depreciation used and the rate of inflation. This implies that leasing becomes the preferred alternative with lower d-values in the case of a serial loan, as compared with an annuity loan.

The effect of d on the quantity $\partial G/\partial d$ is always negative.[42] In the present model, this effect does not depend on the depreciation method chosen, which can also be ascertained by examining the analytical expression for $\partial G/\partial d$, equation (3.13). *Increasing inflation strengthens the effect of d.* Choice of the loan type has a negligible impact on $\partial G/\partial d$ at all levels of inflation. In the case of a serial loan, the effect of d is smaller than in the case of an annuity loan. Inflation increases this difference slightly.

When the purchase is totally or mainly financed from equity/income sources, leasing is always more profitable than buying ($G(1) < 0$). In this case, the present value difference is strongly[43] dependent on the rate of inflation, increasing inflation making the lease alternative even more favourable. Of course, when d equals one the type of loan has no effect on G, and this effect is only slight as long as d is close to unity. The effects of choosing the method of depreciation are dependent on the prevailing rate of inflation. If $s = 0$, the order is as follows: $G(1)_I < G(1)_{III} < G(1)_{II}$; and if $s > 0$, the ranking changes to: $G(1)_I < G(1)_{II} < G(1)_{III}$. As inflation increases, the differences in these G-values become wider, and $G(1)_{II}$ moves towards $G(1)_I$.

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42 See the partial analysis earlier in this paper.

43 Cf. the analysis of the strength of the d-effect above.

Figure 3.3 summarizes the dependence of d_0, the critical structure of finance, on the rate of inflation under different combinations of loan types and depreciation methods. The tax rate is assumed to be fixed at 0.55. If the actual fraction of the purchase price financed from equity/income, d, lies above the graph of d_0 in Figure 3.3, leasing is the preferred alternative, given the assumed values for fixed parameters. *Increasing inflation improves the profitability of leasing, which implies that in order to become the preferred alternative, purchase must be increasingly debt-financed.* Correspondingly, if the actual d lies below the d_0-graph in Figure 3.3, buying is the preferable choice.

Figure 3.3 can be utilized in planning the structure of finance in the purchase case. After fixing all the other parameters, the critical structure of finance, d_0, can be immediately located in the diagram. Should it not be feasible for practical reasons to use adequate debt finance so that d remains below its critical level, leasing should be selected as the method of finance.

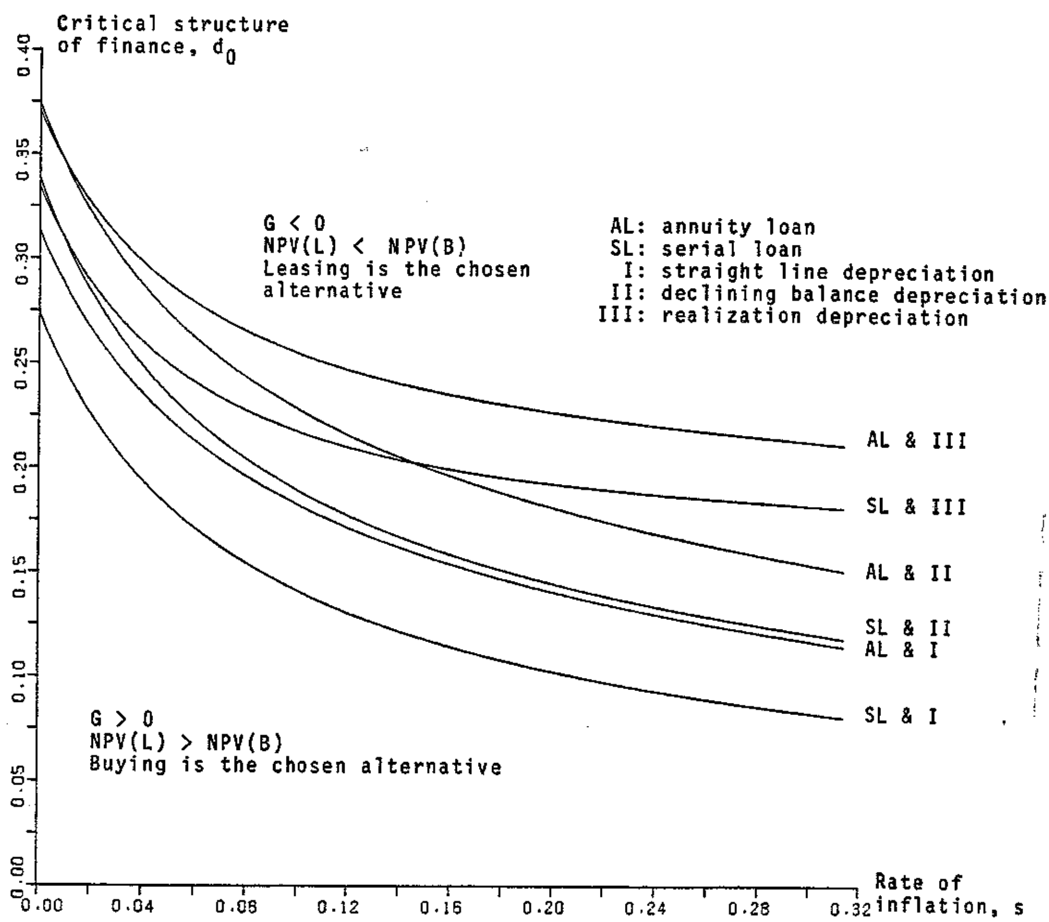

Figure 3.3. The Critical Structure of Finance, d_0, as a Function of the Rate of Inflation, s (f = 0.55)

Figure 3.3 can also be used to determine the magnitude of the permissible rate of inflation, when d is assumed to be fixed. This takes place analogously with the discussion on taxation effects (see 331). If the expected rate of inflation is lower than the rate corresponding to the critical d-value, d_0, purchase is the chosen alternative. Conversely, if the expected rate of inflation exceeds the rate permitted by the selected structure of finance, leasing becomes the preferred option. In addition to the above analyses, Figure 3.3 may be utilized in examining the effects of the type of loan and the depreciation method chosen when the rate of inflation is assumed to be fixed.

4. DISCUSSION

This paper has analysed the lease-vs.-buy profitability comparisons using the present value method. The model does not take into account the scrap value of the investment. This exclusion can be justified by the fact that in practice it is possible for the firm to buy the leased machine at the end of the lease period. If this takes place, the final scrap values of the investment can be assumed to be equal in both alternatives, and do not therefore affect the difference between the respective present values of costs of finance.[44] The price that the firm under the lease alternative actually pays for the machine at the end of the lease period should be incorporated in calculating the present value of leasing costs. However, this item is so negligible in relation to lease payments, that its omission was considered justified for reasons of clarity. Further support for this decision is provided by the fact that the present value of this purchase price becomes a fraction of its nominal magnitude when inflation and time factors are taken into account.

Defining the discount rates to be used in the calculations is an important question in the present value method. Different discount rates have been used in discounting flows with different degrees of riskiness attached to them in order to arrive at one single present value.[45] The methods of determining these discount rates are, however, still controversial.[46] For this reason, and for the sake of expositional clarity, it was considered justified to use one discount rate only in constructing the present model.[47]

Comparing the results of this model with the previous studies is difficult because of the scarcity of lease-vs.-buy comparison models with inflation

44 Cf. Bower (1976), pp. 265—267.

45 See e.g. Schall (1974), p. 1207.

46 Bower (1976), pp. 265—267.

47 For similar decisions, see Mao (1969), p. 325 and Merrett—Sykes (1974), p. 262.

incorporated.[48] This stands in marked contrast with the wealth of studies that analyse inflation and its incorporation into the present value method in general.[49] As is evident from the present paper, inflation can be taken into account in calculating the present values for the alternatives in a relatively clear-cut manner.

The numerical analysis of the model was mainly restricted to one investment decision, in which the structure of finance, tax rate, type of loan and method of depreciation were allowed to take different values. The diagrams or nomographs that were used in summarising the results of the analysis can, however, be seen as important even more generally. If the firm applies the present value model constructed here in its lease-vs.-buy comparisons, it is possible, after the parameters are fixed, to design the nomographs specifically according to the requirements of each investment problem.

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48 The only reference that directly analyses the effects of inflation is Merrett—Sykes, pp. 260—263.

49 Foster (1970), pp. 19—24, gives one example of these.

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APPENDIX 1. List of Symbols

Symbol	Interpretation	Unit
$\bar{a}_{n\,\rceil\,i}$	discount or present value factor for discrete payments made at the end of the relevant period, where n is the number of periods and i the discount rate (discrete payments, continuous discounting)	—
$\bar{A}$	(constant) annuity of an annuity loan	FIM
C	purchase price of the equipment	FIM
$\bar{c}_{n\,\rceil\,r}$	amortization or annuity factor for an annuity loan, where the loan period is n and r the loan interest rate (discrete payments, continuous discounting)	—
d	fraction of equity/income finance in the purchase price	—
d_0	critical d-value (which makes the lease and buy alternatives equally profitable)	—
D_t	depreciation in year t	FIM
f	income tax rate	—
f_0	critical tax rate (see discussion on d_0 above)	—
G	present value difference between the costs of lease and buy alternatives	FIM
i	real discount rate	$year^{-1}$
i_s	inflation adjusted or nominal discount rate $= i + s$	$year^{-1}$
I_t	interest payments in year t	FIM
I^*	present value of interest payments after tax when the magnitude of the loan is C	FIM
j	rate of depreciation under the declining balance method	—
k	annual lease payment coefficient	—
k′	monthly lease payment coefficient	—
K_t	amortization in year t	FIM
K^*	present value of amortizations for a loan whose magnitude is C	FIM
L_t	lease payments in year t	FIM
n	number of lease periods	—
NPV(B)	present value of the costs of finance after tax under the purchase alternative	FIM
NPV(D)	present value of the depreciation tax shield	FIM

Symbol	Interpretation	Unit
NPV(I)	present value of interest payments after tax	FIM
NPV(K)	present value of amortizations + finance from equity/ income sources	FIM
NPV(L)	present value of lease payments after tax	FIM
r	nominal rate of interest on debt capital	$year^{-1}$
s	rate of inflation (continuous)	$year^{-1}$
s_0	critical rate of inflation	$year^{-1}$
t	sub-index denoting the number of the year	—
t	length of the discounting period	years

APPENDIX 2. Numerical results concerning the choice criterion G(f).

Table 1. Serial loan, straight line depreciation, d = 0

s	∂G/∂f	G(0)	G(1)	f_0
0.00	—11172	11172	0	1
0.01	—11252	11252	0	1
0.02	—11321	11321	0	1
0.03	—11382	11382	0	1
0.04	—11434	11434	0	1
0.05	—11478	11478	0	1
0.06	—11514	11514	0	1
0.07	—11543	11543	0	1
0.08	—11566	11566	0	1
0.09	—11582	11582	0	1
0.10	—11593	11593	0	1
0.12	—11600	11600	0	1
0.14	—11587	11587	0	1
0.16	—11559	11559	0	1
0.18	—11518	11518	0	1
0.20	—11464	11464	0	1
0.30	—11074	11074	0	1
0.40	—10572	10572	0	1
0.50	—10035	10035	0	1
0.60	—9498	9498	0	1
0.70	—8979	8979	0	1
0.80	—8482	8482	0	1
0.90	—8010	8010	0	1
1.00	—7562	7562	0	1

Table 2. Annuity loan, straight line depreciation, d = 0

s	∂G/∂f	G(0)	G(1)	f_0
0.00	—9791	11464	1673	1.170
0.01	—9917	11679	1761	1.177
0.02	—10031	11875	1843	1.183
0.03	—10135	12054	1918	1.189
0.04	—10228	12217	1989	1.194
0.05	—10312	12366	2054	1.199
0.06	—10386	12501	2114	1.203
0.07	—10453	12622	2169	1.207
0.08	—10511	12731	2220	1.211
0.09	—10562	12829	2266	1.214
0.10	—10606	12916	2309	1.217
0.12	—10675	13059	2383	1.223
0.14	—10722	13165	2443	1.227
0.16	—10748	13240	2491	1.231
0.18	—10758	13286	2528	1.235
0.20	—10751	13308	2556	1.237
0.30	—10554	13135	2580	1.244
0.40	—10189	12673	2483	1.243
0.50	—9750	12075	2325	1.238
0.60	—9285	11425	2140	1.230
0.70	—8817	10766	1949	1.221
0.80	—8359	10123	1763	1.210
0.90	—7916	9505	1588	1.200
1.00	—7490	8917	1427	1.190

Table 3. Serial loan, declining balance depreciation, d = 0

s	∂G/∂f	G(0)	G(1)	f_0
0.00	—9022	11172	2150	1.238
0.01	—8967	11252	2284	1.254
0.02	—8909	11321	2411	1.270
0.03	—8848	11382	2533	1.286
0.04	—8784	11434	2649	1.301
0.05	—8717	11478	2760	1.316
0.06	—8648	11514	2865	1.331
0.07	—8577	11543	2965	1.345
0.08	—8505	11566	3061	1.359
0.09	—8431	11582	3151	1.373
0.10	—8356	11593	3237	1.387
0.12	—8204	11600	3395	1.413
0.14	—8051	11587	3536	1.439
0.16	—7898	11559	3661	1.463
0.18	—7745	11518	3772	1.487
0.20	—7595	11464	3869	1.509
0.30	—6894	11074	4180	1.606
0.40	—6302	10572	4269	1.677
0.50	—5823	10035	4211	1.723
0.60	—5440	9498	4057	1.745
0.70	—5132	8979	3846	1.749
0.80	—4878	8482	3603	1.738
0.90	—4663	8010	3347	1.717
1.00	—4472	7562	3089	1.690

Table 4. Annuity loan, declining balance depreciation, d = 0

s	∂G/∂f	G(0)	G(1)	f_0
0.00	—7641	11465	3824	1.500
0.01	—7633	11679	4045	1.530
0.02	—7619	11875	4255	1.558
0.03	—7601	12054	4452	1.585
0.04	—7578	12217	4639	1.612
0.05	—7551	12366	4814	1.637
0.06	—7521	12501	4979	1.662
0.07	—7487	12622	5135	1.685
0.08	—7450	12731	5281	1.708
0.09	—7411	12829	5418	1.731
0.10	—7369	12916	5546	1.752
0.12	—7280	13059	5778	1.793
0.14	—7186	13165	5979	1.832
0.16	—7087	13240	6153	1.868
0.18	—6985	13286	6301	1.901
0.20	—6882	13308	6425	1.933
0.30	—6374	13135	6760	2.060
0.40	—5919	12673	6753	2.140
0.50	—5539	12075	6536	2.180
0.60	—5227	11425	6197	2.185
0.70	—4971	10766	5795	2.165
0.80	—4755	10123	5367	2.128
0.90	—4568	9505	4936	2.080
1.00	—4400	8917	4517	2.026

Table 5. Serial loan, realization depreciation, d = 0

s	∂G/∂f	G(0)	G(1)	f_0
0.00	—9149	11172	2023	1.221
0.01	—8944	11252	2307	1.257
0.02	—8721	11321	2600	1.298
0.03	—8481	11382	2900	1.341
0.04	—8226	11434	3207	1.389
0.05	—7958	11478	3519	1.442
0.06	—7678	11514	3835	1.449
0.07	—7388	11543	4154	1.562
0.08	—7090	11566	4475	1.631
0.09	—6784	11582	4798	1.707
0.10	—6472	11593	5120	1.791
0.12	—5835	11600	5764	1.987
0.14	—5185	11587	6402	2.234
0.16	—4531	11559	7028	2.551
0.18	—3877	11518	7640	2.970
0.20	—3229	11464	8235	3.550
0.30	—191	11074	10882	57.715
0.40	2335	10572	12908	—4.526
0.50	4282	10035	14318	—2.343
0.60	5686	9498	15184	—1.670
0.70	6623	8979	15602	—1.355
0.80	7181	8482	15664	—1.181
0.90	7443	8010	15454	—1.076
1.00	7479	7562	15042	—1.011

Table 6. Annuity loan, realization depreciation, d = 0

s	∂G/∂f	G(0)	G(1)	f_0
0.00	—7768	11465	3697	1.475
0.01	—7610	11679	4068	1.534
0.02	—7431	11875	4443	1.597
0.03	—7234	12054	4819	1.666
0.04	—7020	12217	5196	1.740
0.05	—6792	12366	5573	1.820
0.06	—6551	12501	5949	1.908
0.07	—6298	12622	6324	2.004
0.08	—6035	12731	6696	2.109
0.09	—5764	12829	7065	2.225
0.10	—5485	12916	7430	2.354
0.12	—4911	13059	8148	2.659
0.14	—4230	13165	8845	3.047
0.16	—3720	13240	9520	3.558
0.18	—3117	13286	10169	4.262
0.20	—2516	13308	10791	5.288
0.30	328	13135	13463	—40.031
0.40	2718	12673	15391	—4.661
0.50	4567	12075	16643	—2.643
0.60	5899	11425	17324	—1.936
0.70	6784	10766	17551	—1.586
0.80	7304	10123	17427	—1.385
0.90	7537	9505	17042	—1.261
1.00	7551	8917	16469	—1.180

Table 7. Serial loan, straight line depreciation, d = 0.35

s	∂G/∂f	G(0)	G(1)	f_0
0.00	—19069	9381	—10228	0.478
0.01	—19510	8609	—10900	0.441
0.02	—19405	7855	—11550	0.404
0.03	—19296	7118	—12177	0.368
0.04	—19183	6398	—12784	0.333
0.05	—19066	5696	—13370	0.298
0.06	—18946	5009	—13936	0.264
0.07	—18823	4338	—14484	0.230
0.08	—18698	3683	—15014	0.197
0.09	—18570	3044	—15526	0.163
0.10	—18441	2419	—16022	0.131
0.12	—18178	1212	—16966	0.066
0.14	—17910	60	—17850	0.003
0.16	—17639	—1039	—18678	—0.058
0.18	—17366	—2089	—19456	—0.120
0.20	—17094	—3091	—20186	—0.180
0.30	—15755	—7476	—23231	—0.474
0.40	—14505	—10994	—25498	—0.757
0.50	—13367	—13849	—27127	—1.036
0.60	—12345	—16197	—28542	—1.312
0.70	—11427	—18153	—29581	—1.588
0.80	—10601	—19807	—30408	—1.868
0.90	—9854	—21222	—21076	—2.153
1.00	—9173	—22448	—31622	—2.447

Table 8. Annuity loan, straight line depreciation, d = 0.35

s	∂G/∂f	G(0)	G(1)	f_0
0.00	—18712	9572	—9140	0.511
0.01	—18642	8887	—9755	0.476
0.02	—18566	8214	—10352	0.442
0.03	—18485	7555	—10930	0.408
0.04	—18399	6908	—11491	0.375
0.05	—18308	6273	—12035	0.342
0.06	—18213	5650	—12562	0.310
0.07	—18114	5040	—13074	0.278
0.08	—18012	4441	—13571	0.246
0.09	—17907	3854	—14053	0.215
0.10	—17800	3278	—14521	0.184
0.12	—17577	2160	—15417	0.122
0.14	—17347	1085	—16261	0.062
0.16	—17112	52	—17059	0.003
0.18	—16872	—939	—17812	—0.055
0.20	—16630	—1893	—18524	—0.113
0.30	—15417	—6136	—21553	—0.398
0.40	—14255	—9628	—23884	—0.675
0.50	—13182	—12523	—25706	—0.950
0.60	—12206	—14945	—27151	—1.224
0.70	—11322	—16991	—28314	—1.500
0.80	—10521	—18740	—29262	—1.781
0.90	—9792	—20250	—30043	—2.067
1.00	—9126	—21568	—30694	—2.363

Table 9. Serial loan, declining balance depreciation, d = 0.35

s	∂G/∂f	G(0)	G(1)	f_0
0.00	—17459	9381	—8077	0.537
0.01	—17225	8609	—8616	0.499
0.02	—16993	7855	—9138	0.462
0.03	—16762	7118	—9643	0.424
0.04	—16533	6398	—10134	0.387
0.05	—16305	5696	—10609	0.349
0.06	—16080	5009	—11071	0.311
0.07	—15857	4338	—11518	0.273
0.08	—15637	3683	—11953	0.235
0.09	—15419	3044	—12375	0.197
0.10	—15204	2419	—12785	0.159
0.12	—14783	1212	—13571	0.081
0.14	—14374	60	—14313	0.004
0.16	—13977	—1039	—15017	—0.074
0.18	—13594	—2089	—15684	—0.153
0.20	—13224	—3091	—16316	—0.233
0.30	—11575	—7476	—19051	—0.645
0.40	—10234	—10994	—21229	—1.074
0.50	—9156	—13849	—23006	—1.512
0.60	—8287	—16197	—24485	—1.954
0.70	—7581	—18153	—25735	—2.394
0.80	—6997	—19807	—26804	—2.830
0.90	—6506	—21222	—27728	—3.261
1.00	—6083	—22448	—28532	—3.690

Table 10. Annuity loan, declining balance depreciation, d = 0.35.

s	∂G/∂f	G(0)	G(1)	f_0
0.00	—16561	9572	—6989	0.577
0.01	—16358	8887	—7471	0.543
0.02	—16154	8214	—7940	0.508
0.03	—15951	7555	—8396	0.473
0.04	—15749	6908	—8841	0.438
0.05	—15548	6273	—9274	0.403
0.06	—15347	5650	—9696	0.368
0.07	—15148	5040	—10108	0.332
0.08	—14951	4441	—10510	0.297
0.09	—14756	3854	—10901	0.261
0.10	—14562	3278	—11284	0.225
0.12	—14182	2160	—12021	0.152
0.14	—13811	1085	—12725	0.078
0.16	—13450	52	—13397	0.003
0.18	—13100	—939	—14040	—0.071
0.20	—12761	—1893	—14655	—0.148
0.30	—11237	—6136	—17373	—0.546
0.40	—9985	—9628	—19614	—0.964
0.50	—8971	—12523	—21494	—1.395
0.60	—8148	—14945	—23093	—1.833
0.70	—7476	—16991	—24468	—2.272
0.80	—6917	—18740	—25658	—2.709
0.90	—6445	—20250	—26695	—3.142
1.00	—6036	—21568	—27604	—3.573

Table 11. Serial loan, realization depreciation, d = 0.35

s	∂G/∂f	G(0)	G(1)	f_0
0.00	—17536	9381	—8204	0.533
0.01	—17202	8609	—8593	0.500
0.02	—16805	7855	—8950	0.467
0.03	—16395	7118	—9277	0.434
0.04	—15975	6398	—9576	0.400
0.05	—15546	5696	—9850	0.366
0.06	—15110	5009	—10101	0.331
0.07	—14669	4338	—10330	0.295
0.08	—14222	3683	—10538	0.259
0.09	—13772	3044	—10728	0.221
0.10	—13320	2419	—10901	0.181
0.12	—12413	1212	—11201	0.097
0.14	—11508	60	—11448	0.005
0.16	—10610	—1039	—11650	—0.097
0.18	—9726	—2089	—11815	—0.214
0.20	—8858	—3091	—11950	—0.349
0.30	—4872	—7476	—12348	—1.534
0.40	—1596	—10994	—12590	—6.887
0.50	950	—13849	—12899	14.573
0.60	2839	—16197	—13358	5.704
0.70	4174	—18153	—13979	4.348
0.80	5062	—19807	—14744	3.912
0.90	5599	—21222	—15622	3.789
1.00	5868	—22448	—16580	3.825

Table 12. Annuity loan, realization depreciation, d = 0.35

s	∂G/∂f	G(0)	G(1)	f_0
0.00	—16688	9572	—7116	0.573
0.01	—16335	8887	—7448	0.544
0.02	—15966	8214	—7751	0.514
0.03	—15585	7555	—8029	0.484
0.04	—15191	6908	—8283	0.454
0.05	—14789	6273	—8515	0.424
0.06	—14378	5650	—8727	0.393
0.07	—13960	5040	—8919	0.361
0.08	—13537	4441	—9095	0.328
0.09	—13109	3854	—9255	0.294
0.10	—12679	3278	—9400	0.258
0.12	—11812	2160	—9652	0.182
0.14	—10945	1085	—9859	0.099
0.16	—10083	52	—10030	0.005
0.18	—9232	—939	—10171	—0.101
0.20	—8395	—1893	—10289	—0.225
0.30	—4534	—6136	—10671	—1.353
0.40	—1347	—9628	—10976	—7.146
0.50	1135	—12523	—11388	11.030
0.60	2977	—14945	—11967	5.018
0.70	4279	—16991	—12712	3.970
0.80	5142	—18740	—13598	3.644
0.90	5661	—20250	—14589	3.577
1.00	5915	—21568	—15652	3.645