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The impact of OPT and TPM on the economic production quantity

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In manufacturing systems, the material flow is influenced by a number of factors, such as batching policies, capacity of machines, machine breakdowns, etc. Realizing the role of batching policies and reliability of machines in production systems, a mathematical model is presented here for determining optimal batching policies with the objective of improving the speed of material flow considering machine breakdowns and batch splitting and forming. This model is employed for studying (i) the significance of total preventive maintenance (TPM); (ii) the use of the optimized production technology (OPT) concept in batching policies; and (iii) the influence of a set-up cost reduction programme on the performance of manufacturing systems. The basic criterion considered for optimizing the batch sizes is the minimization of total system cost (TSC). An example problem is solved to explain the application of the model.

Notation

i	product index ($i = 1, 2, \ldots, M$)
j	stage index ($j = 1, 2, \ldots, N$)
S_j	number of machines at stage j
D_i	demand for product i per unit time or per year
A_{ij}	set-up cost per set-up for product i at stage j
Q_{ij}	batch size for product i at stage j (decision variable)
ϵ_{ij}	priority assigned in processing (capacity allocation) product i at stage j
α_{ij}	mean breakdown rate of a machine while processing product i at stage j
β_{ij}	mean service rate for bringing the machine to normal operating conditions for product i at stage j
t_{ij}	processing time per unit of product i at stage j
C_{ij}	cost per unit product i after processing at stage j
C_{i0}	raw material cost per unit product i
H	inventory cost per unit investment per unit time period
R_{ij}	number of product cycles for the given demand for product i at stage j
T_{ij}	processing time for a batch of product i at stage j
L_{ij}	average completion time for a batch of product i at stage j
G_{ij}	average cost per unit of product i between stages j and $j+1$
λ_{ij}	production rate for product i at stage j
d_{ij}	lower bound on the value of batch size for product i at stage j
u_{ij}	upper bound on the value of batch size for product i at stage j
Z	total system cost

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1. Introduction

Most multi-stage production–inventory operations are complex systems with many items and production stages, through which these items are processed in batches. The main characteristic of such systems is that they often exhibit congestion phenomena with batch queueing at machines. The queues occur because of the complex flows in the production system and the heterogeneity of items; these result respectively in arrival patterns at work centres/machines that are not uniform and in a variability in processing times. An immediate result of queues is an increase in the manufacturing lead times, implying high levels of work-in-process inventories and poor inventory 'turn'. This clearly emphasizes the need for considering the in-process inventory carrying cost due to queueing of batches between stages. Nevertheless, the batch size optimization with respect to each stage may result in significant cost savings in total system cost. Furthermore, there is a number of mathematical models available in the literature to deal with lot-sizing problems in multi-stage production inventory systems (Imo and Das 1983). However, there are very few models that deal with the problems in lot-sizing considering the machine breakdowns (Goyal and Gunasekaran 1990). However, in practice, the machine breakdowns significantly influence the lot-sizing policies and hence the inventory cost.

Nowadays, the Optimized Production Technology (OPT) concept has been extensively used in determining the capacity required and loading policies. OPT identifies and isolates bottleneck operations and focuses on these bottlenecks to determine production plans and schedules for the entire shop. In this paper, we have used the idea of batch splitting and forming with an objective to employ the concept of OPT in minimizing the inventory cost due to queueing of batches. However, there is limited literature available to consider the application of OPT in lot-sizing policies (Goldratt 1980, 1988). This kind of application will be useful at the planning level and it can also be used as the input to the operational level decision making. The machine breakdowns affect productivity of the manufacturing system to a great extent. The purpose of this paper is to develop a model for a multi-stage production–inventory system with an objective of determining economic production quantities by minimizing total system cost. The basic idea behind this modelling effort is to consider the concept of OPT in batching policies and study the implications of total preventive maintenance (TPM) on the performance of production–inventory systems (Jacobs 1983).

The organization of this paper is as follows: § 2 presents the background for the research. A mathematical model is developed for a multi-stage production system in § 3. Section 4 presents an example to study the behaviour and application of the model. Solution methodology is presented in § 5. Section 6 contains results and analysis from the example. The conclusions of this research are presented in § 7.

2. Background for the research

A number of mathematical and simulation models have been reported in the literature. Most deal with a simple multi-stage production situation, such as each stage consists of only one machine, only one item is processed in the multi-stage production system, no breakdown of the machines, etc. However, these situations do not really represent a realistic situation. Therefore, an attempt has been made in this paper to develop a model for planning-level decision making, considering most of these realistic aspects. In this attempt, a number of approximations and assumptions have been made to model a more realistic production system. However, a balance between the accuracy of modelling and modelling of more realistic situations should be considered. Szendrovits (1975) presented a mathematical model for determining the manufacturing cycle time as a function of the lot-size in a multi-stage production–inventory system. The model assumes that a constant lot size is manufactured through all operations with only one set-up at each stage. He evaluated the economic production quantity (EPQ) using the functional relationship between manufacturing cycle time and work-in-process inventory. Goyal and Gunasekaran (1990) presented a literature review on multi-stage production–inventory systems.

Zipkin (1983) developed an $M/M/1$ queue model, where the facility consists of a single server, and production time is distributed exponentially for all products. This model is constructed by linking standard queueing formulae for that system with standard inventory formulae. Zipkin studied the effect of specifying the inventory sub-models in different ways, including the replacement of penalty costs by constraints on stockouts. An approach that is similar to Zipkin (1983) has been developed by Karmarkar (1983) to investigate the relationships between lot-sizing, manufacturing lead times, and in-process inventories through standard queueing models and the resulting effect on waiting times of batches. Korgaonker (1979) extended the integrality condition for batch sizes to be optimal, as suggested by Crowston *et al.* (1973), to the multi-item case. Imo and Das (1983) investigated the effects of scheduling according to the optimality of various production stages. The results of their study indicate that scheduling according to the batch size optimality of various production stages can be of real advantage. They do show remarkable superiority over the batch schedule with respect to the measures of performance tested. However, they do not account for the inventory carrying cost due to queueing of batches in their model.

Karmarkar *et al.* (1983) presented a model for the multi-item production facility, at which jobs for work at a machine behave as an $M/G/1$ queue. The model is an extension of the model for a single item developed by Karmarkar (1983). A model of the queueing behaviour of a multi-item multi-machine job-shop in which item lot sizes appear as explicit parameters, has been developed by Karmarkar *et al.* (1985). The advantages of unequal batch sizes over constant batch sizes for a simple multi-stage production system are discussed by Szendrovits and Drezner (1980) and Goyal and Szendrovits (1986). Moiley (1986), formulating the problem of lot-sizing in a multi-stage production system under a specified component lot-splitting policy, has shown that considerable cost savings can be achieved if the component lot-splitting policy is followed in multi-stage production systems.

However, none of the above research deals with the case of varying lot sizes for each item with respect to different stages and machine breakdowns while also considering queueing inventory cost with multiple machines at each stage in a multi-stage production system. The varying lot-sizes might help us to employ the concept of OPT in reducing the waiting time of batches. The proposed model will determine the batch sizes with respect to each stage based on the bottleneck stages with an objective of minimizing the multi-stage production system is (i) to model a real-life situation; and (ii) to understand the significance of TPM on the performance of the production system. Taking into account the importance of such situations in most of the real-life manufacturing systems, a mathematical model is presented here for such a system considering machine breakdowns and batch splitting and forming. Furthermore, a search optimization technique is used for determining the EPQs with respect to each stage that would result in least total system cost.

3. The mathematical model

The production system considered for modelling has multiple processing stages to process multiple products. At each stage, there are multiple machines performing the same type of operations. The items are processed and transported between stages only in whole batches. At the first stage, the number of machines to be loaded for a particular product are decided by the priority (ϵ_{ij}) assigned to that product. In the case where more than one product is competing for the same machine, then the items are sequenced according to the value of priority assigned (ϵ_{ij}) to those products for processing at the stage. Products are produced in a cyclical pattern following the priority assigned to those items. Once processing of a batch is over at a particular stage, then it will be transported to the buffer in case no machine is free at the next stage. If the batch size is smaller than the batch size to be processed at the next stage, then the batch has to wait until sufficient quantity has arrived from the previous stage to form a suitable batch for processing at the next stage. Nevertheless, the main objective here is to reduce this waiting time with the help of OPT. In the case where the batch size at the next stage is less than the batch size at the previous stage, then the batch is split into appropriately sized batches for processing at the next stage. The remaining quantity of the batch, if there is any, waits at the buffer to form the next batch suitable for processing at the next stage. This operation is carried out for all products at all stages until all the demand units are processed. The material flow between stages j and $j + 1$ through a buffer is shown in Fig. 1. The development of the model is presented next.

3.1. *Assumptions*

The following assumptions are made in developing the model.

(i) Demand per unit time of a product is deterministic and known.
(ii) Set-up cost per set-up is constant, independent of set-up sequence and batch sizes.
(iii) Machine breakdown follows Poisson distribution and the service time required for each breakdown follows an exponential distribution with average breakdown and repair-service rates respectively.

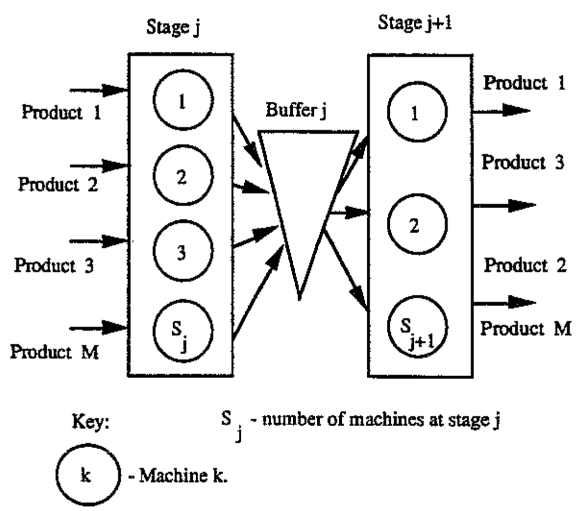

Figure 1. The product system.

(iv) Machines at each stage have identical capacities.

(v) There is no finished product inventory as the products will be dispatched once processing is completed at the final stage.

3.2. *The basic model*

The total system cost consists of following costs: (i) set-up cost; (ii) inventory cost due to machine breakdowns; and (iii) inventory cost due to queueing of batches. These costs are derived here.

3.2.1. *Set-up cost.* The total set-up cost considering all products and stages is given by

$$\sum_{i=1}^{M} \sum_{j=1}^{N} \left\{\frac{D_j}{Q_{ij}}\right\} A_{ij} \tag{1}$$

3.2.2. *Inventory cost due to machine breakdowns.* This cost arises from the waiting of batches, due to stopping and restarting the machines for repair processes. In most occasions, the start-up and shut down of the machines for repair-service processes leads to an in-process inventory carrying cost due to waiting for the machine being brought to the normal operating condition.

The processing time for a batch is given by

$$T_{ij} = Q_{ij} \times t_{ij} \tag{2}$$

It has been assumed here that the time between two successive breakdowns of a machine follows an exponential distribution. Hence, the number of breakdowns per unit time follows the Poisson process with mean breakdown rate. The breakdown rate is a function of machine age, and the nature of the process, etc. Depending upon the nature of service required and the skill of the repair-service worker, the service time required to bring the machine to a normal working condition varies. Therefore, it is assumed that the service time required for each machine breakdown task follows an exponential distribution with mean service rate. The purpose of incorporating machine failures is to point out the importance of TPM in improving the performance of the production system.

Suppose the operator is an $M/M/1$ server (or a server when there are S_j machines at stage j). Every time a machine fails it has to be serviced by the maintenance. From $M/M/1$ queueing theory, the total time spent (waiting time plus service time per breakdown) by a batch per breakdown can be estimated using the $M/M/1$ (infinite source) queueing formula (Panico 1968) as

$$\frac{1}{(\beta_{ij} - \alpha_{ij})} \tag{3}$$

The average time spent by the batch due to machine breakdown while processing that batch can be obtained as

$$T_{ij}\left\{\frac{\alpha_{ij}}{(\beta_{ij} - \alpha_{ij})}\right\} \tag{4}$$

The total time required (processing time for batch + average time spent by the batch due to machine breakdowns) for product i at stage j to complete the processing of a batch is given by

$$L_{ij} = T_{ij}\left\{\frac{\beta_{ij}}{(\beta_{ij} - \alpha_{ij})}\right\} \tag{5}$$

The number of production cycles per unit time (per year) for product i at stage j is represented by

$$R_{ij} = \left\{\frac{D_i}{Q_{ij}}\right\} \tag{6}$$

Since a breakdown of the machine may occur at any time during the processing of a batch of a product, the value of the per unit product at which the breakdown occurs may be difficult to obtain. Hence, the average cost per unit product has been accounted for to compute the cost due to machine breakdowns. This cost can be calculated as

$$G_{ij} = \left\{\frac{C_{ij-1} + C_{ij}}{2}\right\} \tag{7}$$

The total inventory cost due to machine breakdowns is estimated as

$$\sum_{i=1}^{M} \sum_{j=1}^{N} \left[\{R_{ij} Q_{ij} T_{ij} G_{ij}\}\left\{\frac{\alpha_{ij}}{(\beta_{ij} - \alpha_{ij})}\right\}\right] H \tag{8}$$

3.2.3. *Inventory cost due to queueing of batches.* The production rate for a particular product depends upon the number of machines actually used for a product. If more than one machine is used for a product, then the production rate will be much higher than with only one machine. The production rate for a product at a stage is a function of the total process completion time of a batch, and the number of machines used for processing at that stage, etc.

The product rate for product i from stage j can be calculated using (8) as

$$\lambda_{ij} = \left[\frac{\{\epsilon_{ij} \times S_j\}}{L_{ij}}\right] \tag{9}$$

where $$\sum_{i=1}^{M} \epsilon_{ij} = 1, \text{ for } j = 1, 2, \ldots, N$$

The parameter ϵ_{ij} indicates the number of machines to be assigned for a particular product at a stage. In practice, the value of ϵ_{ij} is estimated based on marketing and financial factors.

To find the waiting time for each batch and hence the inventory carrying cost of product i between stages j and $j + 1$, the material flow for a product through a buffer with two adjacent production stages is modelled as an $M/M/1$ production–inventory queueing system. In this system, the waiting bay constitutes the buffer storage. The arrival process of batches of a product at the buffer storage depends on the output rate at stage j, while the discharge process from the buffer depends on the output rate of the stage $j + 1$. Since these two rates for a product may not be equal, this would lead to an imbalance in arrival and departure of batches and hence random forming and depletion of the in-process inventory at the buffer from time to time.

Here, we use the concept of OPT in order to distribute the effect of the bottleneck stage to other stages by suitably adjusting the batch sizes by forming and splitting processes. This concept helps to reduce the effect of bottleneck stages and hence reduced the waiting time of batches due to bottleneck stages.

Using the standard $M/M/1$ queueing formulae (see Panico 1968), the average waiting time for a batch is given by

$$W_{ij} = \left\{ \frac{\lambda_{ij}}{[\lambda_{ij+1}(\lambda_{ij+1} - \lambda_{ij})]} \right\} \tag{10}$$

In order to maintain the stability of material flow and hence the system, the condition $\lambda_{ij+1} > \lambda_{ij}$ has to be satisfied. In case the restriction on batch sizes (due to technological and operational constraints) may not permit the condition $\lambda_{ij+1} > \lambda_{ij}$ to be satisfied, then the capacities at each stage should be arranged to satisfy this condition and reduce the effect of bottleneck operations.

Hence, the in-process inventory carrying cost due to queueing of batches between stages/at buffer considering all products and buffers is obtained using (8) and is given as

$$\sum_{i=1}^{M} \sum_{j=1}^{N} \{R_{ij} W_{ij} Q_{ij} C_{ij}\} H \tag{11}$$

3.2.4. *Problem formulation.* The total system cost can be obtained by the summation of cost equations (1), (8) and (11). The formulation of the batching problem can be given as

$$\text{minimize } Z = \sum_{i=1}^{M} \sum_{j=1}^{N} \left\{ \frac{D_i}{Q_{ij}} \right\} A_{ij} + \sum_{i=1}^{M} \sum_{j=1}^{N} \left[\{R_{ij} Q_{ij} T_{ij} G_{ij}\} \left\{ \frac{\alpha_{ij}}{(\beta_{ij} - \alpha_{ij})} \right\} \right] H$$
$$+ \sum_{i=1}^{M} \sum_{j=1}^{N-1} \{R_{ij} W_{ij} Q_{ij} C_{ij}\} H \tag{12}$$

subject to the following constraints

$$\beta_{ij} > \alpha_{ij} \qquad \text{for all } i \text{ and } j \tag{13}$$

$$d_{ij} \leq Q_{ij} \leq u_{ij} \quad \text{for all } i \text{ and j} \tag{14}$$

Constraint (13) indicates that the service rate for the process control must be greater than the drift rates for a product at any stage. The lower and upper bounds on batch sizes are represented by the constraint (14).

4. Example

A three-stage production system manufacturing three products is considered to illustrate the application of the model. The objective here is to determine optimal batch sizes by minimizing the total system cost. The input to the example problem is presented in Table 1. The value of ϵ_{ij} has been determined here arbitrarily. However, this value should be determined based on economical and technical factors.

	Product 1			Product 2			Product 3		
Parameter/variables	Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3
D		3000			2000			1500	
s	2	4	6	2	4	6	2	4	6
C_0		30·0			25·00			4·00	
C	50·000	70·00	85·00	40·00	55·0	75·00	65·00	80·00	95·00
t	0·0001	0·00006	0·00003	0·00008	0·00004	0·00002	0·0001	0·00005	0·00003
A	200·0	150·00	100·00	150·00	100·00	150·00	100·00	80·00	150·00
ϵ	0·2	0·4	0·2	0·4	0·4	0·4	0·4	0·2	0·4
α	100	60	30	120	80	50	80	40	20
β	120	100	50	150	110	70	100	60	40

Table 1. Input to the analytical model ($M = 3$; $N = 3$; $H = 0{\cdot}20$; and $d_{ij} = 100$, $u_{ij} = 600$ for all i and j).

5. Solution methodology

The structure of the objective function is discontinuous in nature and therefore it is not easily amenable to the classical optimization procedure, wherein the objective function should be differentiable at least once. The direct pattern search method (DPSM) of Hooke and Jeeves (1966) is used for the present problem. The search method is not unidirectional in nature as it moves in both directions. The search is performed either by increasing or decreasing the value of the decision variables 'Q_{ij}' by a constant factor, called the step size. Different starting values may produce different optimal batch size values. For more details about this search method, the readers are encouraged to refer to Hooke and Jeeves (1966). The batch size optimization, with respect to each product-stage, embodies the feature of batch splitting and forming and results in better system performance by reducing the total manufacturing cost and increasing the utilization level of facilities. The model has been coded in Fortran IV together with the DPSM. The computational time required for each run is only a few seconds on a Vax 3400 computer system, which has a memory capacity of 29 MB. It should also be noted that larger problems can also be solved using this DPSM.

6. Results and Analysis

The results obtained by the DPSM are presented in Table 2. This table also contains the comparison of results obtained for the cases of uniform batch sizes of each product at all stages and of batch size with respect to each stage. A saving

Iteration	Batch sizes ($Q_{ij}, j = 1, N; i = 1, M$)	Set-up cost (\$) ($\times 10^4$)	Cost due to machine breakdowns (\$) ($\times 10^4$)	Cost due to queueing of batches (\$) ($\times 10^4$)	Total cost (Z) (\$) ($\times 10^4$)
1	{400, 400, 400, 200, 200, 200, 100, 100, 100.}	1·23250	0·935658	0·24667	2·414825
16	{200, 200, 400, 200, 200, 400, 100, 100, 300.}	1·19500	0·750358	0·133114	2·088472
49	{200, 400, 200, 300, 300, 400, 200, 200, 400.}	0·995417	0·850050	0·111178	1·956645
...	...	...	...	...	...
229	{231, 409, 225, 284, 306, 362, 181, 163, 391.}	0·971002	0·864291	0·106681	1·941974
263	{230, 410, 224, 285, 307, 361, 180, 164, 390.}	0·9772263	0·863203	0·106491	1·941957
299	{230, 409, 224, 286, 307, 361, 181, 164, 390.}	0·971851	0·863846	0·106258	1·941955

Cost savings in total system cost = ((2·414825 − 1·941955)/2·414825) × 100 = 19·58%

Table 2. The results obtained by the search method.

of 19.58% in total system cost has been achieved using the lot-sizing with respect to each stage, as compared with that of uniform batch sizes at all stages for a product. However, there are some administrative problems associated with keeping track of various batch sizes with respect to each stage while the work is in progress. The ideas of batch splitting and forming fully support the concept of OPT in distributing the bottleneck stages or operations with the objective of reducing the queueing time between stages and hence helping to increase the speed of material flow within the production system.

The optimal batch sizes and the corresponding costs obtained for the cases with and without machine breakdowns are presented in Table 3. Comparing the results for the cases without machine breakdowns and with machine breakdowns (for the same set of batch sizes) indicates an increase in TSC by 23·97%. Obviously, one could save this cost by proper preventive maintenance and this explains the importance of the TPM system. This model can also be extended to consider the level of scrappage due to machine failures while optimizing batch sizes. Nevertheless, the cost of service of failed machines should also be included in the modelling effort.

The optimal batch sizes obtained with machine breakdowns facilitate the use of smaller batch sizes as compared with those without machine breakdowns. Also, they lead to an effective balancing of production rates as compared with larger batch sizes in the case without machine breakdowns.

To study the behaviour of the model, a sensitivity analysis has been carried out. The details of the results obtained for different levels of inventory holding rate (H), set-up cost per set-up (A), and repair-service rate (β) are reported in Table 4.

The variation in inventory holding rate leads to significant changes in cost due to machine breakdowns and set-up cost. For example, lowering the inventory holding rate (H) from 0·20 to 0·15 results in a reduction in set-up cost ($9718·51 to $8411.·56). This indicates that the model selects larger batch sizes when $H = 0{\cdot}15$ as compared with those when $H = 0{\cdot}20$, in order to save some set-up cost. Because of the larger batch sizes, the cost due to batch waiting has also increased. This also explains how important it is to reduce the inventory level, due to various disturbances in the system. By doing this, one could achieve an increased productivity by reducing the cycle time of the product using smaller batch sizes.

Situation	Optimal batch sizes ($Q_{ij}, j = 1, N; i = 1, M$)	Set-up cost ($) ($\times 10^4$)	Cost due to machine breakdowns ($) ($\times 10^4$)	Cost due to queueing of batches ($) ($\times 10^4$)	Total cost (Z) ($) ($\times 10^4$)
Without machine breakdowns	600, 600, 301, 600, 600, 600, 600, 328, 600.	0·507212	...	0·123725	0·630937
With machine breakdowns	230, 409, 224, 286, 307, 361, 181, 164, 390.	0·971851	0·863846	0·106258	1·941955
With machine breakdowns	600, 600, 301, 600, 600, 600, 600, 328, 600.	0·507212	1·923170	0·123725	2·554107

Percentage increase in total system cost = (2·554107 − 1·941955) × 100/2·554107 = 23·97%

Table 3. Comparison of the results without and with machine breakdowns.

Parameters/ variables	Optimal batch sizes ($Q_{ij}, j = 1, N; i = 1, M$)	Set-up cost (\$) ($\times 10^4$)	Cost due to machine breakdowns (\$) ($\times 10^4$)	Cost due to queueing of batches (\$) ($\times 10^4$)	Total cost (Z) (\$) ($\times 10^4$)
0·15	{266, 473, 259, 330, 355, 418, 208, 189, 451.}	0·841156	0·748350	0·0922766	1·681783
H = 0·20	{230, 409, 224, 286, 307, 361, 181, 164, 390,}	0·971851	0·863846	0·106258	1·941955
0·25	{206, 366, 200, 256, 275, 323, 162, 147, 276.}	1·08570	0·966712	0·118757	2·171169
0.5A	{163, 290, 159, 202, 218, 256, 128, 116, 276.}	0·685776	0·611929	0·0754651	1·373170
A = 1.0A	{230, 409, 224, 286, 307, 361, 181, 164, 390,}	0·971851	0·863846	0·106258	1·941955
2.0A	{326, 579, 317, 404, 435, 511, 256, 232, 552.}	1·37300	1·222910	0·150419	2·746329
1.0β	{230, 409, 224, 286, 307, 361, 181, 164, 390,}	0·971851	0·863846	0·106258	1·941955
β = 2.0β	{600, 600, 327, 600, 572, 600, 511, 289, 543.}	0·514098	0·345320	0·0901260	0·949544
3.0β	{600, 600, 346, 600, 600, 600, 600, 331, 600.}	0·493918	0·200951	0·0880338	0·782903

Table 4. Results obtained by the sensitivity analysis.

A decrease in set-up cost per set-up (1·0A to 0·5A) may lead to a considerable reduction in total set-up cost (from \$9718·51 to \$6857·76). Because of this reduction, the model selects smaller batch sizes. This lowers the cost due to machine breakdowns when A is at 0·5A (\$8638·46 to \$6119·29) as compared with 1·0A. However, the cost due to the queueing of batches has come down from \$1062·58 to \$754·651. This implies that the model looks for higher savings in set-up cost when A is equal to 0·5A. The model prefers smaller batches to achieve a reasonable reduction in cost due to machine breakdowns and batch queueing. The set-up cost reduction facilitates smaller batch size production and hence reduced throughput time and increased productivity. However, the issue of investment in the set-up cost reduction programme must be included in the modelling effort of a multi-stage production–inventory system.

An increase in the repair-service rate (1·0β to 2·0β) will allow the model to select larger batch sizes. This, perhaps, results in a total set-up cost (\$9718·51 to \$5140·98) and lower total system cost (\$19419·55 to \$9495·44). However, the cost due to investment in machine breakdown rate reduction should also be accounted for in the total system cost. Obviously, the increase in repair-service rate leads to a corresponding reduction in cost due to machine breakdowns, from \$8638·46 to \$3453·20. This also leads to a decrease in inventory cost due to batch queueing. The analysis of the results reveals the significance of preventive maintenance and set-up reduction for an improved productivity and quality.

7. Concluding remarks

A mathematical model is presented in this paper for determining economic production quantities in a multi-stage production system by minimizing the total system cost. The main objective of the model is to determine optimal batch sizes incorporating machine breakdowns and batch splitting and forming, to illustrate the significance of TPM and OPT respectively in improving the effectiveness of the system. A direct pattern search method has been employed for determining the economic production quantities. However, the model developed is based on a number of assumptions and approximations. This implies that there are opportunities for further investigation to enhance the accuracy of the modelling of multi-stage production–inventory systems. The influence of OPT, TPM and set-up cost reduction on lot-sizing policies in a multi-stage production–inventory system has been studied using the model. However, the application of OPT in batching policies, and the importance of total preventive maintenance and set-up cost reduction can be studied in detail further by suitably modifying this model.

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