A Simulation Assessment of Three Methods for Deriving the Long-run Profitability of the Firm as its Internal Rate of Return

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Deriving the Internal Rate of Return from the Accountant's Rate of Return

by

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A Simulation Assesment of Three Methods for Deriving the Long-Run Profitability of the Firm as its Internal Rate of Return

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Introduction

- A central task in accounting theory and practice: to measure the firm’s profitability both in the long-run and in the short-run
- In the economics literature the internal rate of return (IRR) is the widely used theoretical long-run profit-ability concept. IRR is a well-established measure also among practitioners (in capital budgeting decisions)
- Methods to estimate the IRR
  - methods based on the accountant’s rate of return (ARR): Kay’s model, the average ARR method
  - methods based on the cash recovery rate (CRR): Ijiri’s and Salamon’s models
  - methods based on market values (Lawson) etc.

Research Problem and Methodology

- Research problem in general:
  - To develop an objective and operational methodology for assessing the various long-run profitability (IRR) estimation methods presented in literature
  - To use this methodology for finding out which of the methods works best both in practice and in theory
- Evaluation based on simulated financial statements
  - Evaluations using actual financial data from business enterprises suffer from missing an objective profitability benchmark (true IRR unknown)
  - Results based on an analytic deduction are valid only under strict assumptions and have arrived at conflicting conclusions
  - Using simulated data allows one to know the true IRR in advance for providing the objective benchmark needed

Specific Research Questions

1. Are the methods sensitive to cyclical fluctuations in the capital investment activities?
   - earlier research typically based on a constant growth approach
2. Are the methods sensitive to the underlying, assumed cash contribution patterns and duration of the firm's capital investments?
   - sensitivity analysis of the contribution distribution and the life span
3. Are the methods sensitive to disparities between the firm's growth and profitability?
   - verification of analytic results under more general assumptions
4. Are the methods sensitive to the depreciation choice used to produce the financial statements?
   - sensitivity analysis of the firm's accounting practice

Capital Investment Simulation Model

(1) $$g_t = g_0(1+k)^t\{1 + A\sin[(2\pi t / C) + \phi]\}[1 + \delta_{t\tau} S],$$

where

g_t = capital expenditures in year t

k = growth rate of the investments

A = amplitude of the business cycle

C = length of the business cycle

ϕ = phase adjustment for the business cycle

$\delta_{t\tau}$ = Kronecker's delta ($\delta_{t\tau} = 1$, when $t = \tau$, and 0 otherwise

S = capital investment shock coefficient

Some Basic Formulas

The total contribution f_t in year t cumulated from the contributions of the earlier years' investments:

$$(2) \qquad f_t = \sum_{i=1}^{\min(N,t)} f_{ti} = \sum_{i=1}^{\min(N,t)} b_i g_{t-i}$$

f_t = cash inflow in year t
f_{ti} = contribution in year t from capital investment i years back
b_i = relative contribution from capital investment i years back
N = life-span of every capital investment project

The true internal rate of return r:

$$(3) \qquad \sum_{i=1}^{N} b_i (1+r)^{-i} = 1$$

The accountant's profit p_t is defined as the cash inflow f_t less the depreciation d_t:

(4) $$p_t = f_t - d_t$$

The book value v_t of the firm at the end of period t is defined as:

(5) $$v_t = v_{t-1} + g_t - d_t$$

The accountant's rate of return ARR is defined as:

(6) $$ARR = p_t / v_{t-1} = \left(f_t - d_t\right) / v_{t-1}$$

Depreciation Methods

Annuity depreciation (theoretical):

(7) $$d_t = f_t - p_t = f_t - r\ v_{t-1}$$

Straight-line depreciation:

(8) $$d_t = \sum_{i=1}^{\min(N,t)} (1/N)\, g_{t-i}$$

Double declining balance method:

(9) $$d_t = \sum_{i=1}^{\min(N,t)} (2/N)(1-2/N)^{i-1} g_{t-i}$$

Contribution Distributions

The uniform distribution

(10) $$b_i = 1/N,\ i = 1, 2, \ldots, N$$

The negative binomial distribution

(11) $$b_i = s(i+1)p^2(1-p)^i,\ i = 1, 2, \ldots, N$$

s = a scaling parameter
p = a shape parameter

The Anton distribution

(12) $$b_i = 1/N + (N - i + 1)r/N,\ i = 1, 2, \ldots, N$$

Kay's Model

(13) $$IRR = \left[\sum_{t=2}^{n} p_t \left(1 + IRR\right)^{-t}\right] / \left[\sum_{t=2}^{n} v_{t-1} \left(1 + IRR\right)^{-t}\right]$$

where

n = the length of the observation period

Average ARR Method

(14) $$ARR = \left[1 / \left(n - 1\right)\right] \sum_{t=2}^{n} p_t / v_{t-1}$$

Ijiri's Model

Cash recovery rate CRR_t:

(15) $$CRR_t = f_t \,/\, V_{t-1}$$

(16) $$V_t = v_t + \sum_{i=0}^{N/2-1} d_{t-i}$$

where V_t denotes the gross assets at the end of year t.

IRR estimate (by iteration):

(17) $$CRR = IRR \,/ \left[1 - \left(1 + IRR\right)^{-N}\right]$$

Results (Kay's Model)

- The estimates are insensitive to the cyclical fluctuat-ions and their amplitude (Table 2)
- When the growth rate and the true IRR are equal, the profitability estimates contain no error (Table 3)
- When the annuity method of depreciation is used, the estimates are perfectly accurate
- The main source of error in the estimates is the dis-parity between the firm's growth and profitability
- The firm's capital investment opportunities (the cont-ribution distribution) and accounting choice (the depreciation method used) also affect the estimates
- The three sources of error interact with each others

Results (Ijiri's Model)

- Two additional sources of error:
  - the estimate of the cumulated depreciations for the GBV
  - the estimate of the life-span N
- Ijiri's model fares numerically on the average at least as well as does Kay's model (Table 4). However
  - perfectly accurate estimates do not exist (cf. Kay's model in the case k = r and under annuity depreciation)
  - there is no clear pattern (direction / magnitude) in the errors
  - the individual error components can be much larger than the resulting total error due to compensating effect (Table 5)
  - \+ the method is insensitive to cyclical fluctuations

Results (Average ARR Method)

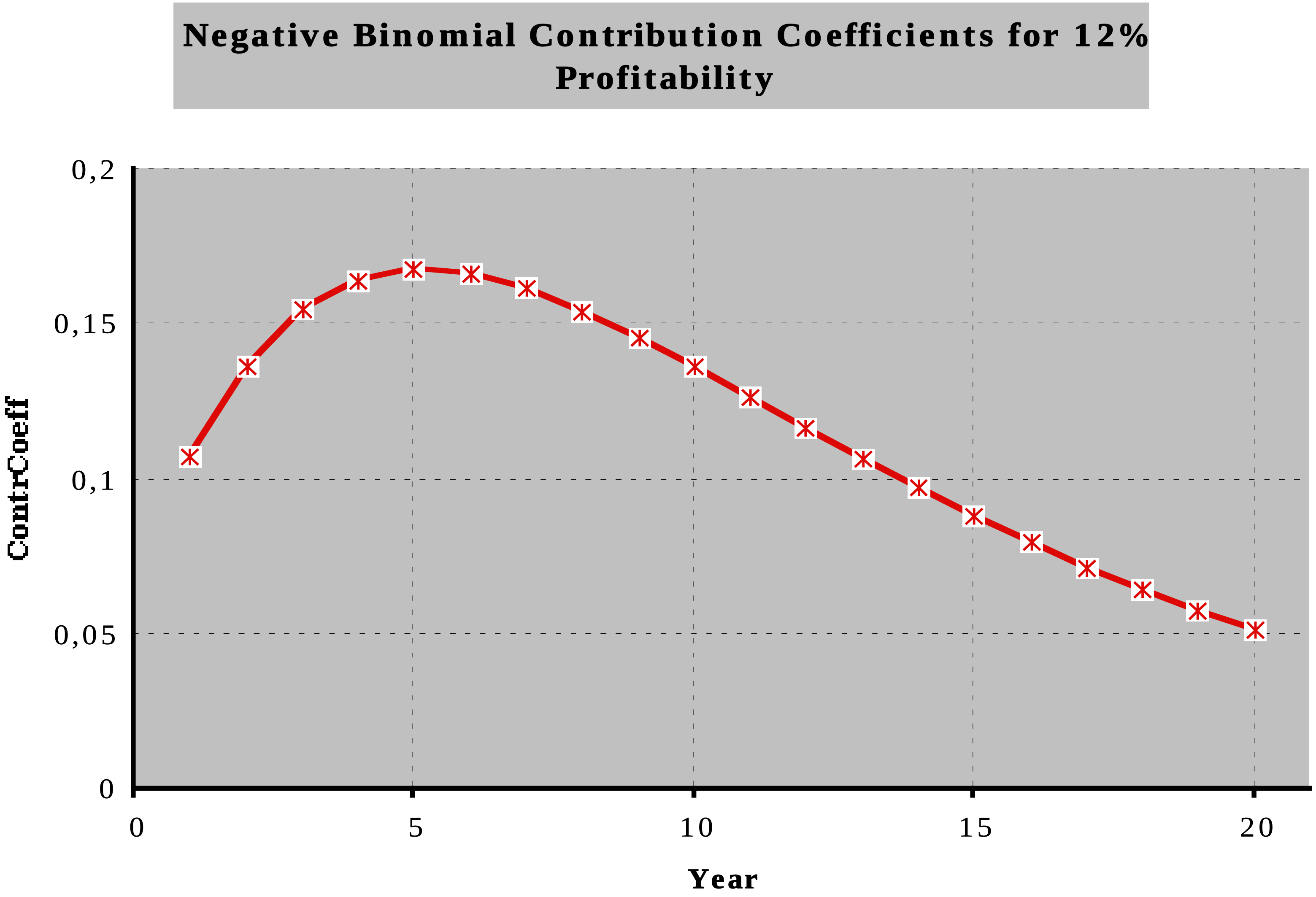
- The results produced by the average ARR method are very similar to the results by Kay's model (Table 6)
  - maximum difference in the estimates of the two methods is 0.1 per cent
  - perfect accuracy in the case k = r does not hold (but holds in the case of annuity depreciation)
  - the method is insensitive to cyclical fluctuations

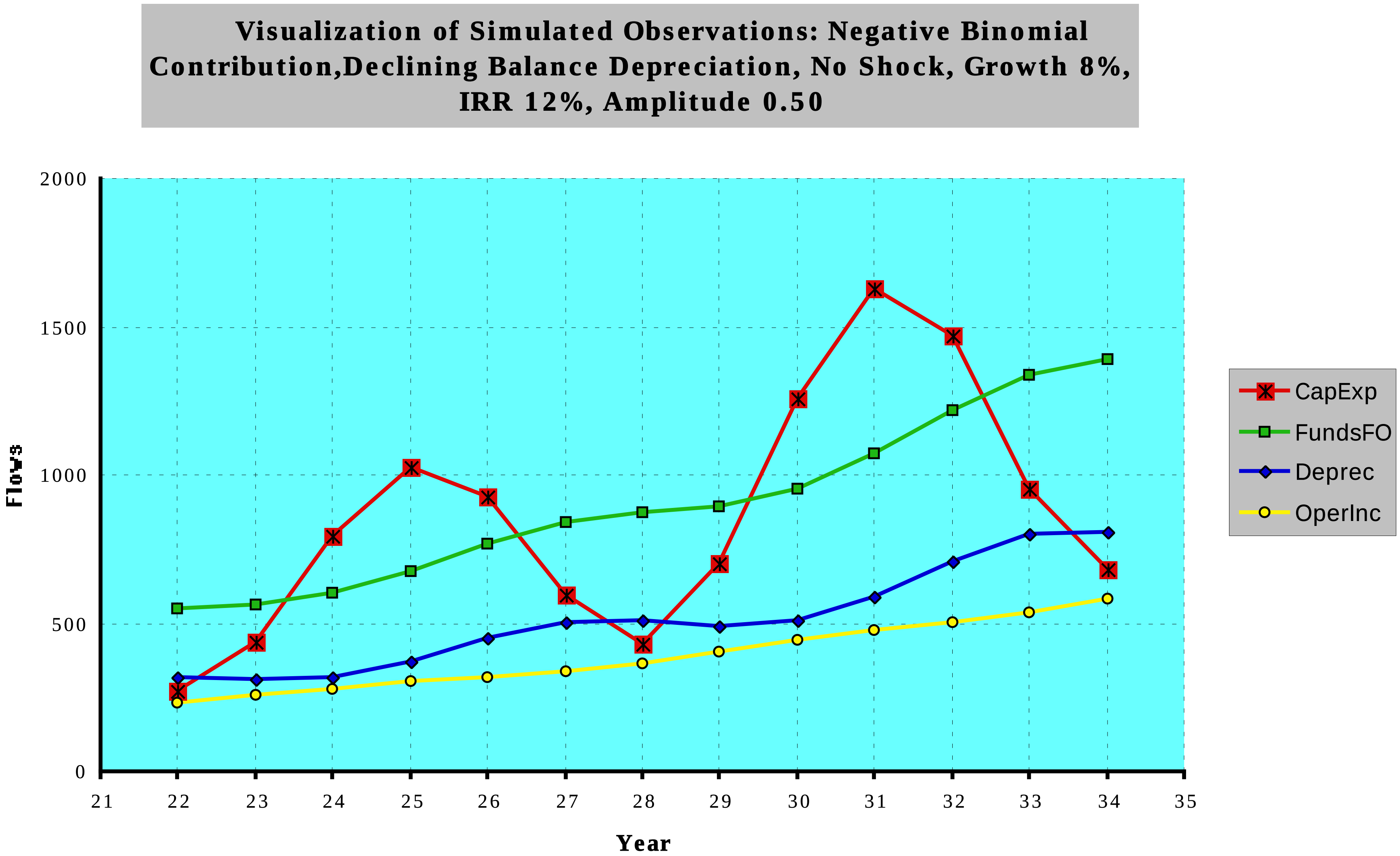
Comparison of Results

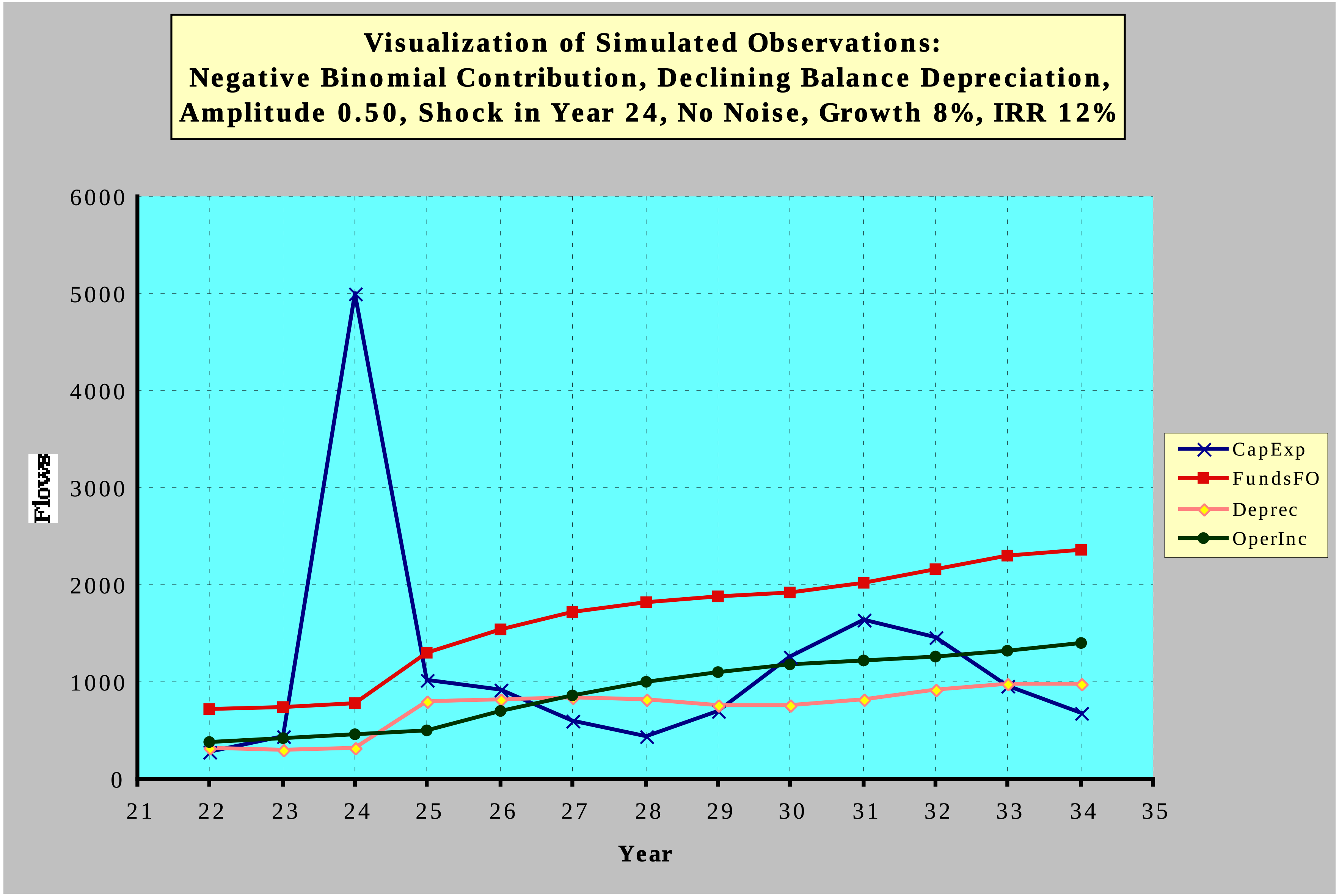
- Numerical performance
  - none of the methods clearly outperforms the others
  - the errors in Kay's and the average ARR method are more regular and predictable than in Ijiri's method (Ijiri's method has more, although compensating, sources of error)
- Theoretical foundations
  - Kay's model is theoretically best founded, with the average ARR method very close by
  - Ijiri's model can be regarded more as a good rule of thumb
- Practical applicability
  - the average ARR method has the outstanding merit of being directly based on an established accounting practice
  - Kay's and Ijiri's methods difficult to "sell" to practitioners

Conclusion

- The simulation approach makes it possible to know the true IRR in advance and provides an objective benchmark for an assessment of the different methods
- Due to the inclusion of the investment cycles into the models the applicability of the models beyond the steady state assumptions is confirmed
- The discrepancy between the the growth and true profitability is the dominating source of error in all the three methods
- The numerical performance of the methods is roughly at par. Kay’s method is theoretically best founded and the average ARR method most easy and straight-forward to apply in practice


Negative Binomial Contribution Coefficients for 12% Profitability
ContrCoeff
0,2
0,15
0,1
0,05
0
0
5
10
15
20
Year


Visualization of Simulated Observations: Negative Binomial Contribution,Declining Balance Depreciation, No Shock, Growth 8%, IRR 12%, Amplitude 0.50
2000
1500
1000
500
0
Flows
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
Year
CapExp
FundsFO
Deprec
OperInc


Visualization of Simulated Observations:
Negative Binomial Contribution, Declining Balance Depreciation,
Amplitude 0.50, Shock in Year 24, No Noise, Growth 8%, IRR 12%
6000
5000
4000
3000
2000
1000
0
Flows
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
Year
CapExp
FundsFO
Deprec
OperInc