# Association between Accounting and Market-Based Variables <br> A Canonical Correlation Approach with U.S. Data 

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## Abstract

- The nature of the association between the firm's accounting and market-based variables is investigated using canonical correlation analysis.
- The data consists of NYSE and AMEX firms for 197693.
- A clear relationship between the firm's accounting and stock-market variables is observed.
- However, the accounting variables making up the relationship vary along time.
- The decomposed analysis of the association suggests that accrual-based financial ratios are crucial for security analysis. Cash-based financial ratios show increasing relevance over time.
- The effect of a reduction of the original set of the accounting variables into six key financial ratios is observed.
- The inclusion of the variance of the stock return into the market-based variable set is found to crucially increase the strength of the association.


## The general research problem

- The question of the existence, strength and stability of the association between the firm's accounting and market-based variables.
- The question of the relative usefulness of alternative accounting variables in explaining and predicting market behavior.
$\Rightarrow$ Which of the two, accrual-based or cashbased accounting variables, contain more relevant information for security analysis.
- The question of a sufficient reduced set of accounting variables for successful security analysis.
- The discussion of whether the market beta is a sufficient measure of securities' riskiness.


## Some earlier studies

- The association between the firm's accounting beta and its security market beta.
$\Rightarrow \quad$ Ball and Brown (1969)
$\Rightarrow \quad$ Beaver et al. (1970)
$\Rightarrow \quad$ Gonedes (1973)
$\Rightarrow \quad$ Bowman (1979)
$\Rightarrow \quad$ Ismail and Kim (1989)
- Correlations between a single financial ratio, or cluster of financial ratios, and a security's return and risk.
$\Rightarrow \quad$ Beaver et al. (1970)
$\Rightarrow \quad$ Pettit and Westerfield (1972)
$\Rightarrow \quad$ Martikainen (1991)
$\Rightarrow \quad$ Kim and Lipka (1991)
- A general correlation between the two sets of variables, i.e. between the set of accounting variables and the set of market-based variables.
$\Rightarrow \quad$ Salmi, Virtanen and Yli-Olli (1997)


## Specific research questions

1) Is there a temporally stable general association (measured by canonical correlations) between the firm's accounting ratios and its stock return and risk?
2) Is the association with stock return and risk stronger for accrual-based or cash-based financial ratios? Does the potentially weaker set yet contain incremental information?
3) Does a reduced set of key financial ratios essentially retain the correlation between the firm's accounting ratios and its stock return and risk?
4) Is the general correlation between the firm's accounting ratios and its stock return and risk strengthened when the risk is measured by alternative risk measures?

## Data description

- The annual accounting data is retrieved from Compustat tapes for all the December firms listed on the NYSE and AMEX.
- The stock returns are obtained from CRSP tapes.
- The research period is from 1976 to 1993 and is divided into three subperiods 1976-1981, 1982-1987, and 1988-1993.
- The accounting variables
$\Rightarrow$ The twelve accrual-based financial ratios are the ones given in Foster (1978).
$\Rightarrow$ The eight cash-based financial ratios follow Gombola and Ketz (1983).
$\Rightarrow$ The values of the financial ratios are calculated separately for each subperiod by averaging the annual values of each individual ratio for each firm


## - The market-based variables

$\Rightarrow$ The five market-based variables are the security's beta, return on the security, total risk (variance), skewness of the return, and kurtosis of the return.
$\Rightarrow$ The variance, kurtosis and skewness of the stock returns are calculated from the time series of $\mathbf{7 2}$ monthly returns for each of the three subperiods. The market betas are estimated by regressing the stock returns on the market return using the market model. The market return is obtained as the return on the SP500 index.

- The sample contains 613 firms in 1976-81, 627 in 1982-87 and 612 in 1988-93.


## Results

## - Empirical association between accounting and market-based variables (research question 1).

## Table 1:

$\Rightarrow$ There is a clear general association between the firms' accounting and market-based variables.
$\Rightarrow$ The two canonical correlations between the variable sets are highly significant for all the three periods.
$\Rightarrow$ The numerical values of the first canonical correlation vary from $\mathbf{0 . 4 3 1}$ to 0.619 , and from 0.293 to 0.499 for the second.

Table 2:
$\Rightarrow$ The structure of the existing association is not stable, the association is generated by different accounting variable combinations in the different periods.
$\Rightarrow$ The contribution of the financial leverage variables, however, is fairly stable (the coefficients of $\mathbf{v}_{\mathbf{1}}$ for the variables $\mathrm{x}_{4}-\mathrm{x}_{6}$ ).

## - Relative influence of accrual-based and cash-based variables in the association (research question 2).

Tables 3 and 4:
$\Rightarrow$ Both the accrual-based and cash-based accounting variables are significantly associated with the market-based variables.
$\Rightarrow$ The accrual-based set outperforms the cash-based set (in the period 1988-93 they are, however, at par).

Table 3 vs. Table 1:
$\Rightarrow$ Incremental information provided by the cash-based variables

* Period 1976-81: relatively small
* Period 1982-87: negligible
* Period 1988-93: crucial


## - Association with the key financial ratios (research question 3).

Table 5:
$\Rightarrow$ Six key ratios in the predictor set

* Liquidity (quick ratio $\mathbf{x}_{2}$ )
* Dynamic liquidity (defensive interval measure $\mathbf{x}_{3}$ )
* Solvency (debt to equity $\mathbf{x}_{4}$ )
* Profitability (return on equity $\mathrm{x}_{9}$ )
* Turnover (total asset turnover $\mathbf{x}_{10}$ )
* Cash-flows (cash-flow to sales $\mathbf{x}_{18}$ )

Table 5 vs. Table 1:
$\Rightarrow$ The reduced set holds a crucial amount of information for security analysis, although a natural decrease in the information content appears

## - Effect of additional risk measures on the association (research question 4).

Table 6 vs. Table 1:
$\Rightarrow$ The inclusion of the variance to the set of the market-based variables increases the association between the accounting variable set and the market-based variable set remarkably; also the emerging third canonical correlation is highly significant for all the three periods.

Table 7 vs. Table 6:
$\Rightarrow$ The inclusion of skewness and kurtosis does not enhance the association between the accounting and market-based variable sets.

## Conclusion

- There exists a general association between the firm's accounting ratios and its stock return and risk, but the association is structurally unstable: the accounting variables making up the relationship vary along time.
- When taken alone, both the accrual-based and the cash-based variables are significantly associated with the market-based variables. The accrual-based variable set has a stronger relationship with the market-based variable set than the cash-based set. With the exception of the period 1988-93 the incremental information of the cash-based set is not notable.
- The reduction of the original set of the accounting variables ( 20 financial ratios) into six key ratios retains a significant association. The resulting drop in strength of the association is about one third.
- The inclusion of the variance of the stock return into the market-based variable set as a measure of the total risk crucially increases the strength of the association. The inclusion of the higher moments (skewness and kurtosis) have no influence on the association.

Table 1. Canonical correlations and their statistical significance:
return and beta vs. all financial ratios

|  | Canonical <br> variable | Canonical <br> correlation | Approxim. F | p-value |
| :--- | :---: | :---: | :---: | :---: |
| Panel A | 1 | 0.438 | 5.430 | 0.0001 |
| 1976-81 | 2 | 0.341 | 4.113 | 0.0001 |
| Panel B | 1 | 0.619 | 14.207 | 0.0001 |
| 1982-87 | 2 | 0.499 | 10.588 | 0.0001 |
| Panel C | 1 | 0.431 | 4.693 | 0.0001 |
| 1988-93 | 2 | 0.293 | 2.923 | 0.0001 |

Table 2. Standardized canonical coefficients

|  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A: Coefficients for the predictor (the accounting) variables1976-81 1982-87 1988-93 |  |  |  |  |  |  |  |
|  | $\mathrm{v}_{1}$ | $\mathrm{V}_{2}$ | $\mathrm{v}_{1}$ | $\mathrm{v}_{2}$ | $\mathrm{v}_{1}$ | $\mathrm{v}_{2}$ |  |
| $\mathrm{x}_{1}$ | -0.063 | 0.295 | -0.128 | 0.547 | 0.191 | -0.678 | current ratio |
| $\mathrm{X}_{2}$ | 0.111 | 0.430 | -0.053 | -0.597 | -0.114 | 0.530 | quick ratio |
| $\mathrm{x}_{3}$ | 0.901 | -0.578 | -0.350 | 0.529 | 0.381 | 0.171 | defensive interval measure |
| $\mathrm{X}_{4}$ | 0.057 | 0.096 | 0.062 | -0.320 | 0.162 | -0.759 | debt to equity |
| $\mathrm{x}_{5}$ | 0.220 | 0.312 | 0.179 | 0.189 | 0.492 | 0.795 | long-term debt to equity |
| $\mathrm{x}_{6}$ | 0.428 | -0.136 | 0.070 | -0.121 | 0.013 | 0.185 | times interest earned |
| $\mathrm{x}_{7}$ | -0.361 | -0.177 | 0.588 | -0.557 | 0.434 | -0.548 | earnings to sales |
| $\mathrm{x}_{8}$ | 0.798 | 0.283 | 0.408 | 0.912 | -0.336 | 1.315 | return on assets |
| $\mathrm{X}_{9}$ | -0.258 | 0.736 | 0.064 | -0.327 | 0.054 | -0.527 | return on equity |
| $\mathrm{x}_{10}$ | 0.009 | 0.493 | -0.018 | 0.311 | -0.186 | 0.562 | total assets turnover |
| $\mathrm{x}_{11}$ | 0.154 | 0.294 | -0.057 | 0.136 | -0.063 | -0.142 | inventory turnover |
| $\mathrm{X}_{12}$ | -0.147 | 0.112 | 0.004 | -0.122 | -0.251 | 0.005 | accounts receivable turnover |
| $\mathrm{X}_{13}$ | 0.336 | -1.208 | -0.115 | 0.072 | 0.281 | 0.520 | cash / current debt |
| $\mathrm{X}_{14}$ | -0.455 | 0.677 | 0.049 | -0.396 | -0.778 | -0.826 | cash / sales |
| $\mathrm{x}_{15}$ | 0.507 | 0.329 | 0.103 | 0.412 | 0.658 | -0.298 | cash / total assets |
| $\mathrm{x}_{16}$ | -0.671 | 0.363 | 0.230 | -0.194 | -0.239 | 0.929 | cash / total debt |
| $\mathrm{X}_{17}$ | 0.216 | -0.572 | 0.074 | -0.048 | -0.159 | 0.054 | cash flow / equity |
| $\mathrm{x}_{18}$ | -0.495 | 1.205 | -0.140 | -0.383 | -1.039 | 0.468 | cash flow / sales |
| $\mathrm{X}_{19}$ | -0.195 | -0.137 | 0.259 | 0.472 | 1.327 | -0.712 | cash flow / total assets |
| $\mathrm{x}_{20}$ | -0.477 | -0.075 | -0.362 | -0.304 | -0.467 | -0.502 | cash flow / total debt |

Panel B: Coefficients for the criterion (the market-based) variables

$$
\begin{array}{lll}
1976-81 & 1982-87 & 1988-93
\end{array}
$$

|  | $\mathrm{W}_{1}$ | $\mathrm{~W}_{2}$ | $\mathrm{~W}_{1}$ | $\mathrm{~W}_{2}$ | $\mathrm{~W}_{1}$ | $\mathrm{~W}_{2}$ |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | :--- |
| $\mathrm{x}_{21}$ | 0.594 | 0.969 | 0.820 | 0.581 | 0.241 | 0.970 | security's beta |
| $\mathrm{X}_{22}$ | 0.569 | -0.984 | -0.492 | 0.876 | 0.972 | -0.232 | return |

Table 3. Canonical correlations and their statistical significance: return and beta vs. accrual-based financial ratios

|  | Canonical <br> variable | Canonical <br> correlation | Approxim. <br> F | p-value |
| :--- | :---: | :---: | :---: | :---: |
| Panel A | 1 | 0.363 | 5.441 | 0.0001 |
| 1976-81 | 2 | 0.252 | 3.689 | 0.0001 |
| Panel B | 1 | 0.604 | 21.609 | 0.0001 |
| 1982-87 | 2 | 0.471 | 15.950 | 0.0001 |
| Panel C | 1 | 0.313 | 3.275 | 0.0001 |
| 1988-93 | 2 | 0.154 | 1.322 | 0.2075 |

Table 4. Canonical correlations and their statistical significance: return and beta vs. cash-based financial ratios

|  | Canonical <br> variable | Canonical <br> correlation | Approxim. <br> F | p-value |
| :--- | :---: | :---: | :---: | :---: |
| Panel A | 1 | 0.212 | 2.734 | 0.0003 |
| 1976-81 | 2 | 0.158 | 2.216 | 0.0314 |
| Panel B | 1 | 0.409 | 13.054 | 0.0001 |
| 1982-87 | 2 | 0.349 | 12.239 | 0.0001 |
| Panel C | 1 | 0.347 | 6.234 | 0.0001 |
| 1988-93 | 2 | 0.175 | 2.734 | 0.0084 |

Table 5. Canonical correlations and their statistical significance: return and beta vs. reduced set of financial ratios.

|  | Canonical <br> variable | Canonical <br> correlation | Approxim. <br> F | p-value |
| :--- | :---: | :---: | :---: | :---: |
| Panel A | 1 | 0.324 | 8.263 | 0.0001 |
| 1976-81 | 2 | 0.214 | 5.821 | 0.0001 |
| Panel B | 1 | 0.445 | 17.758 | 0.0001 |
| 1982-87 | 2 | 0.304 | 12.637 | 0.0001 |
| Panel C | 1 | 0.285 | 4.771 | 0.0001 |
| 1988-93 | 2 | 0.088 | 0.937 | 0.4561 |

Table 6. Canonical correlations and their statistical significance:
return, beta and variance vs. all financial ratios

|  | Canonical <br> variable | Canonical <br> correlation | Approxim. <br> F | p-value |
| :--- | :---: | :---: | :---: | :---: |
| Panel A | 1 | 0.618 | 8.630 | 0.0001 |
| $1976-81$ | 2 | 0.422 | 4.806 | 0.0001 |
|  | 3 | 0.295 | 3.147 | 0.0001 |
| Panel B | 1 | 0.782 | 17.727 | 0.0001 |
| $1982-87$ | 2 | 0.450 | 7.759 | 0.0001 |
|  | 3 | 0.372 | 5.398 | 0.0001 |
| Panel C | 1 | 0.600 | 8.113 | 0.0001 |
| $1988-93$ | 2 | 0.423 | 4.707 | 0.0001 |
|  | 3 | 0.286 | 2.928 | 0.0001 |

Table 7. Canonical correlations and their statistical significance: return, beta, variance, skewness and kurtosis vs. all financial ratios.

|  | Canonical <br> variable | Canonical <br> correlation | Approxim. <br> F | p-value |
| :--- | :---: | :---: | :---: | :---: |
| Panel A | 1 | 0.620 | 5.608 | 0.0001 |
| $1976-81$ | 2 | 0.435 | 3.138 | 0.0001 |
|  | 3 | 0.296 | 1.978 | 0.0001 |
|  | 4 | 0.216 | 1.490 | 0.0357 |
| Panel B | 5 | 0.188 | 1.357 | 0.1575 |
| 1982-87 | 1 | 0.783 | 10.189 | 0.0001 |
|  | 2 | 0.501 | 4.261 | 0.0001 |
|  | 3 | 0.375 | 2.485 | 0.0001 |
| Panel C | 4 | 0.213 | 1.126 | 0.2845 |
| 1988-93 | 5 | 0.126 | 0.609 | 0.8781 |
|  | 1 | 0.613 | 5.473 | 0.0001 |
|  | 2 | 0.431 | 3.100 | 0.0001 |
|  | 3 | 0.309 | 1.975 | 0.0001 |
|  | 4 | 0.196 | 1.335 | 0.0961 |
|  | 5 | 0.189 | 1.362 | 0.1549 |

## APPENDIX A.

## A Brief Review of the Canonical Correlation Analysis

Canonical correlation analysis is a more general case of the usual multiple regression analysis. In multiple regression the aim is to find a linear combination of the independent (or predictor) variables such that the composite has the maximum correlation with the dependent (or criterion) variable. In canonical correlation the interest centers on the linear association between one battery of variables, the predictor variables $x_{1}, x_{2}, \ldots, x_{p}$ and another battery of variables, the criterion variables $\mathrm{y}_{1}, \mathrm{y}_{2}, \ldots, \mathrm{y}_{\mathrm{q}}$.

The pairwise correlations within and between the $\mathrm{x}_{\mathrm{i}}$ and the $\mathrm{y}_{\mathrm{j}}$ variable sets can be presented as matrix

$$
\begin{equation*}
\mathbf{R}=\binom{\mathbf{R}_{\mathbf{y y}} \mathbf{R}_{\mathbf{y x}}}{\mathbf{R}_{\mathbf{x y}} \mathbf{R}_{\mathbf{x x}}} \tag{1}
\end{equation*}
$$

The $x$ and $y$ variables can be assumed to have been routinely standardized to a zero mean and a unit standard deviation.

The objective in canonical correlation analysis is to find a linear composite of the $\mathrm{x}_{\mathrm{i}}$-variables, $i=1,2, \ldots, p$, and a (different) linear composite of the $y_{j}$-variables, $j=1,2, \ldots, q$, such that when this pair of derived variables (linear composites) is correlated, the resulting bivariate correlation is the highest attainable. The two linear composites are

$$
\begin{equation*}
v=\sum_{i=1}^{p} a_{i} x_{i} \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
w=\sum_{j=1}^{q} b_{j} \mathbf{y}_{j} \tag{3}
\end{equation*}
$$

where the canonical coefficients $a_{i}$ and $b_{j}$ are adjusted to make the $v$ and $w$ variables standardized as well. To solve the canonical correlation the ordinary bivariate correlation between the composites v and w

$$
\begin{equation*}
\mathbf{R}_{C}=\frac{1}{m-1} \sum_{k=1}^{m} \mathbf{v}_{k} \mathbf{w}_{k} \tag{4}
\end{equation*}
$$

is maximized. In Formula (4) m is the number of observations, and $\mathrm{v}_{\mathrm{k}}$ and $\mathrm{w}_{\mathrm{k}}$ are the observed values for the v and w variables.

Having done this, it is (generally) possible to find a second pair of linear composites, chosen to be uncorrelated with the first pair, such that the correlation between this second pair of derived variables is, conditionally for the first pair, maximal. In general, with p predictors and q criteria we can obtain $\mathrm{r}=\min (\mathrm{p}, \mathrm{q})$ different pairs of linear composites. The correlations between successive pairs will, in general, decline in size.

APPENDIX B: Definitions of the variables using Compustat codes.

| Symbol | Variable | Compustat Definition |
| :---: | :--- | :--- |
| $\mathrm{x}_{1}$ | current ratio | $(4) /(5)$ |
| $\mathrm{x}_{2}$ | quick ratio | $[(1)+(2)] /(5)$ |
| $\mathrm{x}_{3}$ | defensive interval measure | $[(4)-(3)] /$ |
| $\mathrm{x}_{4}$ | debt to equity | $(181) /(216)$ |
| $\mathrm{x}_{5}$ | long-term debt to equity | $(9) /(216)$ |
| $\mathrm{x}_{6}$ | times interest earned | $(13) /(15)$ |
| $\mathrm{x}_{7}$ | earnings to sales | $(172) /(12)$ |
| $\mathrm{x}_{8}$ | return on assets | $[(18)+(15)] /[(6)]$ |
| $\mathrm{x}_{9}$ | return on equity | $(18) /[(216)+(50)]$ |
| $\mathrm{x}_{10}$ | total assets turnover | $(12) /(6)$ |
| $\mathrm{x}_{11}$ | inventory turnover | $(12) /(3)$ |
| $\mathrm{x}_{12}$ | accounts receivable turnover | $(12) /(2)$ |
| $\mathrm{x}_{13}$ | cash / current debt | $(1) /(5)$ |
| $\mathrm{x}_{14}$ | cash / sales | $(1) /(12)$ |
| $\mathrm{x}_{15}$ | cash / total assets | $(1) /(6)$ |
| $\mathrm{x}_{16}$ | cash / total debt | $(1) /(181)$ |
| $\mathrm{x}_{17}$ | cash flow / equity | $[(13)-\{\mathrm{d}[(4)-(1)]-\mathrm{d}(5)\}] /(216)$ |
| $\mathrm{x}_{18}$ | cash flow / sales | $[(13)-\{\mathrm{d}[(4)-(1)]-\mathrm{d}(5)\}] /(12)$ |
| $\mathrm{x}_{19}$ | cash flow / total assets | $[(13)-\{\mathrm{d}[(4)-(1)]-\mathrm{d}(5)\}] /(6)$ |
| $\mathrm{x}_{20}$ | cash flow / total debt | $[(13)-\{\mathrm{d}[(4)-(1)]-\mathrm{d}(5)\}] /(181)$ |
| $\mathrm{x}_{21}$ | security's beta |  |
| $\mathrm{x}_{22}$ | return on the security |  |
| $\mathrm{x}_{23}$ | security's total risk (variance $)$ |  |
| $\mathrm{x}_{24}$ | skewness of the return |  |
| $\mathrm{x}_{25}$ | kurtosis of the return |  |

APPENDIX C: The Basic Statistics of the Variables

|  | 1976-81 |  | 1982-87 |  | 1988-93 |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | mean | std | mean | std | mean | std |
| $\mathbf{x}_{1}$ | $\mathbf{2 . 2 0 2}$ | $\mathbf{0 . 8 9 1}$ | $\mathbf{1 . 9 4 9}$ | $\mathbf{0 . 9 7 1}$ | $\mathbf{1 . 8 5 4}$ | $\mathbf{1 . 1 7 5}$ |
| $\mathbf{x}_{2}$ | $\mathbf{1 . 1 7 7}$ | $\mathbf{0 . 4 7 2}$ | $\mathbf{1 . 1 2 5}$ | $\mathbf{0 . 6 3 3}$ | $\mathbf{1 . 1 1 2}$ | $\mathbf{0 . 9 0 9}$ |
| $\mathbf{x}_{3}$ | $\mathbf{0 . 2 7 7}$ | $\mathbf{0 . 2 2 3}$ | $\mathbf{0 . 2 9 4}$ | $\mathbf{0 . 1 7 5}$ | $\mathbf{0 . 3 1 2}$ | $\mathbf{0 . 2 4 0}$ |
| $\mathbf{x}_{4}$ | $\mathbf{1 . 3 9 6}$ | $\mathbf{1 . 9 3 0}$ | $\mathbf{1 . 5 6 2}$ | $\mathbf{2 . 0 1 7}$ | $\mathbf{1 . 7 8 6}$ | $\mathbf{1 . 3 7 1}$ |
| $\mathbf{x}_{5}$ | $\mathbf{0 . 5 6 2}$ | $\mathbf{0 . 6 3 7}$ | $\mathbf{0 . 6 5 3}$ | $\mathbf{0 . 9 0 3}$ | $\mathbf{0 . 7 6 0}$ | $\mathbf{0 . 7 5 0}$ |
| $\mathbf{x}_{6}$ | $\mathbf{1 5 . 3 4 1}$ | $\mathbf{2 8 . 0 3 9}$ | $\mathbf{1 1 . 6 2 7}$ | $\mathbf{3 1 . 0 7 7}$ | $\mathbf{1 1 . 7 8 3}$ | $\mathbf{3 0 . 3 2 5}$ |
| $\mathbf{x}_{7}$ | $\mathbf{0 . 0 5 3}$ | $\mathbf{0 . 0 5 9}$ | $\mathbf{0 . 0 4 6}$ | $\mathbf{0 . 0 6 9}$ | $\mathbf{0 . 0 4 1}$ | $\mathbf{0 . 0 9 4}$ |
| $\mathbf{x}_{8}$ | $\mathbf{0 . 0 8 6}$ | $\mathbf{0 . 0 3 1}$ | $\mathbf{0 . 0 7 0}$ | $\mathbf{0 . 0 4 0}$ | $\mathbf{0 . 0 6 5}$ | $\mathbf{0 . 0 4 7}$ |
| $\mathbf{x}_{9}$ | $\mathbf{0 . 1 1 3}$ | $\mathbf{0 . 1 4 9}$ | $\mathbf{0 . 0 5 9}$ | $\mathbf{0 . 2 7 3}$ | $\mathbf{0 . 0 7 1}$ | $\mathbf{0 . 1 8 3}$ |
| $\mathbf{x}_{10}$ | $\mathbf{1 . 4 1 6}$ | $\mathbf{0 . 7 4 0}$ | $\mathbf{1 . 1 3 6}$ | $\mathbf{0 . 6 5 8}$ | $\mathbf{1 . 0 8 8}$ | $\mathbf{0 . 7 1 5}$ |
| $\mathbf{x}_{11}$ | $\mathbf{1 1 . 6 3 3}$ | $\mathbf{1 6 . 0 1 8}$ | $\mathbf{1 5 . 1 5 2}$ | $\mathbf{2 4 . 9 3 0}$ | $\mathbf{1 8 . 9 2 5}$ | $\mathbf{3 2 . 8 4 8}$ |
| $\mathbf{x}_{12}$ | $\mathbf{1 0 . 8 2 3}$ | $\mathbf{2 7 . 3 7 8}$ | $\mathbf{9 . 3 4 2}$ | $\mathbf{1 7 . 0 1 2}$ | $\mathbf{1 1 . 5 0 3}$ | $\mathbf{4 1 . 1 8 7}$ |
| $\mathbf{x}_{13}$ | $\mathbf{0 . 3 3 3}$ | $\mathbf{0 . 2 8 8}$ | $\mathbf{0 . 3 7 2}$ | $\mathbf{0 . 4 6 3}$ | $\mathbf{0 . 3 8 0}$ | $\mathbf{0 . 7 2 8}$ |
| $\mathbf{x}_{14}$ | $\mathbf{0 . 0 6 4}$ | $\mathbf{0 . 0 8 0}$ | $\mathbf{0 . 0 8 3}$ | $\mathbf{0 . 1 2 7}$ | $\mathbf{0 . 0 8 6}$ | $\mathbf{0 . 1 6 5}$ |
| $\mathbf{x}_{15}$ | $\mathbf{0 . 0 7 2}$ | $\mathbf{0 . 0 5 7}$ | $\mathbf{0 . 0 7 1}$ | $\mathbf{0 . 0 7 4}$ | $\mathbf{0 . 0 6 6}$ | $\mathbf{0 . 0 8 0}$ |
| $\mathbf{x}_{16}$ | $\mathbf{0 . 1 6 6}$ | $\mathbf{0 . 1 6 9}$ | $\mathbf{0 . 1 7 1}$ | $\mathbf{0 . 2 7 5}$ | $\mathbf{0 . 1 6 3}$ | $\mathbf{0 . 3 2 2}$ |
| $\mathbf{x}_{17}$ | $\mathbf{0 . 3 2 9}$ | $\mathbf{0 . 2 9 0}$ | $\mathbf{0 . 3 1 7}$ | $\mathbf{0 . 2 1 0}$ | $\mathbf{0 . 3 4 6}$ | $\mathbf{0 . 2 3 8}$ |
| $\mathbf{x}_{18}$ | $\mathbf{0 . 1 3 5}$ | $\mathbf{0 . 1 0 2}$ | $\mathbf{0 . 1 6 5}$ | $\mathbf{0 . 1 2 4}$ | $\mathbf{0 . 1 6 5}$ | $\mathbf{0 . 1 2 5}$ |
| $\mathbf{x}_{19}$ | $\mathbf{0 . 1 5 0}$ | $\mathbf{0 . 0 6 0}$ | $\mathbf{0 . 1 3 7}$ | $\mathbf{0 . 0 6 0}$ | $\mathbf{0 . 1 3 0}$ | $\mathbf{0 . 0 6 4}$ |
| $\mathbf{x}_{20}$ | $\mathbf{0 . 3 3 3}$ | $\mathbf{0 . 2 0 3}$ | $\mathbf{0 . 2 8 7}$ | $\mathbf{0 . 1 8 8}$ | $\mathbf{0 . 2 5 6}$ | $\mathbf{0 . 1 9 6}$ |
| mean $^{\text {beta }}$ | $\mathbf{0 . 0 2 0}$ | $\mathbf{0 . 0 1 2}$ | $\mathbf{0 . 0 1 5}$ | $\mathbf{0 . 0 0 9}$ | $\mathbf{0 . 0 1 5}$ | $\mathbf{0 . 0 1 5}$ |
| beta | $\mathbf{0 . 4 5 1}$ | $\mathbf{0 . 9 8 3}$ | $\mathbf{0 . 3 8 8}$ | $\mathbf{0 . 8 4 7}$ | $\mathbf{0 . 4 6 4}$ |  |
| vari | $\mathbf{0 . 0 1 1}$ | $\mathbf{0 . 0 0 7}$ | $\mathbf{0 . 0 1 0}$ | $\mathbf{0 . 0 0 7}$ | $\mathbf{0 . 0 1 0}$ | $\mathbf{0 . 0 1 8}$ |
| skew | $\mathbf{0 . 1 8 2}$ | $\mathbf{0 . 5 1 1}$ | $\mathbf{- 0 . 2 5 7}$ | $\mathbf{0 . 6 3 8}$ | $\mathbf{0 . 0 8 2}$ | $\mathbf{0 . 6 2 6}$ |
| kurt | $\mathbf{1 . 1 6 6}$ | $\mathbf{1 . 8 1 4}$ | $\mathbf{1 . 9 5 4}$ | $\mathbf{2 . 4 7 0}$ | $\mathbf{1 . 1 8 4}$ | $\mathbf{2 . 1 8 9}$ |
|  |  |  |  |  |  |  |

