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Abstract: Past research into the evolution of Finnish stock returns focuses on modeling linear and nonlinear dependence using various ARIMA and GARCH formulations, respectively. This paper extends the extant work by using Grassberger-Procaccia correlation dimensions to explore the nature of the nonlinear dynamics in daily Finnish stock returns during the 1970s and 1980s. Nonlinear behavior in both periods is evident. A simple GARCH model removes the nonlinearity in the first decade and dramatically reduces the nonlinearity in the second period. This supports the notion that Finnish stock returns exhibit nonlinear dependence but that the form of dependence is not chaotic.

Keywords: Finance; Stock returns; Nonlinear dependence; Chaos; GARCH

1. Introduction

Previous studies investigating the evolution of Finnish stock returns have documented the stylized facts that these returns are linearly and nonlinearly dependent and their unconditional distribution is thicker tailed than a normal distribution. Martikainen, Yli-Olli, and Gunasekaran (1991) review these papers. Booth, Hatem et al. (1992) suggest that Finnish stock returns can be modeled by a generalized autoregressive conditional heteroscedastic (GARCH) model with the mean of the conditional distribution being specified as an autoregressive process. Their modeling, however, does not explore the possibility that the observed nonlinearity may be the result of Finnish stock returns being generated by some type of a chaotic process.

The attraction of chaos as a model of stock returns is that, in addition to explaining the aforementioned stylized facts, it is capable of describing abrupt changes. Moreover, as opposed to a martingale or instantaneous martingale process (Lehmann, 1990), returns may be predicted over short but not long time intervals. That a chaotic process is an explanation for the behavior of stock returns has been explored for different US stock indexes, portfolios, and a small number of individual stocks and for a German stock index. The empirical evidence supporting the con-

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tention that these return series are chaotic is mixed. For example, Scheinkman and LeBaron (1989), Brock and Malliaris (1989), Gennotte and Marsh (1986) report evidence that chaos is present in US stock returns, while Ramsey, Sayers and Rothman (1990) and Hsieh (1991) dispute this claim. Booth, Chowdhury et al. (in press) reject chaos as a general model of German stock returns.

The purpose of this paper is to provide additional evidence from the Finnish stock market concerning whether returns are generated by a chaotic process. The use of Finnish data provides a different perspective because the Finnish stock market is significantly smaller in scale and scope than the US and German stock markets. In addition, each of the three markets has a different market architecture. Thus the inclusion of the Finnish market provides evidence on the role of institutional structure in the return generating process. Moreover, given the importance of increased globalization in the securities markets, evidence from small markets, such as those in Finland, are of potential interest, for example, to those considering developing financial products for these markets.

The plan of the paper is as follows. Section 2 provides a brief discussion of the chaos methodology used. The next section describes the data. Section 4 presents the empirical results, and conclusions are offered in the final section.

2. Research method

Several papers have addressed the potential importance of chaos in economic and financial modeling and the corresponding statistical tests that may be used to detect its presence. Baumol and Benhabib (1989), Brock and Malliaris (1989), and Rosser (1991) provide useful insights into the role of chaos in economics. Key references concerning various statistical tests include Grassberger and Procaccia (1983), Brock (1986), Brock, Hsieh and LeBaron (1991), and Hsieh (1991). Since the exposition of these papers is comprehensive, only a brief discussion of their contents is presented below.

A chaotic process is a nonlinear, deterministic process that has first and second moment properties identical to those of a stochastic series. A special case occurs if these properties also indicate white noise. If this is the case and if the time series is analyzed using conventional time series techniques, the chaotic series may be mistakenly identified as a stochastic white noise process. A common example of such a chaotic series is the tent map. In particular,

$$x_{t+1} = \begin{cases} 2x_t, & x_t \le 0.5, \\ 2 - 2x_t, & x_t \ge 0.5, \end{cases} \tag{1}$$

where x_0 is a number between zero and one. The spectrum and autocovariance function of this nonlinear model are identical to that of an i.i.d., uniform, [0,1] random variable. Other examples include pseudo random number generators, certain logistic functions, the Hénon map, the Mackey–Glass equation, the quasi-periodic five-torus process and the Lorenz map. The above examples encompass univariate and multivariate as well as difference and delayed differential equation systems.

A chaotic time series has two characteristics. First, as the time series evolves, no point is ever reached twice, although in actual data this may not be strictly true because of rounding. Second, the evolutionary pattern of the time series is extremely sensitive to the system's initial conditions. Thus, if at the time a forecast is made the information set contains errors or is imprecise, the forecast based on this information is inaccurate, with the level of inaccuracy growing exponentially as the time period covered by the forecast increases.

The key concept in determining whether a process is chaotic is the correlation dimension, a statistic which helps obtain topological information about the underlying system generating the data without knowing (or assuming) a particular structural model. For truly random data, the correlation dimension monotonically increases with the dimension of the space within which these data are contained. This latter dimension is called the embedding dimension. In contrast, for chaotic data the correlation dimension remains small even when the embedding dimension increases. In other words, the correlation dimension measure indicates the 'nonlinear degrees of freedom' or complexity of the time series. Its value, however, is not related in any way to the number of variables (factors) that describe the system. That is, a univariate process, for example, may have high (but finite) or low correlation dimension values.

The concept of correlation dimension is based on the correlation integral. Let the ordered series $\{r_t, t=1,\ldots,T\}$ represent a linearly independent time series containing T observations. Define M (m) to be the embedding dimension such that M is the symbol for the M-th embedding dimension and m is the value of M used in mathematical operations. It is then possible to create M-dimensional vectors, labeled M-histories, such that

$$r_t^M = \{r_t, r_{t+1}, \ldots, r_{t+m-1}\}.$$

Thus, a single time series is converted to a set of time series with overlapping entries. The dynamics of the original time series are depicted by the M-histories if $m \geq 2n+1$, where n is the time series' true dimensionality and $T \to \infty$. The correlation integral measures the spatial correlation among the points in the M-histories for a specific embedding dimension and it is defined as

$$C^M(\varepsilon, T) = 2[(T-m+1)(T-m)]^{-1} \times \sum_{1 \leq i < j \leq (T-m+1)} I_\varepsilon\left(r_i^M - r_j^M\right), \quad (2)$$

where I_ε is a Heaviside step function that equals one if $\left\| r_i^M - r_j^M \right\| < \varepsilon$ and zero otherwise. $\| \cdot \|$ is a measure of the distance between r_i^M and r_j^M, and the distance measure employed herein is the *sup-norm*. Thus, the correlation integral measures the fraction of the pairs of all points that are within a distance ε of each other.

For an infinite series, the correlation dimension is specified as

$$D^M(T) = \lim_{\epsilon \to 0} \{\ln C^M(\varepsilon, T)/\ln \varepsilon\}. \quad (3)$$

The choice of values for ε and M are interrelated and is influenced by the length of the series. For a finite series and a given M, it is necessary to find an ε that is neither too large nor too small. If it is too large, $C^M(\varepsilon, T) = 1$ and no information is gained. A similar problem occurs if ε is too small so that $C^M(\varepsilon, T) = 0$. Operationally, the procedure involves selecting various values of ε so that neither of these two extremes occur. In practice, $0.5\sigma \leq \varepsilon \leq 1.5\sigma$, where σ is the original series' standard deviation, usually provides appropriate values. Further, since the estimate of the series' true dimensionality is constrained by the size of M, M should be as large as possible for a given ε. In addition, since r is an observable realization of the true state variable, the larger the M, the more closely the dynamical behavior of the M-histories of r will mimic the law of motion of the true state variable. Once the set of ε and M are selected, for a given M the convention is to regress various $\ln C^M(\varepsilon, T)$ on their corresponding $\ln \varepsilon$ and the resulting slope is the estimate of $D^M(T)$. In particular,

$$\ln C^M(\varepsilon, T) = \hat{a} + \hat{d} \ln \varepsilon + e, \quad (4)$$

where $\hat{d}$ is the estimate of $D^M(T)$. Ramsey and Yaun (1989) report that the value of $\hat{d}$ is biased downward for small sample sizes. In chaos terminology, where physical science studies involve 100 000 or more observations, a 'long' economic series of 2 500 observations (the equivalent of 10 years of daily data) is considered extremely small. This means that there is a bias toward concluding that chaos is present when it may not be.

By itself, the correlation dimension may be used in one of two ways. First, as Scheinkman and LeBaron (1989) and Frank and Stengos (1989) suggest, it may be compared to the $D^M(T)$ of the shuffled (randomized) original series (shuffling maintains the existing distribution but presumably destroys any existing time series structure). Comparison to a Gaussian series of approximately the same length has also been suggested. Simulation experiments, however, indicate that random series with thick-tailed (low entrophy) distributions (e.g., Student-t and power exponential) are often characterized by a relatively lower $D^M(T)$ than for a Gaussian series. See Ramsey and Yaun (1989) and Booth, Chowdhury et al. (in press). In any case, if nonlinearities exist, the $D^M(T)$ for the shuffled or Gaussian series should be noticeably higher than the $D^M(T)$ of the original series. In fact, the $D^M(T)$ for a truly random series is infinite. It is worthwhile to note that the shuffled series are not really random but are very high dimension chaotic series. This is because the shuffling, or randomization, is accomplished using a pseudo random number generator. Thus the $D^M(T)$ of the shuffled series are finite. Nevertheless, for analytic purposes, if the original series cannot be distinguished from a pseudo random series, little is to be gained by not considering it to be truly random. Thus the null hypothesis is that the original series is random.

Second, the original series' $D^M(T)$ may be compared to the $D^M(T)$ of the residuals of a transformed linear or smooth nonlinear series. Brock (1986) shows that if a series is generated by deterministic chaos, the residuals of such a transformation have the same $D^M(T)$ as the original series. Therefore, if the $D^M(T)$ of the transformed series is greater than the $D^M(T)$ of the original series, the original series is not chaotic. The equality of the two $D^M(T)$ indicates that chaos may or may not be present, since there may still be a transformation for which the $D^M(T)$ is greater than that of the original series.

3. Data description

The data series used to investigate Finnish stock behavior is the Helsinki Stock Index (HSI) (Berglund, Wahlroos and Grandell, 1983). The HSI is a value-weighted index consisting of 134 stocks. The index is adjusted for stock splits and dividends (stock or cash) of its component stocks. The adjustment assumes that cash dividends are reinvested without incurring any transactions costs. The index is a daily series and usually transaction prices are used in its construction. If more than one transaction occurs during the day, the mean price of the transactions is used. If a transaction does not occur, the bid price is used. Since the HSI is reported in natural logarithms, first differencing the series results in continuously compounded returns, which is the variable of interest.

The data period begins on the first trading day in January 1970 and ends on the last trading day in December 1989. For analytical purposes, this 20 year period is split into two consecutive 10 year periods. Although the timing of this split is arbitrary and by nature the split is dichotomous, this division does recognize that the financial market in the two periods was, on average, noticeably different. The 1970s witnessed high inflation rates and regulated low interest rates. Moreover, the private sector was not subject to significant international competitive pressures, and the financial markets were heavily dominated by banks. Nineteen hundred and eighty marked the beginning of liberalization of financial markets for many countries, including Finland. As documented by Malkamäki and Solttila (1990) volume increased more than ten-fold during the 1980s and the influence of banks on the financial markets, although remaining important, lessened considerably. Hietala (1989) suggests that this increased activity was due in part to a significant increase in Finnish stock purchases by foreign investors and to Finnish investors becoming more active in their own financial market, either because of additional liberalization of investment restrictions or because Finnish investors caught up with the behavior of their foreign counterparts. The decade ended with the establishment of the Securities Market Act, which provided the infrastructure necessary to meet international stock exchange standards. Thus, the 1980s, in contrast to the 1970s, may be thought of as a period in which the Finnish stock market matured into a stock market on the verge of becoming fully developed. Indeed, subsequently the Helsinki Stock Exchange established the HETI system for trading stocks. This is an automated trading and information mechanism that permits a continuous market (see Malkamäki, 1989).

The returns series for the 1970s contains 2345 daily returns, and the 1980s return series is comprised of 2443 observations. Hereafter the return series are labeled HSI-70 and HSI-80, respectively. This econometrically large sample has implications for the statistical tests that are subsequently applied. In particular, the 1-percent level is used to determine statistical significance. The selection of this value recognizes that statistical tests used on large samples reject the null hypothesis more often than they should. For discussion on this point in the context of financial research, see Connolly (1989).

Descriptive statistics for both these series are reported in the first two columns of Table 1. Both series are characterized by statistically significant positive means, skewness (positive for HSI-70 and negative for HSI-80), thick-tails, and linear dependence. Interestingly, the linear dependence and thick-tailedness of HSI-80 seem less than HSI-70. To see whether this observation is caused by the October 1987 worldwide stock market crash and the October 1989 mini-crash, the two returns surrounding each crash were removed from HSI-80. The descriptive statistics for this modified series, labeled HSI-80M are presented in the last

Table 1
Empirical characteristics of HSI returns

	HSI-70 $n = 2345$	HSI-80 $n = 2443$	HSI-80M $n = 2439$
Location and shape:			
Mean (10^2)	0.054 *	0.089 *	0.091 * [a]
Standard deviation (10^2)	0.565	0.681	0.560
Skewness	0.730 *	−3.052 *	−0.940 *
Kurtosis	13.458 *	93.094 *	10.829 *
Kolmogorov–Smirnov D	0.085 *	0.134 *	0.099 *
Linear dependence:			
Autocorrelation: Lag (1)	0.351 *	0.195 *	0.464 *
Lag (2)	0.243 *	0.103 *	0.232 *
Ljung–Box Q (24)	504.90 *	201.90 *	747.62 *
Bartlett's Kolmogorov–Smirnov statistic	0.259 *	0.147 *	0.320 *

[a] * indicates significance at least at the 1-percent level for all statistics except the standard deviation and autocorrelations. For the latter, * denotes that the estimated autocorrelation is at least three times its standard error. The null value is zero for the mean, skewness, and kurtosis. Two tests are used to investigate dependence. The Ljung–Box Q examines dependency that is revealed by the autocorrelation function. In contrast, Bartlett's Kolmogorov–Smirnov statistic measures dependency by considering departures from white noise over all frequencies. HSI-80M series is HSI-80 less the returns on 20 and 21 October 1987 and 17 and 18 October 1989.

column of Table 1. The impact of adjusting for the two crashes is dramatic. HSI-80M is much more similar to HSI-70 than HSI-80 with respect to thick-tailedness and linear dependency. Berglund and Liljeblom (1988) attribute the observed low order linear dependence mainly to the trading mechanism ('calling out') system employed by the Helsinki Stock Exchange. In any case, the HSI-80M mean is higher than the HSI-80 mean, but the skewness, although still significantly negative is much less.

Prior to estimating the correlation dimension, it is necessary to remove the linear dependency from the three return series. The residuals of these models are distinguished from their respective original series by a 'w'. For example, the AR(2) residuals of HSI-70 is labeled HSI-70w. Descriptive statistics for HSI-70w, HSI-80w, and HSI-80Mw are displayed in Table 2. A review of this table indicates that these return series are linearly independent and, with the exception of the mean that is zero by construction, the shape characteristics are similar to their linearly non-whitened counterparts.

4. Empirical results

The correlation dimension estimates for HSI-70w, HSI-80w, and HSI-80Mw are presented in Table 3. The $D^M(T)$-values are calculated using the procedure described in Section 2, using values for ε ranging from 0.5σ to 1.5σ in increments of 0.02σ. For each of the three series, the $D^M(T)$ for four embedding dimensions ($M = 2$, 5, 10, and 15) are provided. Results for greater embedding dimensions are not supplied because small sample bias, as evidenced by plots of $\ln C^M(\varepsilon, T)$ vs. $\ln \varepsilon$, tends to become visually noticeable for $M > 15$.

An examination of Table 3 reveals two interesting observations. First, with respect to the magnitude of correlation dimensions, the $D^M(T)$ for HSI-70w are much larger than those of either HSI-80w or HSI-80Mw. The former is around one-fifth smaller than the values found for German stocks by Booth, Chowdhury et al. (in press) and is about one-third larger than the values reported for US stocks by Scheinkman and LeBaron (1989), Brock (1989), and Gennotte and Marsh (1986). Moreover, $D^M(T)$-values for HSI-80w and HSI-80Mw are not only considerably smaller than those for HSI-70w, but also they are both smaller than those of US stocks. Thus it appears that HSI-80w and HSI-80Mw are less

Table 2
Empirical characteristics of linearly independent HSI returns

	HSI-70w $n = 2345$	HSI-80w $n = 2443$	HSI-80Mw $n = 2439$
Location and shape			
Mean (10^2)	−0.000	−0.000	−0.000
Standard deviation (10^2)	0.524	0.661	0.495
Skewness	0.640 *	−1.817 *	−0.801 *
Kurtosis	13.776 *	117.974 *	16.185 *
Kolmogorov–Smirnov D	0.086 *	0.147 *	0.111 *
Linear dependence			
Autocorrelation: Lag (1)	−0.002	−0.001	0.001
Lag (2)	−0.001	0.003	0.019
Ljung–Box Q (24)	12.80	10.04	36.47
Bartlett's Kolmogorov–Smirnov statistic	0.017	0.011	0.025

See Table 1 notes. AR(2), AR(16), and AR(2) models are used to remove the linear dependence from HSI-70, HSI-80, and HSI-80M, respectively. An AR(16) model is necessary to remove the linear dependence in HSI-80 because the presence of the two October crashes caused a autocorrelation spike at lag 16. There is no economic explanation for this statistical phenomenon.

Table 3
Correlation dimensions for linearly independent HSI returns

	Embedding dimension	Returns	Shuffled returns				t-ratio
			Mean	Quantile		Shapiro–Wilk W statistic	
				1%	5%		
HSI-70w	2	1.388	1.450	1.436	1.440	0.979	−9.538 *
	5	3.159	3.625	3.517	3.559	0.990	−11.478 *
	10	5.597	7.248	6.772	6.972	0.984	−9.934 *
	15	7.835	10.669	9.582	9.792	0.973	−5.998 *
HSI-80w	2	0.962	1.096	1.075	1.083	0.982	−17.867 *
	5	1.886	2.736	2.638	2.665	0.969	−20.936 *
	10	2.862	5.474	5.123	5.204	0.974	−18.076 *
	15	3.420	8.247	7.556	7.715	0.982	−14.409 *
HSI-80Mw	2	1.186	1.299	1.281	1.287	0.979	−17.121 *
	5	2.418	3.247	3.157	3.173	0.971	−19.101 *
	10	3.702	6.505	6.076	6.221	0.984	−16.665 *
	15	4.396	9.773	8.776	8.982	0.986	−11.603 *

* and * * denote statistical significance at least at the 1- and 5-percent levels, respectively. The critical values for a one-tail test for 1- and 5-percent are −2.326 and −1.645, respectively. The shuffled series contain 100 observations.

dimensionally complex than HSI-70w. Interestingly, Frank and Stengos (1989) report $D^M(T)$ for London gold that is similar to US stocks. Brock and Malliaris (1989), on the other hand, find a correlation dimension for US Treasury Bills that is even smaller than that of HSI-80w.

Second, that there is some type of nonlinear dependence in HSI-70w, HSI-80w and HSI-80Mw is apparent by comparing the $D^M(T)$ for each of these series to their $D^M(T)$ after they have been randomly shuffled. Each of the series is shuffled 100 times using a uniform pseudo random number generator, and the $D^M(T)$ for each shuffled series are calculated. Brock (1991) suggests that the number of replications be partly determined by the computer power available. This advice recognizes the enormous amount of computer time needed to calculate the correlation dimension. Frank and Stengos (1989) use 30 replications while Booth, Chowdhury et al. (in press) repeat the process 100 times.

The results of this shuffling are contained in Table 3. In particular, for each series the mean and 1- and 5-percentile delimiting values are given for each embedding dimension. Also given is the Shapiro–Wilk W-statistic, which tests the null hypothesis that the distribution of a specific shuffled series is normal, as well as the 't-ratio'. The t-ratio denotes the number of standard deviations that the $D^M(T)$ for the original series is away from the mean $D^M(T)$ of the 100 scrambled series. If the distribution of the $D^M(T)$ for a shuffled distribution is normal, the t-ratio may be interpreted as the t-statistic.

A review of Table 3 indicates that the $D^M(T)$ for each of the three return series are far less than the corresponding 1-percentile value. In addition, relying on the finding that it is not possible to reject the hypotheses that the distributions of the shuffled $D^M(T)$ are normal, in all cases the t-ratio far exceeds the 1-percent level of significance. Specifically, the t-ratios range from −5.998 to −20.936. Both comparisons, percentile and t-ratio, suggest that the shuffling of the data eliminated (or at least dramatically reduced) the initial nonlinearity in HSI-70w, HSI-80w, and HSI-80Mw.

A potential explanation for the observed nonlinearity is that the conditional variance follows an autoregressive conditional heteroscedastic path. This modeling approach, ARCH, was introduced by Engle (1982) and later generalized by Bollerslev (1986) to GARCH. This type of nonlinearity in stock returns has been modeled by Bollerslev (1987), Akgiray (1989), and Booth, Hatem et al. (1992), among many others. Bollerslev, Chou and Kroner (1992) provide a comprehensive survey of autoregressive conditional heteroscedastic modeling of asset returns.

ARCH processes are a form of multiplicative nonlinearity that retain the martingale property. As Hsieh (1989, 1991) points out, however, nonlinearity can be either multiplicative or additive and there is no reason to presuppose one or the other. Fortunately, models are available to handle additive nonlinearity or a combination of additive and multiplicative nonlinearity. For instance, most bilinear and threshold autoregressive models are capable of describing nonlinear behavior. The GARCH in mean model is able to handle both additive and multiplicative nonlinear dependence. However, this model has not performed well in explaining US (Baillie and DeGennaro, 1990; Nelson, 1991) or German (Booth, Chowdhury et al., in press; Koutmos, 1992) stock returns. Koutmos (1992) documents this phenomenon for Australia, Belgium, Canada, France, Britain, Italy, Netherlands, and Switzerland as well. He does find support, however, for the GARCH in mean effect using Japanese data. Thus it appears that additive nonlinearity is unlikely to be present in stock returns. To ascertain whether this assertion holds for Finnish stock returns, Hsieh's (1989) third-moment test is applied. Application of this test to HSI-70w, HSI-80w and HSI-80Mw reveals that the test's null hypothesis of no additive nonlinearity cannot be rejected in any instance.

Thus, to investigate whether the observed nonlinearity in return series r_t may be explained by a simple GARCH model, the following specification is estimated using the method of maximum likelihood using the method suggested by Berndt et al. (1974):

$$r_t|\Omega_t \sim P(\mu, v_t, \delta), \tag{5}$$

$$e_t = r_t - \mu, \tag{6}$$

$$v_t = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 v_{t-1}, \quad \alpha_0 > 0, \quad \alpha_1 > 0, \quad \beta_1 \geq 0, \tag{7}$$

such that

$$P(\mu, v_t, \delta) = 0.5\delta \left[\Gamma\left(\frac{3}{\delta}\right)\right]^{0.5} \left[\Gamma\left(\frac{1}{\delta}\right)\right]^{-1.5} v_t \times \exp\left\{ - \left[\frac{\Gamma(3/\delta)}{\Gamma(1/\delta)}\right]^{0.5\delta} \left|\frac{e_t - \mu}{v_t}\right|^{\delta}, \right. \tag{8}$$

where P denotes a power exponential (sometimes referred to as generalized error) distribution with mean μ, variance v_t, and degrees of freedom $\infty \leq \delta \leq 1$, all of which are conditioned on information set Ω_t. P has two special cases. The first arises if $\delta = 1$. In this case, P is labeled double exponential and is extremely peaked. The second case occurs if $\delta = 2$. In this instance, P simplifies to a normal distribution. Thus, the smaller the value of δ (or conversely the larger value of $1/\delta$), the thicker are the distribution's tails. A detailed description of the power exponential distribution's properties is provided in Box and Tiao (1973). Although Bollerslev, Chou and Kroner (1992) point out that the inclusion of only one period lag for the squared innovations (e_t^2) and conditional variance (v_t) in the variance equation is usually sufficient to capture the dynamics of the process, undoubtedly the specification of this model can be improved if institutional changes are explicitly recognized. For examples of such modeling efforts see Akgiray, Booth and Loistl (1989), Lamoureux and Lastrapes (1990), and especially Booth, Hatem et al. (1992) since they model Finnish stock returns.

The GARCH model estimation results for HSI-70w and HSI-80Mw are presented in Table 4. Results for estimating a GARCH model for HSI-80w are not reported. This is because the impact of the crash is not only linear, as indicated previously, but also nonlinear. Recall that an AR(16) was required to obtain HSI-80w from HSI-80. An examination of the autocorrelation function for squared HSI-80w reveals that a significant autocorrelation at lag 16 exists, signifying that HSI-80w is a particularly difficult series to model using GARCH because of convergence difficulties. Eliminating HSI-80w from analysis at this stage, however, should not effect this study's conclusions, since as Ramsey, Sayers and Rothman (1990) point out, the elimination of a few data points should not affect the correlation dimension of a chaotic process because of the self-similar nature of such a process.

An examination of Table 4 indicates that for both return series the GARCH model appears to fit reasonably well. With the exception of μ for HSI-80Mw all parameters are significant at the 1-percent level. Particularly noteworthy is that the persistence parameters, α_1 and β_1, and the degrees of freedom for the conditional distribution are similar for both HSI-70w and HSI-80Mw. (These values are also not dissimilar from the corresponding values reported by Booth, Hatem

Table 4
GARCH model estimates

	HSI-70w	HSI-80Mw
μ (10^4)	1.778 *	−0.494
	(0.689)	(0.498)
α_0 (10^4)	0.029 *	0.013 *
	(0.005)	(0.002)
α_1	0.371 *	0.366 *
	(0.034)	(0.029)
β_1	0.528 *	0.569 *
	(0.033)	(0.024)
$1/\delta$	0.770 *	0.773 *
	(0.016)	(0.014)
log-likelihood value	9389	10170
Derivative statistics:		
K	1.341	1.360
s (10^2)	0.539	0.444
HL	7.489	11.232

* indicates significance at least at the 1-percent level. Standard errors are contained within the parentheses. Conditional excess kurtosis, K, equals $\Gamma(5/d)\Gamma(1/d)[\Gamma(3/d)]^{-2}-3$. Unconditional standard deviation, s, equals $[\alpha_0/(1-\alpha_1-\beta_1)]^{-0.5}$. Half-life, HL, is defined as $[-\ln(2)/\ln(\alpha_1+\beta_1)]+1$ and measures the number of time units required for a shock to dissipate by 50%.

Table 5
Diagnostics for HSI standardized GARCH residuals

	HSI-70wg	HSI-80Mwg
Location and shape:		
Mean (10^2)	0.031	0.003
Standard deviation (10^2)	1.079	1.154
Skewness	−0.114	−1.023 *
Kurtosis	8.604 *	12.151 *
Equiprobable $\chi^2_{\text{power exponential}}$	34.183	33.055
Linear dependence:		
Autocorrelation: Lag (1)	0.052	0.056
Lag (2)	0.037	0.078 *
Ljung–Box Q (24)	27.54	60.31 *
Bartlett's Kolmogorov–Smirnov statistic	0.048	0.064 *

See Table 1 notes. The interval size for the equiprobable χ^2 goodness-of-fit test is determined by the procedure described in Kendall and Stuart (1977), with the degrees of freedom being 36.

et al. (1992).) Moreover the calculated unconditional variances for both series are close to their values reported in Table 2. Further, as required, the conditional kurtosis is smaller than the unconditional kurtosis, which is also reported in Table 2. Diagnostic statistics for the standardized GARCH residuals, defined as $(r_t-\mu)v_t^{-0.5}$ and labeled as HSI-70wg and HSI-80Mwg, are presented in Table 5. Both series are correctly characterized by approximately a zero mean and unit variance. In addition, both series are thick-tailed. Application of an equiprobable goodness-of-fit test indicates that the assertion that the conditional distributions are of the power exponential variety cannot be rejected. Somewhat puzzling, however, is that some linear dependence seems to have significantly reasserted itself in HSI-80Mwg.

The correlation dimensions for the standardized GARCH residuals are displayed in Table 6, along with the appropriate comparison information from the shuffled series. Compared to HSI-70w and HSI-80mw, the $D^M(T)$ for HSI-70wg and HSI-80Mwg, respectively, have increased substantially. For instance, $D^{15}(T)$ increases from 7.835 (HSI-70w) to 11.507 (HSI-70wg) and from 4.396 (HSI-80Mw) to 9.624 (HSI-80Mwg). Thus, it does not appear that the requirement of Brock's residual test is met, indicating that HSI-70w and

Table 6
Correlation dimensions for HSI standardized GARCH residuals

	Embedding dimension	Returns	Shuffled returns				t-ratio
			Mean	Quantile		Shapiro–Wilk W statistic	
				1%	5%		
HSI-70wg	2	1.578	1.575	1.562	1.567	0.985	0.555
	5	3.976	3.938	3.943	3.883	0.978	1.118
	10	7.567	7.873	7.451	7.604	0.983	−1.801 * *
	15	11.507	11.381	9.985	10.331	0.977	0.183
HSI-80Mwg	2	1.501	1.511	1.500	1.502	0.968	−1.786 * *
	5	3.741	3.777	3.683	3.713	0.982	−0.970
	10	7.142	7.550	7.123	7.281	0.971	−2.525 *
	15	9.624	10.997	9.816	10.086	0.979	−2.625 *

See Table 3 notes.

HSI-80mw are not generated by deterministic chaos.

There is little evidence supporting the notion that the nonlinearity in HSI-70w is not attributable to GARCH effects. Some evidence, however, indicates that the simple GARCH model employed in this paper markedly reduced the nonlinearity present in HSI-80Mw but it did not remove all of it. For instance, for HSI-80Mw the t-ratio is significant at the 1-percent level for $M=10$ and 15. Using the quantile statistics, $D^{10}(T)$ falls between the 1- and 5-percentiles, while $D^{15}(T)$ is lower than the 1-percentile delimiter. This evidence, however, may be the result of the linear dependence present in the standardized residuals and the downward bias in $D^M(T)$ caused by small sample size. It may also be the result of a misspecified GARCH model, since Booth, Hatem et al. (1992) show that a GARCH model augmented with dummy variables to account for structural changes in the conditional mean and variance is capable of explaining Finnish stock returns from the beginning of 1980 up to but not including the October 1987 crash. Modeling HSI-80Mw in the framework employed by Booth, Hatem et al. (in press) was explored. However, the addition of over two years of data, even after removing the crash related 'outliers', rendered this approach ineffective. This is probably the result of the market maturing even more rapidly in 1988–89 than it did in the previous eight years. An example of this is the establishment of a stock futures market in 1988. Booth, Martikainen and Puttonen (1992) demonstrate that this market permits international stock price movements to significantly influence Finnish stock returns. In any case, it may be appropriate to specify the persistence parameters, α_1 and β_1, to be time dependent. It may also be appropriate to use a GARCH model that is capable of recognizing that linear and nonlinear dependence may be related (LeBaron, 1992). In any case, the weight of evidence makes it plausible to conclude that the nonlinear dependence that is observed in Finnish stock returns for both decades examined is not chaotic but appears to be able to be attributed to GARCH effects.

This conclusion is buttressed by the BDS test. The BDS statistic sometimes called the w-statistic, was developed by Brock, Dechert and Sheinkman (1987) to test whether $C^M(\varepsilon, T)$ is significantly greater than $C^1(\varepsilon, T)^m$, and if it is, nonlinearity is present. For $\varepsilon=\sigma$ and $M=15$, the BDS-statistics for HSI-70w and HSI-80Mw are 55.12 and 318.09, respectively, with the corresponding values similarly relatively high for other values of M. The respective test values for HSI-70wg and HSI-80Mwg are -0.238 and 5.507. According to critical value tables provided by Brock, Hsieh, and LeBaron (1991), the BDS-statistics for HSI-70w, HSI-80Mw, and HSI-80Mwg are significant at the 1-percent level, but the statistic for HSI-70wg is insignificant. (note that for HSI-70wg the BDS-statistic for $M=10$ is -0.685, which is also insignificant. This result conflicts with the observation the corresponding t-ratio reported in Table 6 is significant at the 5-percent level.) Thus a simple GARCH model is able to remove the nonlinear dependence in this latter series. For HSI-80Mw, the GARCH model markedly reduces the nonlinearity but does not remove it completely.

It is worthwhile to mention that Frank and Stengos (1989) report that GARCH models do not remove the nonlinear dependence contained in gold and silver prices. In contrast, Hsieh (1989) finds that these models describe the nonlinear behavior of the Canadian dollar, Swiss franc, and German mark but perform poorly for the British pound and the Japanese yen. Regardless of the fit, however, Hsieh (1989) maintains that diagnostics show that his GARCH models account for most of the observed nonlinearity.

5. Summary and conclusion

Finnish stock returns in the 1970s and 1980s exhibit significant linear and nonlinear dependence. In both periods, linear dependence can be removed by an autoregressive process, but this does not eliminate the observed nonlinearity. A simple GARCH model seems to remove most, if not all, of the nonlinearity. This removal, as indicated by the comparison of the $D^M(T)$ to their shuffled counterparts and the failed Brock residual test, suggests that the observed nonlinear dependence is not generated by deterministic chaos. Instead, it can be modeled using an autoregressive conditional heteroscedastic approach, although it may be necessary to explicitly account for shifting parameter values that may be occurring throughout the 1980s.

These findings contradict the conclusions of Scheinkman and LeBaron (1989), Brock and Malliaris (1989), and Gennotte and Marsh (1986) for US stock returns. Further, they are in sharp contrast with Scheinkman and LeBaron's (1989) observation that GARCH models are not capable of eliminating the nonlinear dependence that they observe in US stock returns. They also provide evidence against Brock and Malliaris' (1989, p.339) contention that GARCH processes are not capable of accounting for all nonlinear dependence found in asset prices. Nevertheless, the findings complement Hsieh's (1991) contention that US stock returns are not chaotic and that the observed nonlinearities are effectively removed by GARCH processes. It also supports a similar assertion by Booth, Chowdhury et al. (in press) concerning German stock returns.

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