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Theory and Methodology

Stochastic modeling of security returns: Evidence from the Helsinki Stock Exchange

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Abstract: This paper documents the presence of linear and nonlinear dependencies in Finnish stock returns and models these dependencies using autoregressive conditional heteroscedastic methods. Three conditional distributions (normal, Student-*t*, and the power exponential) are explored. The statistical estimates and the corresponding diagnostic tests indicate that a GARCH (1, 1) model with a power exponential conditional distribution, which is characterized by an autoregressive mean, represents the data better than any of the other models examined.

Keywords: Time series, stochastic, finance, distributions, GARCH

1. Introduction

Since the works of Mandelbrot (1963) and Fama (1965), a vast literature has developed that is concerned with the probabilistic modeling of the behavior of common stock returns. Most of the efforts to determine the 'best' model have been restricted to considering those stochastic processes that are consistent with the empirical observations that shock return distributions are thicker-tailed than a normal distribution and that successive observations of these returns exhibit little or no dependence. Popular contenders for the appropriate process have been the stable laws or some type of mixed distribution (e.g., the Student-*t*, compound normal, and mixed diffusion–jump models). Key references to these models include Blattberg and Gonedes (1974), Oldfield, Rogalski, and Jarrow (1977), Kon (1984), and Bookstaber and McDonald (1987).

Recently, however, several studies have cast doubt on the validity of these models by pointing out that stock returns often exhibit significant levels of nonlinear dependence; see, e.g., Bollerslev (1987), Akgiray (1989), and Schienkman and LeBaron (1989). Regardless of the reason for the existence of this nonlinear phenomenon, a common way of modeling it is through the use of *autoregressive conditional heteroscedastic* (ARCH) models. Examples of such modeling efforts include Bollerslev (1987), Akgiray (1989), and Lamoureux and Lastrapes (1990). ARCH applications generally have been restricted to the U.S. stock market, with Akgiray, Booth, and Loistl's (1989) modeling of German stock returns as an exception.

The purpose of this study is to provide additional international evidence on the existence of nonlinear dependence in stock returns and the corresponding suitability of ARCH modeling techniques. The stock market chosen to accomplish this task is the Helsinki Stock Exchange. This market is chosen primarily for two reasons. First, the Helsinki stock market differs from the major exchanges of the United States, Japan, and Western Europe. It is a relatively small market comprised of generally thinly-traded stocks. Further, certain laws exist that curtail stock ownership, although in recent years these regulations have been significantly liberalized. Second, research concerning the use of probability models to explain Finnish stock prices is theoretically limited and quantitatively sparse. For example, Korhonen's (1977) and Berglund's (1986) conclusions are that Finnish stock returns do not fully support the random walk hypothesis. Further, Virtanen and Yli-Olli (1987) exploit this nonrandomness by using an ARIMA model to explain and forecast monthly stock returns.

The remainder of this paper is divided into three sections. The first section introduces and empirically describes the data. The second develops the generalized ARCH (GARCH) models to be employed and provides the appropriate parameter estimates and relevant diagnostics. Conclusions are offered in the final section.

2. Data and preliminary analysis

The raw data series is the Helsinki daily value-weighted price index, which consists of 134 stocks. Prices are expressed in natural logarithms and are adjusted for stock splits and dividends (stock and cash). The adjustment assumes that cash dividends are re-invested with no transaction costs. In most cases transaction prices are used to construct the index. If there is more than one transaction during the day, the mean price is used. If, however, a transaction did not occur on a given day, the bid price is used. The data period is the beginning of January 1980 to the end of September 1987. Data prior to 1980 are excluded for two reasons. First, in the 1970's the Finnish economy was characterized by high inflation and interest rates as well as low international competitiveness of the private sector. Beginning in 1980, however, the economy improved radically. Second, during the same period, the Finnish financial markets were strictly dominated by banks. Data availability and the October 1987 worldwide stock market crash are responsible for the ending date. The series analyzed is the return series, which is defined as the first difference of the price series.

Relevant location, shape, and dependency statistics for the return series are presented in the first column of Table 1. In terms of distributional characteristics, the return series is similar to most asset return series. It is non-normal, thick-tailed, and somewhat skewed. More interesting is that the series is both linearly and nonlinearly dependent. The linear dependence is demonstrated by the autocorrelation function of the returns and is consistent with the results reported by Berglund (1986) and Virtanen and Yli-Olli (1987). The autocorrelation coefficients denoted as statistically significant are equal to at least three times their standard errors, thereby mitigating rejecting the null hypothesis of zero autocorrelation by mistake because of the non-normal characteristics of the distribution. For instance, the autocorrelation coefficient for the first lag is over 18 times its standard error. Ljung–Box Q–statistics confirm the finding of linear dependence. Moreover, as pointed out by Akgiray (1989) and others, nonlinear dependence is evinced by the significant autocorrelation coefficients and Q-statistics for the squared return series. These statistics indicate that large returns are followed by large returns, regardless of sign. It is possible, however, that these distributional and dependency characteristics are caused by a mixture of independent distributions rather than a single nonlinear stochastic process. Possible mixing candidates are structural changes, day-of-the-week effects, and monthly effects. Monthly effects have been identified by Berglund (1986) and Virtanen and Yli-Olli (1987). Berglund (1986), however, investigates day-of-the-week effects but finds none. In a tangential work, Hietala (1989) indicates that structural changes may have occurred around the end of 1982 and the end of 1985. The former is associated with a substantial increase in Finnish stock purchases by foreign investors. The latter is the result of a significant increase in participation by Finnish investors, possibly because restrictions concerning Finnish investors were liberalized or because Finnish investors finally followed the lead of their foreign counterparts. As a result, transaction volume during the eight-year period of the study showed a more than ten-fold increase. That both posited structural changes are reflected by an increase in trading volume is noteworthy because, as is suggested by Karpoff (1987) and Lamoureux and Lastrapes (1990), changes in volume have the potential to explain the non-normality of daily returns. This is because volume and volatility often have been found to be positively related.

To test for the possible existence of these three effects, returns are regressed (General Linear Method) on dummy variables representing Hietala's (1989) somewhat arbitrary designated structural periods (January 2, 1980–December 30, 1982; January 3, 1983–December 30, 1985; January 2, 1986–September 30, 1987), day of the week (Monday,...,Friday) and month (January,...,December). The residuals of this regres-

Table 1
Empirical characteristics of returns

		1980–87 $n=1874$	1980–82 $n=716$	1983–85 $n=726$	1986–87 $n=432$
Location and shape – Returns					
Mean		0.0012 **	0.0008 *	0.0009 **	0.0022 **
Standard Deviation		0.0049	0.0028	0.0056	0.0062
Skewness		−0.5270 **	0.7231 **	−1.103 **	−0.1329
Kurtosis		12.55 **	6.202 **	13.77 **	4.300 **
Equiprobable χ^2_{normal}		428.8 **	161.4 *	144.1 **	81.17 **
Dependence					
Returns					
Autocorrelation:	lag 1	0.5267 **	0.4477 **	0.5325 **	0.5237 **
	lag 2	0.2953 **	0.3412 **	0.3245 **	0.2146 **
	lag 3	0.1426 **	0.2671 **	0.1580 **	0.0498
	lag 4	0.0501	0.1619 **	0.0089	−0.0849
	lag 5	0.0190	0.1632 **	0.0380	−0.0946
	lag 6	0.0000	0.1252	0.0079	−0.0928
Ljung–Box:	$Q(6)$	728.2 **	328.9 **	308.7 **	153.1 **
	$Q(12)$	745.1 **	365.5 **	312.9 **	163.4 **
	$Q(24)$	750.8 **	426.1 **	318.1 **	167.4 **
Squared returns					
Autocorrelation:	lag 1	0.2516 **	0.2850 **	0.1642 **	0.4158 **
	lag 2	0.1279 **	0.1587 **	0.0659	0.2123 **
	lag 3	0.0732 **	0.1704 **	0.0407	0.0660
	lag 4	0.0497	0.0948	0.0304	0.0109
	lag 5	0.0293	0.0830	0.0226	−0.0460
	lag 6	0.0339	0.0733	0.0247	−0.0334
Ljung–Box:	$Q(6)$	168.0 **	112.8 **	25.53 **	98.25 **
	$Q(12)$	236.6 **	113.1 **	41.13 **	147.1 **
	$Q(24)$	281.7 **	162.4 **	100.8 **	170.1 **

(*) and (**) indicate significance at least at the 5- and 1-percent level, respectively, for the mean (= 0), skewness (= 0), kurtosis (= 0), equiprobable χ^2, and the Q-statistics for various lags. For the autocorrelation coefficients for various lags, (**) indicates that an estimated autocorrelation is at least three times its standard error. Interval size for the equiprobable χ^2 goodness-of-fit test is determined using the procedure developed in Kendall and Stuart (1977); degrees of freedom for the total and each consecutive period are 31, 21, 21, and 17, respectively.

sion are squared and regressed on the same 20 dummy variables. The first regression measures the effect of the potential mixing variables on the mean of the return series (shifting mean), and the second portrays their impact on its variance (heteroscedasticity).

The results of this procedure confirm previous conjectures. First, generalized pairwise testing of the significance of the difference of the regression coefficients reveals that the means of the first two structural periods are not statistically dissimilar but are different from the third period's mean. In the case of variance, the last two periods are similar but different from and much larger than the first, which supports the notion that volatility and volume are positively related. For both the mean and the variance, there is a monthly but not a daily effect. Particularly noteworthy is the existence of the so-called January effect. This phenomenon has been documented by Berglund (1986) for Finland and by Gultekin and Gultekin (1983) for 18 other countries. Nevertheless, the usefulness of months as a mixing variable in analyzing the evolution of daily returns is debatable. This is because their use assumes that the effect is spread equally throughout the month. If this is not the case (e.g., the measured January effect may be instead a turn-of-the-year effect), the analysis may be compromised.

In view of the above observations, this analysis explicitly concerns itself only with the impact of the identified structural changes. Relevant descriptive statistics for each of the structural periods are displayed in the last three columns of Table 1. The information contained therein once more illustrates the previously documented differences in means and variances among the periods. However, It also illustrates that the return distribution in each period is shaped like the distribution for the total period, i.e., non-normal with thick tails and some evidence of skewness. Perhaps even more important is that linear and nonlinear dependence is present in all cases. As pointed out by Akgiray (1989), this characteristic is suggestive of a thin market.

3. Models

ARCH models are capable of modeling these dependence phenomena. The ARCH specification was developed by Engle (1982) and later generalized to (G)ARCH by Bollerslev (1986). The GARCH model can also be augmented to handle linear dependence. Symbolically, the augmented model may be stated as:

$$r_t \mid \Omega_t \sim f(\mu_t, h_t, d), \tag{1}$$

$$\mu_t = \phi_0 + \sum_{k=1}^{m} \phi_k r_{t-k}, \tag{2}$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}, \tag{3}$$

and

$$\varepsilon_t = r_t - \mu_t, \tag{4}$$

where the parameters satisfy the conditions $\alpha_0 > 0$, $\alpha_i \geq 0$ for $i = 1, \ldots, p$, and $\beta_j \geq 0$ for $j = 1, \ldots, q$. The parameters p, q and m represent the orders of the process such that it is referred to as a GARCH(p, q, m) process. To illustrate the specifications generality, disregarding ϕ_k, if $q = 0$ and α_0, $\alpha_i > 0$, the model is labeled ARCH. If $\phi_k \neq 0$, the model is a GARCH with an autoregressive component. If $p = q = 0$ and $m \neq 0$, the process is solely autoregressive; but if $m = 0$ as well, the process is strict white noise.

Equation (1) describes the conditional density function $f(\mu_t, h_t, d)$ of r_t, the return series, given the set of information at time t, i.e., $\Omega_t = r_{t-1}, \ldots, r_{t-n}$, and d degrees of freedom. Equation (2) specifies that the mean $\{\mu_t\}$ follows a simple autoregressive process of order m, thereby modeling the linear dependence phenomenon. Thus μ_t is the conditional mean., To permit μ_t to shift among the three structural periods, ϕ_0 is specified to equal $\phi_{01}S_1 + \phi_{02}S_2 + \phi_{03}S_3$, where S_1, S_2, and S_3 are dummy variables corresponding to the three periods. Equation (3) gives the conditional variance $\{h_t\}$. This variance is conditioned on past realized variances $\{h_{t-j}\}$ and on past squared forecast errors of the mean $\{\varepsilon_{t-i}^2\}$, the latter being defined by (4). The former equation is used to accommodate the nonlinear dependence phenomenon plus any possible persistence in the conditional variances. To allow h_t to vary because of the structural period in which it is in, α_0 in (3) is set equal to $\alpha_{01}S_1 + \alpha_{02}S_2 + \alpha_{03}S_3$.

The parameters of (1)–(4) are estimated by the method of maximum likelihood, suggested by Berndt et al. (1974), for the parameters $\theta =$

$\phi_0, \ldots, \beta_q$. Conditional on the initial values of r_s, μ_s, and h_s for $s = 0, \ldots, \tau = \max(p, q, m)$, the log-likelihood function is:

$$L(\theta \mid p, q, m) = \sum_{t=\tau}^{T} \log f(\mu_t, h_t, d).$$

The values of p, q, and m are prespecified. The likelihood function is maximized for several combinations of these three values and the maxima are compared to obtain the optimal orders of the process. The particular optimal combination is obtained by applying a likelihood test. In particular, let z^* denote the set of parameters under the null hypothesis and z the same under the alternative hypothesis ($z^* < z$), then $-2[L(\theta \mid z^*) - L(\theta \mid z)]$ is asymptotically χ^2 distributed with degrees of freedom equalling $z - z^*$. In addition, a benchmark model with $p = q = 0$ is estimated. This model is an autoregressive model of order m and denoted by AR(m).

Although the purpose of the preceding model is to remove linear and nonlinear time dependence, their removal does not necessarily imply that any particular conditional distribution f is the true one. This is because distributional shape may be partially determined by factors other than GARCH effects. To accommodate this phenomenon, three conditional distributions are used: the normal (N), the Student-t (T), and the power exponential (PE), which is articulated by Box and Tiao (1973). The conditional density functions in respective order are:

$$\mathrm{N}(\mu_t, h_t) = (2\pi h_t)^{-1/2} \exp\left[-(r_t - \mu_t)^2/2h_t\right], \quad (5)$$

$$\begin{aligned}\mathrm{T}(\mu_t, h_t, d) &= \Gamma[(d+1)/2]\Gamma(d/2)^{-1}[h_t(d-2)\pi]^{-1/2} \\ &\quad \times \left[1 + \left((r_t - \mu_t)^2/(h_t(d-2))\right)\right]^{-(d+1)2}, \\ d > 2, \end{aligned} \quad (6)$$

and

$$\begin{aligned}\mathrm{PE}(\mu_t, h_t, d) &= 1/2[\Gamma(3/d)]^{1/2}[\Gamma(1/d)]^{-3/2}h_t \\ &\quad \times \exp\left\{-[\Gamma(3/d)/\Gamma(1/d)]^{d/2} \right. \\ &\quad \left. \times |(r_t - \mu_t)/h_t|^d\right\}. \end{aligned} \quad (7)$$

The normal distribution, defined in (5), is a special case of the t distribution, (6), and occurs when $d = \infty$. The normal distribution is also a special case of the power exponential distribution, i.e., (7) simplifies to that of a normal distribution when $d = 2$. Both the t and power exponential distributions are more peaked and thicker-tailed than the corresponding normal distribution and the t distribution permits even thicker tails than does the power exponential. Thus, since d is endogenous, either of these two non-normal distributions provide a great deal of modeling flexibility. To distinguish among the three GARCH models, the notation GARCH-f is used with f denoting the particular conditional distribution (f = N, T, or PE).

To determine whether either the GARCH-T or the GARCH-PE provides more explanation than the GARCH-N is straightforward. This is because the GARCH-N is nested within the other two models. Thus, the 'best' model can be determined using a likelihood ratio test similar to the one described above. Unfortunately, neither the GARCH-T nor the GARCH-PE model is nested within the other, although McDonald and Newey (1988) show that the two distributions are both special cases of the generalized t distribution. Thus, a log-likelihood comparison of models containing the two distributions is inappropriate. Their relative goodness-of-fit, however, may be judged by the degree to which the standardized residuals $\{\hat{\varepsilon}_t/(\hat{h}_t)^{1/2}\}$ match each model's underlying distributional assumptions.

The GARCH estimation results for each of the three conditional distributions are presented in columns 2–4 in Table 2. The estimates for the benchmark autoregressive model are presented in the first column. In Table 3, calculations of GARCH derivative statistics, including conditional kurtosis and standard deviations are also provided. Because of the autoregressive component in (2), there are two conditional variances (standard deviations) to consider: one (σ_ε^2) associated with the forecast errors after the autoregressive component is eliminated and the other (σ_r^2) associated with the return series itself.

For all models, the optimal order for the linear autoregressive component is $m = 2$. For the GARCH models the optimal orders for the error-squared and variance terms are $p = q = 1$. These values of p and q are commonplace in

Table 2
Model estimates

Statistic	AR(2)	GARCH (1, 1, 2)		
		Normal (N)	Student-t (T)	Power Exponential (PE)
ϕ_{01} ($\times 10^2$)	0.0377 **	0.0265 **	0.0286 **	0.0311 **
	(0.0088)	(0.0058)	(0.0067)	(0.0063)
ϕ_{02} ($\times 10^2$)	0.0993 **	0.0919 **	0.0841 **	0.0854 **
	(0.0245)	(0.0196)	(0.0199)	(0.0178)
ϕ_{03} ($\times 10^2$)	0.0380 *	−0.0041	0.0237 *	0.0148
	(0.0193)	(0.0078)	(0.0120)	(0.0108)
ϕ_1	0.4124 **	0.4136 **	0.4870 **	0.4866 **
	(0.0104)	(0.0189)	(0.0268)	(0.0250)
ϕ_2	0.0904 **	0.1006 **	0.0999 **	0.0767 **
	(0.0144)	(0.0198)	(0.0253)	(0.0233)
α_{01} ($\times 10^4$)	0.0558 **	0.0098 **	0.0128 **	0.0124 **
	(0.0016)	(0.0019)	(0.0033)	(0.0036)
α_{02} ($\times 10^4$)	0.2235 **	0.0361 **	0.0425 **	0.0444 **
	(0.0039)	(0.0046)	(0.0084)	(0.0095)
α_{03} ($\times 10^4$)	0.2587 **	0.0495 **	0.0651 **	0.0572 **
	(0.0091)	(0.0081)	(0.0146)	(0.0148)
α_1		0.5736 **	0.3893 **	0.4766 **
		(0.0332)	(0.0598)	(0.0696)
β_1		0.4003 **	0.4091 **	0.4020) **
		(0.0316)	(0.0575)	(0.0611)
$1/d$		0.0000	0.1943 **	0.8903 **
			(0.0138)	(0.0246)
$L(\theta \mid p, q, m)$	7769.9	7955.5	8157.4	8134.8

(*) and (**) indicate significance at least at the 5- and 1-percent level, respectively. Standard errors are contained within parentheses. The sample size is 1872.

Table 3
GARCH derivative statistics

Statistic	GARCH (1, 1, 2)		
	Normal (N)	Student-t (T)	Power Exponential (PE)
$\alpha_1+\beta_1$	0.9739	0.7984	0.8768
HL	27.23	4.078	6.355
σ_ε (1980–82)	0.0061	0.0025	0.0032
(1983–85)	0.0118	0.0046	0.0060
(1986–87)	0.0138	0.0057	0.0069
σ_r (1980–82)	0.0069	0.0030	0.0038
(1983–85)	0.0133	0.0055	0.0070
(1986–87)	0.0156	0.0067	0.0081
K	0.0000	5.232	4.147

Half-life, HL, equals $[-\log[2]/\log(\alpha_1+\alpha_1)]+1$. $\sigma_r^2=[(1-\phi_2)\sigma_\varepsilon^2]/[(1+\phi_2)\{(1-\phi_2)^2-\phi_1^2\}]$, where $\sigma_\varepsilon^2=\alpha_0/(1-\alpha_1-\beta_1)$ and σ_r^2 and σ_ε^2 are the estimated unconditional variances of r and ε, respectively. For GARCH-T, $K=[3(d-2)/(d-4)]-3$ is the estimated conditional excess kurtosis; for GARCH-PE, $K=\Gamma(5/d)\Gamma(1/d)[\Gamma(3/d)]^{-2}-3$; and for GARCH-N, $K=0$ by construction.

empirical studies. Note that in all four models, all the parameters except for ϕ_{03} for the GARCH-T and -PE models are significantly different from zero. The insignificance of the two parameters, however, does not detract from the model because they simply indicate that the mean in the third period is determined by the autoregressive component without a constant term. Note that for the GARCH models, $\alpha_1+\beta_1$ are less than one, indicating that the fitted models are second order stationary or that at least second moments exist. This sum, however, is much smaller for the GARCH-T and -PE models than for the GARCH-N model. An economic interpretation of $\alpha_1+\beta_1$ is apparent when the sum is transformed into a statistic indicating the half-life of an innovation on the return's conditional variance. For the GARCH-N, the half-life exceeds one month. The other two GARCH models demonstrate that this impact is approximately one week. With respect to the theoretical return standard deviation σ_r, those associated with the GARCH-T and -PE are somewhat similar to their empirical counterparts. In contrast, σ_r for the GARCH-N model is noticeably larger.

Since the AR(2) model is nested within the three GARCH models, log-likelihood tests are able to indicate if these latter models provide more explanation than the benchmark model. Application of these tests show that the AR(2) model is inferior to the three GARCH models. The reason for this conclusion lies with the observation that the model's residuals are not white noise. This is documented in the first column of Table 4. Note that the Q-statistics show modest linear independence but they also show substantial nonlinear dependence. As McLeod and Li (1983) observe, this nonlinear dependence in AR models of economic and hydrological time series is commonplace.

Comparing the GARCH-N with the GARCH-T and GARCH-PE models using the log-likelihood test shows that in both comparisons the GARCH-N provides significantly less explanation. As noted above, even though the log-likelihood value for the GARCH-T is slightly greater than that for the GARCH-PE, this differential cannot be used to determine the better model. Instead, other comparisons must be exploited. In particular, to be acceptable, (1) the model's standardized residuals should have a zero mean and unit variance, (2) they should be linearly and nonlinearly dependent, and (3) they should conform to the assumed distributional shape. The importance of meeting these conditions is documented by Tauchen's (1985) assertion that the

Table 4
Model diagnostics for standardized residuals

Statistic		AR(2) residuals	GARCH (1, 1, 2) standardized residuals		
			Normal (N)	Student-t (T)	Power Exponential (PE)
Location and shape-residuals					
Mean		0.0158	0.0634 **	0.0144	0.0247
Standard Deviation		1.0320	1.0387	1.0739	1.0365
Skewness		−0.8006 **	−0.6868 **	−0.9612 **	−0.8523 *
Kurtosis		14.66 **	10.47 **	15.15 **	13.32 *
Equiprobable $\chi^2(31)$		236.3 **	123.6 **	160.6 **	41.74
Dependence					
Residuals					
Autocorrelation:	Lag 1	0.0532	0.0670	0.0251	0.0226
	Lag 2	0.0154	0.0505	0.0225	0.0396
	Lag 3	0.0144	0.0457	0.0204	0.0304
	Lag 4	−0.0324	0.0154	−0.0079	0.0005
	Lag 5	0.0073	0.0104	−0.0023	0.0033
	Lag 6	−0.112	0.0006	−0.0087	−0.0059
Ljung–Box	$Q(6)$	8.450	17.17 **	3.180	6.080
	$Q(12)$	23.81 **	28.33 **	13.79	16.42
	$Q(24)$	29.09	39.90 *	23.45	26.32
Residuals-squared					
Autocorrelation:	Lag 1	0.2016 **	−0.0188	−0.0098	−0.0056
	Lag 2	0.0564	−0.0207	−0.0144	−0.0118
	Lag 3	0.0518	0.0043	0.0038	0.0044
	Lag 4	0.0084	−0.0162	−0.0141	−0.0120
	Lag 5	−0.0041	−0.0237	−0.0193	−0.0179
	Lag 6	0.0053	0.0017	−0.0014	−0.0008
Ljung-Box:	$Q(6)$	87.39 **	3.06	1.23	1.67
	$Q(12)$	91.95 **	3.48	2.40	2.44
	$Q(24)$	160.2 **	18.36	7.61	9.82

See Table 1 notes. Note, however, that the degrees of freedom for the Q-statistics for the residuals, but not the residuals-squared, are reduced by two to account for AR(2) component (McLeod and Li, 1983). The theoretical distribution associated with the equiprobable χ^2 test corresponds to the model's assumed error distribution of residuals. The sample size is 1872.

diagnosis of a maximum likelihood model's residuals is biased toward acceptance of the model.

A review of Table 4 indicates that both the GARCH-T and -PE meet the first two criteria. This is not the case with respect to distributional shape, however. Both GARCH residuals exhibit significant negative skewness (undesirable since both hypothesized distributions are symmetric) and kurtosis (desirable since both distributions are thicker-tailed than a normal distribution), when these two characteristics are evaluated separately. The kurtosis values for both distributions are slightly smaller than the corresponding values for the original returns. According to Hsieh (1989), this relationship is indicative of proper GARCH model fitting. Moreover, from an overall perspective, application of the equiprobable χ^2 goodness-of-fit tests indicates that the null hypothesis that (1) the GARCH-T standardized residuals are described by a Student-t distribution defined by $d = 5.147$ is rejected and (2) the GARCH-PE residuals are characterized by a power exponential distribution defined by $d = 1.1232$ is not rejected. The reason for this result appears to be the latter's ability to portray the distribution1s peakedness. In sum, of the four models examined, the GARCH-PE model appears to be the best explainer. Its choice highlights the fact that the removal of GARCH effects does not always remove distributional thick tails.

4. Conclusions

The results in this paper complement earlier work that examined the behavior of Finnish stock prices. It documents again the presence of a monthly factor, the lack of a daily factor, and the existence of linear dependence. It extends the literature by demonstrating that the stochastic process has changed in the 1980's at least twice, possibly via increased volume. It also shows that there is substantial nonlinear dependence. It further extends the literature by estimating for the first time a GARCH model with a power exponential conditional distribution that is capable of handling both linear and nonlinear dependence as well as structural effects on Finnish stock returns.

Despite this modeling advance, other GARCH models, ones with even more flexible distributional shapes, may prove to be superior. Potential candidates or these distributions include the generalized t distribution and the mixed diffusion-jump process. The latter process may be particularly appropriate because it can accommodate distributional skewness. The usefulness of this type of modeling is underscored by noting that GARCH effects may be proxying underlying economic phenomena that may not be measurable. For instance, Lamoureux and Lastrapes (1990) suggest that the explanatory ability of GARCH models may be related to the notion that equal units of calendar time may contain unequal amounts of economic time. In any case, these modeling efforts are left for future research.

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