Applied Financial Economics, 1993, **3**, 189–200

On the existence of common factors in the arbitrage pricing model: international evidence from US and Scandinavian stock markets

G. GEOFFREY BOOTH, TEPPO MARTIKAINEN[‡], ILKKA VIRTANEN[‡], and PAAVO YLI-OLLI[‡]

College of Business Administration, Louisiana State University, Baton Rouge, LA 70803, USA and [‡]*School of Business Studies, University of Vaasa, PO Box 297, SF-65101 Vaasa, Finland*

The purpose of this paper is to test the arbitrage pricing model (APM) using monthly data for US, Finnish and Swedish stock returns during the 1977–86 period. First, the intra-country stability of the factor patterns of the APM is researched in two exclusive subperiods. Second, the cross-sectional similarities of the factor patterns of twelve 30-stock samples collected from the three countries are investigated. Empirical evidence indicates the existence of two common factors. In addition, it is found that these factors are often produced in different order in different samples. Studying the association between the estimated factors and equilibrium returns indicates that for US and Swedish stocks one or two factors are priced, and for Finnish stocks only one factor is priced in the first subperiod.

I. INTRODUCTION

Background

The capital asset pricing model (CAPM), developed by Treynor (1961), Sharpe (1963, 1964), Lintner (1965) and Mossin (1966), has been the major framework for analysing the cross-sectional variation of expected security returns. The CAPM assumes that the expected return of a stock is linearly related to its systematic risk, where systematic risk of a security is defined to be the covariance of the asset return with the return of the market portfolio divided by the variance of the market portfolio return. Unfortunately, the empirical tests of the CAPM have not produced significant support to the model (for a review see e.g. Huang and Litzenberger, 1988; Copeland and Weston, 1988). The most serious doubts on the testability of the CAPM were presented by Roll (1977). He suggests that the CAPM is not testable unless the exact composition of the true market portfolio is known.

Because of the restrictive assumptions and the problems of testing the traditional CAPM, several other equilibrium models and extensions of the CAPM have also been presented in the literature (see e.g. Fama, 1971; Black, 1972; Merton, 1973; Kraus and Litzenberger, 1976). However, the most frequently tested equilibrium model in recent literature is the model based on the theory of arbitrage pricing formulated by Ross (1976). The arbitrage pricing model (APM) is based on a similar intuition as the CAPM, but the APM is based on less restrictive assumptions than even the multifactor form of the CAPM-(see e.g. Alexander and Sharpe, 1988, Lee *et al.* 1988, pp. 203–19). The APM assumes that the expected return of a security is a linear function, not of one, but of a set of systematic risk components, i.e. sensitivities to factors that are common to all securities. Even though the empirical tests of the APM on US data have generally supported the model (see e.g. Roll and Ross, 1980; Chen, 1983; Cho, 1984; Cho *et al.*, 1984; Conway and Reinganum, 1988), there is a paucity of research evaluating the validity of the APM for non-US stock

0960–3107

markets. The few studies of European markets include the UK by Diacogiannis (1986) and Abeysekera and Mahajan (1988), Spain by Rubio (1988), Sweden by Östermark (1989), and Finland by Yli-Olli and Virtanen (1989), Östermark (1989) and Martikainen *et al.*, (1991).

International asset pricing and portfolio diversification have reached increasing attention in the recent years (see Solnik (1988) for a review). This is because the major national markets have experienced rapid deregulation and integration. In the context of the international CAPM, a typical approach has been to study the sensitivities of individual assets to world-wide market portfolio. The international arbitrage pricing model was formulated by Solnik (1983). In their seminal paper Cho, *et al*, (1986) study the existence of international common factors in the arbitrage pricing model using inter-battery factor analysis. Their results indicate that the number of common factors between a pair of countries ranged from 1 to 5, depending upon the level of their economic integration. Moreover, Korajczyk and Viallet (1989) emphasize the importance of the institutional development of stock markets in this respect. In addition, significant relationships between different stock markets have been reported in several studies. In their early paper Makridakis and Wheelwright (1974) investigate the short-term stability of the relationships between 14 stock market indices and report that the co-movements of international stock exchanges seem to be random processes. Similar kinds of conclusions have been drawn by Hillard (1979). However, other, more recent, studies provide results that suggest a higher level of stability in international stock markets co-movements (see e.g. Philippatos *et al.*, 1983; Cho and Taylor, 1987, von Furstenberg and Jeon, 1989; Eun and Shim, 1989, Grinold *et al.*, 1989; Meric and Meric, 1989). A general trend seems to be that stock prices in different countries have been tending to move more similarly in the 1980s than before.

The purpose and structure of the study

The purpose of this study is to investigate the potential existence of common factors in the APM. For this purpose, data from three stock markets, the New York, Helsinki and Stockholm Stock Exchanges, have been collected. In the first phase, the factors and the asset sensitivities to these factors of the APM are estimated using factor analysis. Then the stability of the estimated factors over time is studied applying transformation analysis. In addition, the number of priced factors is focused using regression analysis. Finally, the cross-sectional similarity of the factors across different samples is studied. The factor is interpreted as common if substantial stability in the contents of the factors is found over time as well as across different samples.

To study the similarity of factor patterns over time and across samples, a statistical method called transformation analysis is employed. This technique makes it possible (1) to analyse the stability of factor structures across different samples in the same time period or across different time periods in the same sample and (2) to describe the reason for the non-invariant part prevailing in these structures.

The remainder of this article is organized as follows. First, in Section II the background of the APM is briefly reviewed. In Section III the statistical methods used in this article, especially transformation analysis, are discussed. The empirical part of the study, Section IV, is divided into three parts. First, the data are described. Second, the stability of the factor structures in the samples collected from three stock markets is examined. Third, the cross-sectional similarity of the factor structures and the potential existence of the common factors in the three investigated countries are studied. In Section V conclusions are drawn. Finally, a detailed description of transformation analysis is given in the Appendix.

II. ARBITRAGE PRICING MODEL

The APM, based on the theory originally developed by Ross (1976), predicts that on the perfectly competitive and frictionless stock markets the stock return is a linear function of a certain number, say k, of economic factors. Thus, the APM starts with the assumption that returns on any stock, R_i, are generated by a k-factor model of the form (see e.g. Roll and Ross, 1980, pp. 1076–82)

$$R_i = E(R_i) + b_{i1}\,\delta_1 + b_{i2}\delta_2 + \dots + b_{ik}\delta_k + \varepsilon_i \qquad (1)$$

where $E(R_i)$, $i=1, 2, \dots, n$, is the expected return of the stock i, $\delta_j, j=1, 2, \dots, k$, are unobserved economic factors, b_{ij} is the sensitivity of the security i to the economic factor j and ε_i are the idiosyncratic risks of the stocks. In addition, it is assumed that $E(\delta_j)=0$ for $j=1, 2, \dots, k$, $E(\varepsilon_i)=0$ for $i=1, 2, \dots, n$, $E(\varepsilon_i\varepsilon_h)=0$ for $i\neq h$, and $E(\varepsilon_i^2)=\sigma_i^2<\infty$.

Ross (1976) has shown that if the number of stocks is sufficiently large, the following linear risk–return relationship can be written as

$$E(R_i) = \lambda_0 + \lambda_1\, b_{i1} + \lambda_2 b_{i2} + \dots + \lambda_k b_{ik} \qquad (2)$$

where λ_0 is a constant riskless rate of return (the common return on all zero-beta stocks), λ_j, $j=1, 2, \dots, k$, represents, in equilibrium, the risk premium for the jth factor.

In Equation 1 each stock i has a unique sensitivity b_{ij} to each factor δ_j but any factor δ_j has a value that is the same for all stocks. These common factors capture the systematic components of risk in Equation 1. Therefore, any δ_j affects more than one security return. If this were not the case, the effect would be incorporated in the unsystematic component of the risk, i.e. in the residual term ε_i.

To test the model described by Equation 1, there are principally two alternative approaches. The first is to specify *a priori*, on the basis of the theory, the general factors that explain pricing in the stock market (see e.g. Chen *et al.*, 1986). The second approach, which is more common and also much more problematic, is to estimate the b_{ij} and unknown factors δ_j simultaneously by factor analysis. In this case theory does not posit, *a priori*, what the exact content or even the number of relevant factors is. Thus, the number of factors to extract from the data must be made subjectively or by statistical criteria. When the systematic components of the risk, b_{ij}, have been obtained, the risk premia, λ_j, are estimated again using cross-sectional regressions.

In the factor-analytic approach there are several methodological problems. First, there is no meaning to the signs of the factors produced by factor analysis. Second, the scaling of b_{ij}'s and λ_j's is arbitrary. Third, there is no guarantee that factors are produced in a particular order when the analysis is performed on separate samples (see Elton and Gruber, 1987, pp. 336–52). In addition, it has been difficult to decide the correct number of priced factors. Dhrymes *et al.* (1985) use samples of different sizes (30, 60 and 90 stocks), and they report that the number of significant factors is an increasing function of the size of the group analysed.

III. STATISTICAL METHODS

The main statistical methods used in the study are factor analysis, regression analysis and transformation analysis. Factor analysis and regression analysis are usual techniques in business applications. Transformation analysis, on the contrary, has been applied mainly in Finnish research only. Transformation analysis was initiated by Ahmavaara (1963) to compare factor solutions between two different groups of objects, and the first empirical applications of this statistical technique exist in the area of Finnish political and sociological research. In recent years, the methodology has gained considerable attention in business applications also (see e.g. Yli-Olli, 1983; Yli-Olli and Virtanen, 1989; Martikainen, 1991). The non-existence of transformation analysis programmes in world-wide statistical packages has restricted the use of this technique to Finnish empirical research mainly. In the following, the general idea behind transformation analysis in testing the APM is sketched out. A more detailed description of this multivariate technique can be found in the Appendix.

According to the APM, the returns of a security are generated by a number of economic factors. An especially interesting aspect when testing the APM is to study whether the estimated factors are the same across different samples or over time. Let $\mathbf{L}_1$ and $\mathbf{L}_2$ be the factor matrices in the APM describing the asset sensitivities to common factors, i.e. the matrices formed by the b_{ij}'s in Equation 1, in two samples during the same time period or across two different samples in the same period. Further, assume that the economic forces deriving $\mathbf{L}_1$ and $\mathbf{L}_2$ are both orthogonal and have the same dimension, i.e. k (see Equation 1).

If there exists invariance between the two factor structures of economic forces, there exists a non-singular $k \times k$-matrix $\mathbf{T}$ such that equation $\mathbf{L}_2 = \mathbf{L}_1 \mathbf{T}_{12}$ holds. Matrix $\mathbf{T}_{12}$ is called the transformation matrix (between $\mathbf{L}_1$ and $\mathbf{L}_2$). If the equation holds exactly, it means that the contents of the economic forces generating stock returns are exactly the same in the two different samples during the same time period (or across the two different samples in the same period, depending upon the samples in question). Empirically, the equality is not obtained exactly, and a goodness-of-fit criterion for the model may be based on the residual matrix $\mathbf{E}_{12} = \mathbf{L}_1 \mathbf{T}_{12} - \mathbf{L}_2$. Non-zero elements in the residual matrix $\mathbf{E}_{12}$ signify that the empirical meaning of the variables in question has changed. In the APM, when investigating the factor structure of a given sample through two subperiods, this would mean that the behaviour of a given asset would have changed with respect to the economic forces generating stock returns. This is called abnormal transformation. The total amount of abnormal transformation is measured by the total squared residual (see Equation A5 in Appendix). In the following, the part of abnormal transformation appointed to a specific variable i, i.e. the return of stock i, is marked with t_i^2 (see Equation A7), and the abnormal transformation for factor i, i.e. underlying economic force j, with s_j^2 (see Equation A8).

Compared to prior techniques used in this context, transformation analysis has several advantages. With correlation and congruence coefficients, only the degree of similarity of two factor solutions (correlations or congruences among factor loadings, i.e. asset sensitivies to common factors) can be measured. This is also possible via transformation analysis (coefficients of coincidence t_{ii} on the main diagonal of the transformation matrix). In addition to this, a regression-type model obtains for shifting of variables from one factor to another (normal or explained transformation). In the context of APM, this means that it is possible to analyse the behaviour of individual asset returns with economic forces generating stock returns. This is revealed by non-zero off-diagonal elements, t_{ij}, in the transformation matrix and indicates interpretative changes for the factors, i.e. the underlying economic forces, in question. Finally, large elements in the residual matrix indicate abnormal or unexplained transformation between the two factor solutions and that means that the empirical content of the corresponding variables, i.e. individual asset returns, has changed. Further, this abnormal transformation can be employed to separate asset returns or to separate factors. Compared to the inter-battery factor analysis often used in this context, transformation analysis makes it possible to investigate the changes in the behaviour of individual asset returns with respect to the underlying economic factors.

IV. DATA AND EMPIRICAL RESULTS

The data

For analysis purposes, monthly stock returns from the New York, Helsinki and Stockholm Stock Exchanges for January 1977 to December 1986 are used. The selection of these three stock exchanges is based on several criteria. First, the New York Stock Exchange, being one of the leading stock markets in the world, is an apparent choice. Since the empirical tests of the APM are usually carried out with data from large stock markets, much work remains to be done on data collected from less developed capital markets. The use of data from the two other stock exchanges enables the study of international relationships between common factors in different stock markets. The Helsinki and the Stockholm Stock Exchanges, especially Helsinki, are small markets comprised of generally thinly traded stocks. The market values of the listed stocks in these exchanges at the end of the research period were about 6 and 55 Milliard USD for Helsinki and Stockholm, respectively. Using stock returns from these two closely related Scandinavian economies is a fruitful starting point in order to study the cross-country invariances in the APM. In addition, it should be noted that previous results indicate that there exist strong co-movements of stock market returns in these two countries (see Virtanen and Yli-Olli, 1987). Thus, it can be assumed that there exist common factors across these two countries.

In order to determine stock returns, first differences of individual logarithmic stock prices are used. In the US markets returns are collected from the CRSP monthly tape. In Finland and in Sweden individual asset returns are collected from a database developed by Berglund, *et al.* (1983). The empirical verification of the APM and the stability analysis require both a large sample in terms of number of securities and also a long time period. From the New York Stock Exchange, ten 30-stock samples (Samples 1–10), ranked by the size of total assets, are used. The Finnish sample (Sample 11) consists of the returns of the shares for 30 firms. The sample includes stocks quoted on the Helsinki Stock Exchange and are the most frequently traded stocks during the entire sample period. Similarly, the Swedish sample (Sample 12) consists of the 30 most frequently traded firms in the Stockholm Stock Exchange during the research period. Selecting a wider number of stocks from the two Scandinavian markets would probably have led to significant problems related to infrequent trading. In addition, note that for the stability analysis, the whole period is divided into two subperiods of equal length: the first subperiod includes years 1977–81, the second subperiod contains years 1982–86.

Time-series stability of factor patterns

The first step in the empirical analysis is to use factor analysis procedure to identify the number of factors affecting equilibrium returns. The estimation of factors can be carried out by different factor-analytic methods. In this study, the following procedure will be used. The initial factor extraction is carried out by principal component method based on the covariance matrices of stock returns; thereafter, varimax rotation is applied.

To identify the common factors, which are stable for the 1977–81 and 1982–86 subperiods, two-, three-, four-, five- and six-factor solutions for each subperiod are extracted. The selection of these factor solutions is based on intuition from the earlier results on the potential numbers of common factors in the APM. The cumulative proportions of total variance explained (of the unrotated factor patterns) for these factor solutions are presented in Table 1. The results indicate that the total explanatory power of the factors seems to be quite similar in different samples, even across countries.

To solve the problem of how many factors to retain in factor patterns, Cattell's scree test (see e.g. Green, 1978, p. 365) has typically been used in the literature, and it is also used in this paper. To save space, the numerous scree figures are not reported here. These tests showed that two to five different factors for each subperiod could be found. However, there is no absolute guarantee that the factors extracted have the same interpretation when the analysis is performed on separate subperiods. To investigate this issue, transformation analysis is used to measure the stability of factor patterns over time.

The conclusion concerning stability is based on the coefficients on the main diagonal of the transformation matrix (provided that factors in different samples are produced in the same order). The numerical values of those coefficients are very close to one if the factor structure over time is stable. Table 2 presents the transformation matrices of the two-, three- and four-factor solutions for the first sample between the two subperiods.

The results indicate that the stability of the factors in two- and three-factor solutions is very high, and three stable common factors over time in the first sample are found. This is evident from the large non-zero, close-to-one, elements in the main diagonal of the transformation matrices (elements $t_{11}=1.000$ and $t_{22}=1.000$ in the two-factor solution, and elements $t_{11}=0.958$, $t_{22}=1.000$ and $t_{33}=0.958$ in the three-factor solution). However, when studying the stability of the four-factor solutions, it can be seen that the first and third factors in the factor solutions have mixed with each other. This can be seen from the fact that there are now two clearly non-zero elements in the first and third columns and rows ($t_{11}=0.687$, $t_{13}=0.692$, $t_{31}=0.706$ and $t_{33}=-0.701$), and no close-to-one elements exist in the transformation matrix in these columns and rows.

The transformation matrices for the other 11 samples are typically similar in nature (the number of stable factors, however, varies as described in Table 7). Tables 3 and 4 present these transformation matrices for the two Scandinavian samples, i.e. samples 11 and 12. The results reveal

Table 1. *Cumulative proportions of total variance explained*

	Two-factor solutions		Three-factor solutions		Four-factor solutions		Five-factor solutions		Six-factor solutions	
	1977–81	1982–86	1977–81	1982–86	1977–81	1982–86	1977–81	1982–86	1977–81	1982–86
Sample 1	0.555	0.510	0.619	0.588	0.678	0.655	0.723	0.706	0.758	0.743
Sample 2	0.481	0.495	0.557	0.562	0.614	0.613	0.670	0.659	0.714	0.697
Sample 3	0.464	0.425	0.552	0.514	0.616	0.580	0.668	0.631	0.709	0.674
Sample 4	0.554	0.530	0.617	0.586	0.666	0.632	0.706	0.670	0.738	0.706
Sample 5	0.487	0.427	0.560	0.503	0.628	0.565	0.670	0.618	0.709	0.664
Sample 6	0.470	0.414	0.553	0.491	0.604	0.560	0.648	0.614	0.689	0.653
Sample 7	0.446	0.469	0.525	0.540	0.590	0.599	0.635	0.642	0.678	0.681
Sample 8	0.458	0.451	0.546	0.513	0.603	0.569	0.647	0.615	0.686	0.655
Sample 9	0.502	0.439	0.571	0.504	0.621	0.561	0.665	0.611	0.704	0.657
Sample 10	0.450	0.371	0.542	0.449	0.598	0.523	0.649	0.583	0.693	0.638
Sample 11 (FIN)	0.449	0.414	0.520	0.495	0.582	0.565	0.633	0.625	0.680	0.670
Sample 12 (SWE)	0.558	0.544	0.629	0.603	0.693	0.655	0.735	0.693	0.770	0.730

that at least three stable factors can be found for both samples, and four relatively stable factors can be found in the Swedish sample. An interesting observation is that in many factor patterns (e.g., the three- and four-factor solutions for Sweden and Finland) the factors are produced in different order in successive periods (some of the close-to-one elements in the transformation matrix exist in off-diagonal positions). It should be noted, however, that this does not mean any 'real' instability, the corresponding economic forces simply appearing in different positions (as different factors) in the two different factor models.

As indicated above, one of the most fruitful aspects in transformation analysis is that the so-called abnormal transformation can be employed to separate variables or to separate factors. As an illustrative example, Table 5 presents the residual matrix between subperiods 1 and 2 for the three-factor solutions in the first sample (see also Table 6 for a summary of cumulative abnormal transformation in all of the 12 samples). The residual matrix shows that typically any significant abnormal transformation in the first sample over time does not exist in this context (with a couple of exceptions, the most abnormally transformed share being Procter & Gamble, there are no large non-zero elements in the residual matrices).

The other residual matrices for the first sample showed only minor abnormal transformation. The total abnormal transformations for the two-, three-, four-, five- and six-factor solutions for all the samples are presented in Table 6. The total abnormal transformation is obtained by summing up the s_j^2's or t_i^2's from the respective residual matrices. These numbers indicate that the abnormal transformation is typically highest for the Finnish sample. This might be due to the significant structural changes in the Finnish stock market during the period, for example in the form of rapid changes in the trading volume in the Helsinki Stock Exchange.

Next, the number of stable factors over time is evaluated in all the other samples using transformation matrices separately for each sample. Table 7 presents the number of stable factors when stable factors were selected based on the criterion that an element size of at least 0.9 was required for each factor in transformation matrices. This criterion makes the elements clearly separable in the transformation matrices. In addition, it should be emphasized that the interpretation of common factors is not very sensitive to this criterion.

Thus, for each sample at least two stable common factors were detected. The following step involves examining the effect of factors on equilibrium returns (see Equation 2). In cross-sections the monthly mean return is the dependent variable and the independent variables are the factor loadings from factor analysis. The risk-free rate, i.e. the intercept term in the regression model was not restricted but estimated from the stock returns. Thus, the OLS regression coefficients can be interpreted as the estimated risk premia. It should be emphasized that in factor analysis there is no

Table 2. *Transformation matrix between the factor patterns of returns, two-, three- and four-factor solutions. Sample 1*

Two-factor solutions

		Subperiod 2	
	Factor	1	2
Sub-period 1	1	1.000	0.017
	2	−0.017	1.000

Three-factor solutions

		Subperiod 2		
	Factor	1	2	3
Sub-period 1	1	0.958	−0.016	0.287
	2	0.014	1.000	0.009
	3	−0.287	−0.004	0.958

Four-factor solutions

		Subperiod 2			
	Factor	1	2	3	4
Sub-period 1	1	0.687	−0.087	0.692	0.204
	2	0.008	0.989	0.133	−0.058
	3	0.706	0.091	−0.701	0.045
	4	−0.175	0.073	−0.105	0.976

Table 3. *Transformation matrix between the factor patterns of returns, two-, three- and four-factor solutions. Sample 11 (Finland)*

Two-factor solutions

		Subperiod 2	
	Factor	1	2
Sub-period 1	1	0.983	0.186
	2	−0.186	0.983

Three-factor solutions

		Subperiod 2		
	Factor	1	2	3
Sub-period 1	1	0.127	0.987	0.094
	2	0.992	−0.127	−0.002
	3	−0.011	−0.094	0.996

Four-factor solutions

		Subperiod 2			
	Factor	1	2	3	4
Sub-period 1	1	−0.186	0.863	0.467	−0.040
	2	0.867	0.368	−0.328	0.068
	3	0.440	−0.339	0.816	0.160
	4	−0.139	0.064	−0.091	0.984

Table 4. *Transformation matrix between the factor patterns of returns, two-, three- and four-factor solutions. Sample 12 (Sweden)*

Two-factor solutions

		Subperiod 2	
	Factor	1	2
Sub-period 1	1	0.986	−0.165
	2	0.165	0.986

Three-factor solutions

		Subperiod 2		
	Factor	1	2	3
Sub-period 1	1	0.365	−0.931	0.003
	2	0.929	−0.365	0.059
	3	−0.056	0.019	0.998

Four-factor solutions

		Subperiod 2			
	Factor	1	2	3	4
Sub-period 1	1	0.276	0.954	0.116	0.021
	2	0.174	−0.170	0.958	0.150
	3	0.945	−0.247	−0.214	−0.010
	4	−0.023	0.003	−0.150	0.988

absolute meaning to the signs of the parameters and the scaling of the factors and then also the signs of regression coefficients are arbitrary. Therefore, only the statistical significance of regression coefficients is relevant instead of their numerical values.

The summary of the results of the cross-sectional regressions when four-factor solutions were used in the tests is presented in Table 8. The results imply, for example, that from the total 12 samples in nine cases at least one risk premium, i.e. regression coefficient of an estimated factor loading (risk-free rate excluded), was found to be statistically significantly different from zero at 0.05 risk level in the first subperiod. If these results are compared with those obtained by Roll and Ross (1980) concerning the period 1962–72 on US data, the results are found to be similar. Roll and Ross reported that in 69% of their groups at least one risk premium was significant. In the present analysis the corresponding number is 70% on US firms.

When comparing the results between different countries, in the Finnish sample the APM performs relatively poorly. In the first subperiod only one factor was priced and in the second subperiod none of the Finnish factor loadings obtained a non-zero regression coefficient at 0.05 risk level, i.e. became priced with respect to equilibrium returns. In the Swedish sample two factors were priced in both periods.

In this context it is relevant to notice that, compared to transformation analysis, regression analysis finally gives the actual number of priced common factors. Transformation analysis gave the maximum number of priced common factors, i.e. all the factors the content of which remains the same in different time periods. It is possible that some very stable factors are so firm- or industry-specific that they are not common. If this is so, in cross-sectional regression they become insignificant. However, transformation analysis is necessary in testing if the contents of factors in different subperiods are the same.

Cross-sectional similarity of factor patterns

As stated in Section III, transformation analysis makes it possible to compare the factor loadings in different samples. Sample 1 is selected as a benchmark, since the factors in the APM are assumed to be generated by macroeconomic forces and, thus, it is reasonable to assume that those forces are most strongly reflected in the first sample, which represents a significant proportion of the US economy. Therefore, it is plausible to suggest that if there are common factors across samples, these factors are also represented in the factors of the first sample.

The results of the transformation matrices from the two-factor solutions between the first and the other samples are summarized in Table 9. Since the main interest is now in the number of common factors in different samples, the analysis is conducted on the entire 1977–86 period. The results imply the existence of two stable factors across different samples. For certain solutions some evidence on three stable factors

Table 5. *Residual matrix E_{12} and abnormal transformation for subperiod 2, three-factor solution. Sample 1*

Firm	Factor 1	Factor 2	Factor 3	Abnormal transformation for different stocks, t_i^2
IBM	−0.125	−0.339	−0.465	0.347
EXXON	−0.087	0.069	−0.302	0.104
GENERAL ELEC CO	−0.025	−0.025	0.210	0.046
AM. TEL & TELEG CO	−0.016	0.018	−0.015	0.001
MATSUSHITA	−0.130	−0.063	−0.153	0.044
GENERAL MTRS	−0.543	0.041	0.083	0.304
DU PONT	−0.025	−0.371	0.256	0.204
BRITISH PETE	0.153	−0.134	−0.162	0.068
MERCK & CO	0.524	0.030	0.022	0.276
PHILIP MORRIS COS	0.032	0.072	0.058	0.010
AMOCO CORP	0.199	−0.280	−0.177	0.150
MOBIL CORP	0.126	0.366	−0.032	0.151
CHEVRON	−0.091	0.057	−0.004	0.011
EASTMAN KODAK	−0.268	−0.239	−0.106	0.140
SEARS ROEB. & CO	−0.436	−0.040	0.203	0.233
COCA COLA CO	−0.019	0.288	−0.426	0.265
FORD MTR CO DEL	−0.173	−0.204	−0.096	0.081
DIGITAL EQUIP	0.244	0.250	−0.448	0.323
MINNES. MNG & MFG	0.187	0.035	−0.277	0.113
WAL MART STORES	0.113	−0.358	0.241	0.199
PROCTER & GAMBLE	**−0.339**	**−0.465**	**0.598**	**0.688**
GTE CORP	0.069	−0.302	0.303	0.188
RJR NABISCO INC	−0.025	0.210	−0.210	0.089
BRISTOL MYERS	0.018	−0.015	−0.241	0.059
STANDARD OIL CO OHIO	−0.063	−0.153	0.308	0.122
JOHNSON & JOHNSON	0.041	0.083	−0.173	0.038
AMERICAN HOME PRODS	−0.371	0.256	0.232	0.257
DOW CHEM CO	−0.134	−0.162	0.200	0.084
HEWLETT PACKARD CO	0.030	0.022	0.013	0.002
ATLANTIC RICHFIELD CO	0.072	0.058	0.214	0.005
Abnormal transformation for different factors j, s_j^2	1.386	1.357	1.905	
Total abnormal transformation				4.648

Note: $t_i^2 = \sum_{j=1}^{k} e_{ij}^2$, $\quad s_j^2 = \sum_{i=1}^{p} e_{ij}^2$.

Table 6. *Total abnormal transformation for subperiod 2*

	Two-factor solution	Three-factor solution	Four-factor solution	Five-factor solution	Six-factor solution
Sample 1	1.896	4.648	5.805	5.699	6.625
Sample 2	2.086	3.991	4.711	5.846	6.961
Sample 3	3.736	5.386	7.274	7.183	8.404
Sample 4	1.810	3.725	4.232	5.078	5.083
Sample 5	3.989	5.360	5.731	6.564	6.698
Sample 6	5.379	6.458	6.482	7.077	8.979
Sample 7	2.421	3.784	4.188	6.475	7.049
Sample 8	3.376	5.811	7.797	9.752	10.535
Sample 9	3.962	6.101	7.074	7.292	7.384
Sample 10	3.902	4.300	5.984	7.066	8.374
Sample 11	4.640	5.393	8.963	10.399	11.164
Sample 12	2.619	3.443	5.672	7.582	7.421

Table 7. *The number of stable factors in different samples over time*

Sample	1	2	3	4	5	6	7	8	9	10	11	12
Number of stable factors	3	4	5	2	2	2	4	3	2	4	3	4

Table 8. *The number of significant risk premia in cross-sectional regressions in 12 samples. Four-factor solutions ($p=0.05$). USA + Finland + Sweden*

Factors	1	2	3	4
1977–81				
Number of significant risk premia	7+1+1	3+0+1	2+0+0	0+0+0
1982–86				
Number of significant risk premia	7+0+1	4+0+1	1+0+0	0+0+0

Table 9. *Results of the transformation matrices from cross-sectional analysis (two- and three-factor solutions, Sample 1 versus other samples)*

	Sample 1			Sample 1		
	Factor 1	Factor 2		Factor 1	Factor 2	Factor 3
SAMPLE 2						
Factor 1	0.274	0.962	Factor 1	0.797	0.050	0.601
Factor 2	0.962	−0.274	Factor 2	0.221	0.903	−0.368
			Factor 3	−0.562	0.426	0.709
SAMPLE 3						
Factor 1	0.368	0.930	Factor 1	0.475	0.569	0.671
Factor 2	0.930	−0.368	Factor 2	0.505	0.448	−0.737
			Factor 3	0.720	−0.689	0.075
SAMPLE 4						
Factor 1	0.988	0.155	Factor 1	0.843	0.525	−0.121
Factor 2	−0.155	0.988	Factor 2	−0.223	0.544	0.809
			Factor 3	0.490	−0.654	0.576
SAMPLE 5						
Factor 1	0.352	0.936	Factor 1	0.370	0.871	0.324
Factor 2	0.936	−0.352	Factor 2	0.675	−0.492	0.550
			Factor 3	0.638	0.015	−0.770
SAMPLE 6						
Factor 1	0.973	0.241	Factor 1	0.483	0.874	−0.055
Factor 2	−0.250	0.975	Factor 2	0.240	−0.073	0.969
			Factor 3	0.842	−0.481	−0.245
SAMPLE 7						
Factor 1	0.315	0.949	Factor 1	0.847	0.119	0.518
Factor 2	0.949	−0.315	Factor 2	−0.037	0.985	−0.166
			Factor 3	0.530	−0.121	−0.839
SAMPLE 8						
Factor 1	0.304	0.953	Factor 1	0.540	0.842	0.018
Factor 2	0.953	−0.304	Factor 2	0.184	−0.140	0.973
			Factor 3	0.819	−0.521	−0.228
SAMPLE 9						
Factor 1	0.232	0.973	Factor 1	0.893	0.443	−0.015
Factor 2	0.973	−0.232	Factor 2	−0.038	0.109	0.993
			Factor 3	−0.448	0.887	−0.115
SAMPLE 10						
Factor 1	0.995	0.097	Factor 1	0.091	0.621	0.779
Factor 2	−0.097	0.995	Factor 2	0.991	−0.137	−0.007
			Factor 3	0.102	0.772	−0.628
SAMPLE 11						
Factor 1	1.000	−0.005	Factor 1	0.834	0.518	−0.192
Factor 2	−0.005	1.000	Factor 2	0.237	−0.022	0.971
			Factor 3	−0.499	0.855	0.141
SAMPLE 12						
Factor 1	0.997	0.072	Factor 1	0.664	0.723	−0.192
Factor 2	0.072	0.997	Factor 2	0.175	0.099	0.979
			Factor 3	0.727	−0.684	−0.061

is also detected, but generally only two stable factors are found.

An interesting observation in Table 9 is that corresponding common factors are very often produced in different order in different samples. For example, between the first and second samples, the factors in two-factor solutions have apparently changed their positions. This, however, does not mean any instability between factor patterns; the contents of the factors are about the same even if they are produced in different order. In this context, the usefulness of transformation analysis becomes particularly clear. Even if the factors are produced in different order in different samples, the common factors across samples can be found via transformation matrices.

Another important finding is that there seems to exist a high level of similarity across the first US sample and Finnish and Swedish samples in the two-factor solutions. The factors have also been produced in the same order across these three samples. Thus, there seem to exist two international common factors between these three countries.

V. SUMMARY AND CONCLUSIONS

The main purpose of this study was to find international evidence on the common factors in the arbitrage pricing model using monthly time series data of US and Scandinavian firms quoted on the New York, Helsinki and Stockholm Stock Exchanges, and as a part of this, especially to test the stability of the factor structures over time and across different samples. For this purpose, transformation analysis is applied to study if the content of the factors has remained the same in different time periods and also in different samples during the same time period.

The empirical evidence indicates that two stable common factors in different samples could be found. In addition, an interesting observation is that the factors were very often produced in different order in different samples. For this purpose, transformation analysis appears to offer a new and useful method to study the contents of factors in different samples as well as over time.

Another important finding is that there exist two common factors across the first US sample and Finnish and Swedish samples. Thus, the two common factors obtained have been international by nature. This finding is consistent with the results obtained by Cho *et al.* (1986), who reported that the number of common factors between a pair of countries ranges from 1 to 5 depending upon their level of economic integration. In addition, the effects of factors on equilibrium returns are examined to evaluate the number of priced factors in different samples and countries. The results imply that for Finland the APM performed relatively poorly and for US and Swedish data one to two priced factors are identified. This finding is not inconsistent with the earlier empirical results regarding the number of priced factors.

APPENDIX: TRANSFORMATION ANALYSIS

Transformation analysis was initiated by Ahmavaara (1963) and further developed by Ahmavaara (1966) and Mustonen (1966). This method was originally developed to compare factor solutions between two different groups of objects. In finance literature it was first exploited by Yli-Olli (1983) to measure the long-term stability of financial ratio patterns (for a more detailed description of this method, see Yli-Olli and Virtanen, 1990).

Assume p original variables $x_1, x_2, \ldots, x_p$ with mean values $\mu_1, \mu_2, \ldots, \mu_p$ and standard deviations $\sigma_1, \sigma_2, \ldots, \sigma_p$, respectively. The common factor model postulates that each x_i is linearly dependent on a few unobservable variables $f_1, f_2, \ldots, f_k$ $(k<p)$, called common factors, and an additional source of variation u_i, called a specific factor. In matrix notation the model is, thus,

$$\mathbf{x}-\mu=\mathbf{Lf}+\mathbf{u} \qquad \text{(A1)}$$

where $\mathbf{x}'=(x_1, x_2, \ldots, x_p)$, $\mu'=(\mu_1, \mu_2, \ldots, \mu_p)$ $\mathbf{L}=(l_{ij})_{p\times k}$, $\mathbf{f}'=(f_1, f_2, \ldots, f_k)$ and $\mathbf{u}'=(u_1, u_2, \ldots, u_p)$. The coefficient l_{ij} is called the factor loading of the ith variable on the jth factor. Thus, $\mathbf{L}$ is the matrix of factor loadings.

Assume now that we have two groups of objects G_1 and G_2 (two different groups of objects, or one group measured at two different times), with the same variables, both by number and content. Let $\mathbf{L}_1$ and $\mathbf{L}_2$ be the factor matrices (c.f. Equation A1) for G_1 and G_2, respectively. Further, assume that the factor models used in deriving $\mathbf{L}_1$ and $\mathbf{L}_2$ are both orthogonal and have the same dimension, $p \times k$.

If there exists invariance between the two factor structures, there exists a non-singular $k \times k$-matrix $\mathbf{T}$ such that the equation

$$\mathbf{L}_2=\mathbf{L}_1\,\mathbf{T}_{12} \qquad \text{(A2)}$$

holds. Matrix $\mathbf{T}_{12}$ is called the transformation matrix (between $\mathbf{L}_1$ and $\mathbf{L}_2$, or direction from G_1 to G_2). If Equation A2 holds exactly, it means that the factor structures in groups G_1 and G_2 are, up to a linear transformation, invariant, that is, all the variables have the same empirical meaning in different groups. Depending upon the type of the transformation matrix $\mathbf{T}_{12}$, the formation of the factors from the variables and, thereby, the interpretation of the factors is either preserved ($\mathbf{T}_{12}$ is the identity matrix $\mathbf{I}$) or changed ($\mathbf{T}_{12}$ has also non-zero off-diagonal elements). If Equation A2 does not hold, a goodness-of-fit criterion for the model described by the equation may be based on the residual matrix

$$\mathbf{E}_{12}=\mathbf{L}_1\,\mathbf{T}_{12}-\mathbf{L}_2 \qquad \text{(A3)}$$

Non-zero elements in the residual matrix $\mathbf{E}_{12}$ signify that the empirical meaning of the variables in question has changed. The main problem in transformation analysis is the estimation of the matrix $\mathbf{T}_{12}$. The estimation methods are, in general, based on the minimization of the sum of squares of the residuals e_{ij} (the elements of the residual matrix $\mathbf{E}_{12}$). This is the common method of least squares. The problem is to minimize.

$$\|\mathbf{E}_{12}\| = \|\mathbf{L}_1 \mathbf{T}_{12} - \mathbf{L}_2\| = \text{trace}\,[(\mathbf{L}_1 \mathbf{T}_{12} - \mathbf{L}_2)(\mathbf{L}_1 \mathbf{T}_{12} - \mathbf{L}_2)'] \quad (A4)$$

Depending upon additional constraints set for $\mathbf{T}_{12}$, three different estimation methods, i.e. three transformation analysis models, are obtained. If there are no constraints for $\mathbf{T}_{12}$ in minimizing Equation A4, the transformation analysis model is called the naive model. The relativistic model is obtained if the transformation analysis model is required to obey the transitivity property, $\mathbf{T}_{kl}\mathbf{T}_{lm} = \mathbf{T}_{km}$. Finally, if the transformation matrix is required to be orthogonal, the symmetric model is obtained. Of these three techniques, the symmetric transformation analysis is the most popular one. It is also applied herein.

In symmetric transformation analysis especially, abnormal transformation (the total residual) $\|\mathbf{E}_{12}\|$ may be expressed in form

$$\|\mathbf{E}_{12}\| = \sum_{i=1}^{p} \sum_{j=1}^{k} e_{ij}^2 = \sum_{i=1}^{p} t_i^2 \quad (A5)$$

or

$$\|\mathbf{E}_{12}\| = \sum_{j=1}^{k} \sum_{i=1}^{p} e_{ij}^2 = \sum_{j=1}^{k} s_j^2 \quad (A6)$$

where

$$t_i^2 = \sum_{j=1}^{k} e_{ij}^2 \quad (A7)$$

and

$$s_j^2 = \sum_{i=1}^{p} e_{ij}^2 \quad (A8)$$

which represent the portions of abnormal transformation due to the ith variable x_i and jth factor f_j, respectively.

ACKNOWLEDGEMENTS

We are grateful to the participants of the 7th Jyväskylä Summer School of Economics and the International TIMS/SOBRAPO-Meeting in Rio de Janeiro in 1991 for their useful comments and encouragement. In addition, the constructive comments by an anonymous referee are gratefully acknowledged.

REFERENCES

Abeysekera, S. P. and Mahajan, A. (1987) A test of the APT using UK stocks, *Journal of Business Finance and Accounting*, **14**, 377–91.

Ahmavaara, Y. (1963) On the mathematical theory of transformation analysis, Publications of the Social Research Institute of Alcohol Studies No. 1.

Ahmavaara, Y. (1966). Transformation Analysis, Mimeographed notes, Department of Psychology, Ohio State University, Columbus, Ohio.

Alexander, G. J. and Sharpe, W. F. (1989) *Fundamentals of Investing*, Prentice-Hall, Englewood Cliffs, NJ.

Berglund, T., Wahlroos, B. and Grandell, L. (1983) The KOP and UNITAS indexes for the Helsinki stock exchange in the light of value weighted index, *Finnish Journal of Business Economics*, **32**, 30–41 (in Swedish).

Black, F. (1972) Capital market equilibrium with restricted borrowing, *Journal of Business*, **45**, 444–55.

Chen, N.-F. (1983) Some empirical tests of the theory of arbitrage pricing, *Journal of Finance*, **38**, 1393–414.

Chen, N.-F., Roll, R. and Ross, S. A. (1986) Economic forces and the stock market, *Journal of Business*, **59**, 383–403.

Cho, D. (1984) On testing the arbitrage pricing theory: interbattery factor analysis, *Journal of Finance* **39**, 1485–502.

Cho, D. C., Elton, E. J. and Gruber, M. J. (1984) On the robustness of the Roll and Ross APT methodology, *Journal of Financial and Quantitative Analysis*, **19**, 1–10.

Cho, D. C., Eun, C. S. and Senbet, L. W. (1986) International arbitrage pricing theory: an empirical investigation, *Journal of Finance*, **41**, 313–29.

Cho, D. C. and Taylor, W. M. (1987) The seasonal stability of the factor structure of stock returns, *Journal of Finance*, **42**, 1195–211.

Conway, D. A. and Reinganum, M. R. (1988) Stable factors in security returns: identification using cross-validation, *Journal of Business and Economic Statistics*, **6**, 1–15.

Copeland, T. E. and Weston, J. F. (1988) *Financial Theory and Corporate Policy*, Addison-Wesley, Reading MA.

Dhrymes, P. J., Friend, I. F., Gultekin, M. N. and Gultekin, N. B. (1985) New tests of the APT and their implications, *Journal of Finance*, **40**, 659–75.

Diacogiannis, G. P. (1986) Arbitrage pricing model: a critical examination of its applicability for the London stock exchange, *Journal of Business Finance and Accounting*, **12**, 489–504.

Elton, E. J. and Gruber, M. J. (1987) *Modern Portfolio Theory and Investment Analysis*, Wiley, New York.

Eun, C. S. and Shim, S. (1989) International transmission of stock market movements, *Journal of Financial and Quantitative Analysis*, **24**, 241–56.

Fama, E. F. (1971) Risk, return and equilibrium, *Journal of Political Economy*, **79**, 30–55.

von Furstenberg, G. M. and Jeon, B. N. (1989) International stock price movements: links and messages, *Brookings Papers on Economic Activity*, **8**, 125–79.

Green, P. E. (1978) *Analyzing Multivariate Data*, The Dryden Press, Hindsdale, Ill.

Grinold, R., Rudd, A. and Stefek, D. (1989) Global factors: fact or fiction, *Journal of Portfolio Management*, **15**, 79–88.

Grinold, R., Rudd, A. and Stefek, D. (1989) Global factors: fact or fiction, *Journal of Portfolio Management*, Fall, 79–88.

Hilliard, J. E. (1979) The relationship between equity indices on world exchanges, *Journal of Finance*, **34**, 103–114.

Huang, C.-f and Litzenberger, R. H. (1988) *Foundations for Financial Economics*, Elsevier, New York.

Korajczyk, R. A. and Viallet, C. J. (1989) An empirical investigation of international asset pricing, *Review of Financial Studies*, **2**, 553–85.

Kraus, A. and Litzenberger, R. (1976) Skewness preference and the value of risk assets, *Journal of Finance*, **31**, 1084–94.

Lee, C. F., Wei, K. C. J. and Bubnys, E. (1989) The APT versus the multi-factor CAPM: empirical evidence, *Quarterly Review of Economics and Business*, **29**, 6–25.

Lintner, J. (1965) The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *The Review of Economics and Statistics*, **47**, 13–37.

Makridakis, S. G. and Wheelwright, S. C. (1974) An analysis of the inter-relationships among the world stock exchanges, *Journal of Business Finance and Accounting*, **1**, 195–215.

Martikainen, T. (1991) On the significance of the economic determinants of systematic risk. Empirical evidence with Finnish data, *Applied Financial Economics*, **1**, 97–104.

Martikainen, T., Yli-Olli, P. and Gunasekaran, A. (1991) Incremental significance of pre-specified macroeconomic factors in testing the arbitrage pricing theory: empirical evidence with Finnish data, *Applied Financial Economics*, **1**, 139–47.

Meric, I. and Meric, G. (1989) Potential gains from international portfolio diversification and inter-temporal stability and seasonality in international stock market relationships, *Journal of Banking and Finance*, **13**, 627–40.

Merton, R. (1973) An intertemporal capital asset pricing model, *Econometrica*, **43**, September, 867–88.

Mossin, J. (1966) Equilibrium in a capital asset market, *Econometrica*, **34**, 768–83.

Mustonen, S. (1966) Symmetric transformation analysis, Reports from the Social Research Institute of Alcohol Studies No. 24 (in Finnish).

Philippatos, G. C., Christofi, A. and Christofi, P. (1983) The intertemporal stability of international stock market relationships: another view, *Financial Management*, Winter, 63–9.

Roll, R. (1977). A critique of the asset pricing theory's tests, Part I: on past and potential testability of the theory, *Journal of Financial Economics*, **4**, 129–76.

Roll, R. and Ross, S. A. (1980) An empirical investigation of the arbitrage pricing theory, *Journal of Finance*, **35**, 1073–103.

Ross, S. A. (1976) The arbitrage pricing theory of capital asset pricing, *Journal of Economic Theory*, **13**, 341–60.

Rubio, G. (1988) Further international evidence on asset pricing: the case of Spanish capital market, *Journal of Banking and Finance*, **12**, 221–42.

Sharpe, W. F. (1963) A simplified model for portfolio analysis, *Management Science*, **9**, 277–293.

Sharpe, W. F. (1964) Capital asset prices: a theory of market equilibrium under conditions of risk, *Journal of Finance*, **19**, 425–42.

Solnik, B. (1974) An equilibrium model of the international capital market, *Journal of Economic Theory*, **8**, 500–24.

Solnik, B. (1983) International arbitrage pricing theory, *Journal of Finance*, **38**, 449–57.

Solnik, B. (1988) *International Investments*, Addison-Wesley, Reading, MA.

Treynor, J. (1961) Towards a theory of the market value of risky assets, unpublished manuscript.

Virtanen, I. and Yli-Olli, P. (1987) Forecasting stock market prices in a thin security market, *OMEGA International Journal of Management Science*, **15**, 145–55.

Yli-Olli, P. (1983) The empirical classification of financial ratios and the stability of the classification. *Proceedings of the University of Vaasa*, Research Papers No. 95. (in Finnish)

Yli-Olli, P. and Virtanen, I. (1989). Arbitrage pricing theory and its empirical applicability for the Helsinki stock exchange. *Working Paper* 89-07. European Institute for Advanced Studies in Management. Brussels, Belgium.

Yli-Olli, P. and Virtanen I. (1990) Transformation analysis applied to long-term stability and structural invariance of financial ratio patterns: US versus Finnish firms. *American Journal of Mathematical and Management Sciences*, **10**, 73–125.

Östermark, R. (1989) Arbitrage pricing models for two Scandinavian stock exchanges, *OMEGA International Journal of Management Science*, **17**, 437–47.