

Exercises 4:

- Let Υ_k denote the k 'th cross autocovariance matrix of the jointly covariance stationary residual series ϵ_i and ϵ_j with elements

$$\gamma_{ij}(k) = \mathbb{E}(\epsilon_{i,t-k} \cdot \epsilon_{j,t}).$$

Show that $\Upsilon_k = \Upsilon'_{-k}$.

- Show that a shock to any of the components of \mathbf{x}_t in the partitioned VAR system $\Phi(L) \begin{pmatrix} \mathbf{y}_t \\ \mathbf{x}_t \end{pmatrix} = \begin{pmatrix} \boldsymbol{\epsilon}_{1,t} \\ \boldsymbol{\epsilon}_{2,t} \end{pmatrix}$ does not affect any of the components of \mathbf{y}_t , if \mathbf{x}_t is block-exogenous to \mathbf{y}_t .

Hint: It suffices actually to show that sums and products of matrices of the form $\begin{pmatrix} XO \\ YZ \end{pmatrix}$ have a block of zeros in the upper right corner too.

- The excel file behind this exercise sheet contains weekly stock index series for US (S&P500), Canada (S&P/TSX Composite Index), Mexico (IPC), and Brazil (Boseva) from January 2000 to March 2007.
 - Calculate percentual log returns as $r_t = 100 \log(S_t/S_{t-1})$ for all series and discuss their sample statistics.
 - Consider pairwise Granger causality with 2 lags using 'Quick/ Group Statistics/ Granger Causality Test'. Figure out what EViews does in order to generate the test results.
 - Setup a four-variate VAR for the returns using EViews default setting. Why is choosing $p = 2$ as a lag not fully satisfactory?
 - Accept $p = 2$ anyway and consider Granger causality using 'View/ Lag Structure / Block Exogeneity Tests'. Again figure out how EViews generates the results. Why are the results not the same as in (b)? Which output should we use?
 - Consider block exogeneity between the returns of $x = (\text{USA, Canada})$ and $y = (\text{Brazil, Mexico})$ as discussed in the lecture notes. Test cross-dependencies of the blocks (overall, feedback, contemporaneous). *Hint: You may find it helpful to fill out the template which is included with the data Excel file. You get the p value for the χ^2 distribution with the commands `1-@cchisq(value,df)` in EViews and `chidist(value,df)` in Excel.*
 - Derive generalized impulse responses and interpret the results.
 - Find the renormalized variance components $R_{ij}^{g'2}$ based upon the generalized impulse responses 1 period after the shock (period 2 in EViews).