

Exercises 3:

1. Using the calculation rules on page 2 of chapter 3, show that

$$E(XY) = E(X)E(Y) \quad \text{for} \quad \text{Cov}(X, Y) = 0.$$

Note: This is different from calculation rule (6), because $\text{Cov}(X, Y) = 0$ does not necessarily imply that X and Y are independent.

2. Prove formulas (3.20)-(3.22) of the lecture notes.
3. Show that the process S_t is a martingale if and only if

- a) $E_t(S_u - S_t) = 0$ for all $u > t$,
- b) $E_t(S_u/S_t) = 1$ for all $u > t$ (we assume here that $S_t > 0$ for all t).

Hint: S_t at the specific time point t is a random variable only at times $t' < t$, when we don't know yet what the value of S_t is. For $t' \geq t$ on the other hand, S_t is just a known constant, such that $E_{t'}(S_t) = S_t$.

4. Use data on the US consumer price index (CPI_U, quarterly observations, 1950.1 to 2000.4) given in Table F5.2 from Greene's data site.
 - a) i) Plot the series.
 ii) Make the inflation series $\pi_t = 100 \log(p_t/p_{t-1})$, where p_t is the series in levels. Make a plot of the π_t series.
 - b) Plot the autocorrelations and partial autocorrelations of the π_t series. What kind of ARMA model these suggest?
 - c) Work out some alternatives and compute AIC and BIC. What do they suggest.
 - d) Estimate your final model.
 - e) Examine the residuals (autocorrelations, normality, histogram).