Practical Econometrics for Finance and Economics

Course homepage: `www.uwasa.fi/~bepa/Econometrics2.html`

Exercises 1:

1. The goal of this exercise is to convince ourselves, that using the notation of the lecture notes, $s_u^2 = \hat{u}'\hat{u}/(n-k-1)$ is an unbiased estimator of the error variance σ_u^2 in the linear regression model $y = X\beta + u$.

   a) Show that the vector of OLS-residuals may be written as $\hat{u} = Mu$, where $M = I_n - X(X'X)^{-1}X'$ and I_n denotes the identity matrix of size n.

   b) What is the rank of $X(X'X)^{-1}X'$? *Hint:* rank(A) = rank(A') *(= rank(A^{-1}) if A^{-1} exists), and rank$(AB) = k$ if both rank$(A) = k$ and rank$(B) = k$.*

   c) Show that M is symmetric and idempotent, that is, $M' = M$ and $M^2 = M$. *Hint:* $(AB)' = B'A'$ *and* $(A^{-1})' = (A')^{-1}$.

   d) Show that trace$(M) = n - (k+1)$ (trace = sum of diagonal elements). *Hint:* trace(A) = rank(A) *for idempotent matrices* A.

   e) Show that the variance covariance matrix of the residuals $\hat{u}$ is $E(\hat{u}\hat{u}') = M\sigma_u^2$. *Hint: Apply your results from (a) and (c).*

   f) Show that $E(\hat{u}'\hat{u}) = (n - (k+1))\sigma_u^2$. *Hint: Since* $\hat{u}'\hat{u}$ *is a scalar,* $E(\hat{u}'\hat{u}) = \text{trace}E(\hat{u}'\hat{u})$. *Furthermore,* E *is a linear operator, such that* $\text{trace}E(A) = E(\text{trace}A)$. *Finally,* $\text{trace}(AB) = \text{trace}(BA)$, *such that you can use your results from (d) and (e).*

   g) Show that $E(s_u^2) = \sigma_u^2$.

2. a) Download the data for Example 1.5 from the internet and import it into EViews. The location of the data is given in the lecture notes.

   b) Reproduce the analysis of Example 1.5.

   c) Apply the reparametrization method in order to confirm the value of the t-statistic given in the EViews output for (b) by pen and paper calculations.

   d) Estimate the restricted model

   $$\log(Y/L) = \beta_0 + \beta_1 \log(K/L) + u$$

   in order to confirm the F-test and the Wald test statistics given in the EViews output for (b) by pen and paper calculations.

   e) Calculate also the value of the Lagrange Multiplier test and the Likelihood Ratio test for the same restriction.