# 3. Univariate Time Series Models

## 3.1 Stationary Processes and Mean Reversion

Definition 3.1: A time series $y_t$, $t = 1, \ldots, T$ is called (covariance) stationary if

$$
\begin{aligned}
\mathbb{E}[y_t] &= \mu, \text{ for all } t \\
\mathbb{C}\text{ov}[y_t, y_{t-k}] &= \gamma_k, \text{ for all } t \qquad (1)\\
\mathbb{V}\text{ar}[y_t] &= \gamma_0 \ (< \infty), \text{ for all } t
\end{aligned}
$$

Any series that are not stationary are said to be nonstationary.

Stationary time series are mean-reverting, because the finite variance guarantees that the process can never drift too far from its mean.

The practical relevance for a trader is that assets with stationary price series may be profitably traded by short selling when its price is above the mean and buying back when the price is below the mean. This is known as range trading. Unfortunately, by far the most price series must be considered nonstationary.

When deriving properties of time series processes, we shall frequently exploit the following calculation rules, where small and capital letters denote constants and random variables, respectively.

Expected Value:

(2) $E(aX+b) = aE(X)+b$

(3) $E(X+Y) = E(X)+E(Y)$

(4) $E(XY) = E(X)E(Y)$ for $X, Y$ indep.

Variance:

(5) $V(X) = E(X^2) - E(X)^2$

(6) $V(aX+b) = a^2V(X)$

(7) $V(X+Y) = V(X)+V(Y)+2\mathbb{C}\text{ov}(X,Y)$

Covariance:

(8) $\mathbb{C}\text{ov}(X,Y) = E(XY) - E(X)E(Y)$

$\Rightarrow \mathbb{C}\text{ov}(X,Y) = \mathbb{C}\text{ov}(Y,X)$, $\mathbb{C}\text{ov}(X,X) = V(X)$
and for independent $X, Y$ : $\mathbb{C}\text{ov}(X,Y) = 0$

(9) $\mathbb{C}\text{ov}(aX+b, cY+d) = ac\,\mathbb{C}\text{ov}(X,Y)$

(10) $\mathbb{C}\text{ov}(X+Y, Z) = \mathbb{C}\text{ov}(X,Z)+\mathbb{C}\text{ov}(Y,Z)$

Stationarity implies that the covariance and correlation of observations taken $k$ periods apart are only a function of the lag $k$, but not of the time point $t$.

Autocovariance Function

(11)

$$\gamma_k = \mathbb{C}\text{ov}[y_t, y_{t-k}] = E[(y_t - \mu)(y_{t-k} - \mu)]$$

$k = 0, 1, 2, \ldots$.

Variance: $\gamma_0 = \mathbb{V}\text{ar}[y_t]$.

Autocorrelation function

(12) $$\rho_k = \frac{\gamma_k}{\gamma_0}.$$

Autocovariances and autocorrelations of covariance stationary processes are symmetric. That is, $\gamma_k = \gamma_{-k}$ and $\rho_k = \rho_{-k}$ (exercise).

The Wold Theorem, to which we return later, assures that all covariance stationary processes can be built using using white noise as building blocks. This is defined as follows.

Definition 3.2:
The time series $u_t$ is a white noise process if

$$\begin{aligned} \mathbb{E}[u_t] &= \mu, \text{ for all } t \\ \mathbb{C}\text{ov}[u_t, u_s] &= 0, \text{ for all } t \neq s \\ \mathbb{V}\text{ar}[u_t] &= \sigma_u^2 < \infty, \text{ for all } t. \end{aligned} \tag{13}$$

We denote $u_t \sim \mathtt{WN}(\mu, \sigma_u^2)$.

Remark 3.2: Usually it is assumed in (13) that $\mu = 0$.

Remark 3.3: A WN-process is obviously stationary.

As an example consider the so called first order autoregressive model, defined below:

AR(1)-process:

$$y_t = \phi_0 + \phi_1 y_{t-1} + u_t \tag{14}$$

with $u_t \sim \mathtt{WN}(0, \sigma^2)$ and $|\phi_1| < 1$.

Note that the AR(1) process reduces to white noise in the special case that $\phi_1 = 0$.

We shall later demonstrate how the AR(1) process may be written as a linear combination of white noise for arbitrary $|\phi_1| < 1$ and show that this condition is necessary and sufficient for the process to be stationary.

For now we assume that the process is stationary, such that among others

$$E(y_t) = E(y_{t-1}) \quad \text{and} \quad V(y_t) = V(y_{t-1})$$

and use this to derive the expectation and variance of the process together with its first order autocorrelation coefficient.

Taking unconditional expectation and variance of (14) yields

$$(15) \qquad E(y_t) = \frac{\phi_0}{1-\phi_1}, \qquad V(y_t) = \frac{\sigma^2}{1-\phi_1^2}.$$

In deriving the first order autocorrelation coefficient $\rho_1$, note first that we may assume without loss of generality that $\phi_0 = 0$, since shifting the series by a constant has no impact upon variances and covariances. Then $E(y_t) = E(y_{t-1}) = 0$, such that by independence of $u_t$ and $y_{t-1}$:

$$\begin{aligned}\gamma_1 &= \mathbb{C}\text{ov}(y_t, y_{t-1}) = E[(\phi_1 y_{t-1} + u_t) \cdot y_{t-1}] \\ &= \phi_1 E(y_{t-1}^2) + E(u_t) \cdot E(y_{t-1}) \\ &= \phi_1 V(y_{t-1}) = \phi_1 \gamma_0.\end{aligned}$$

Hence

$$(16) \qquad \rho_1 = \gamma_1/\gamma_0 = \phi_1.$$

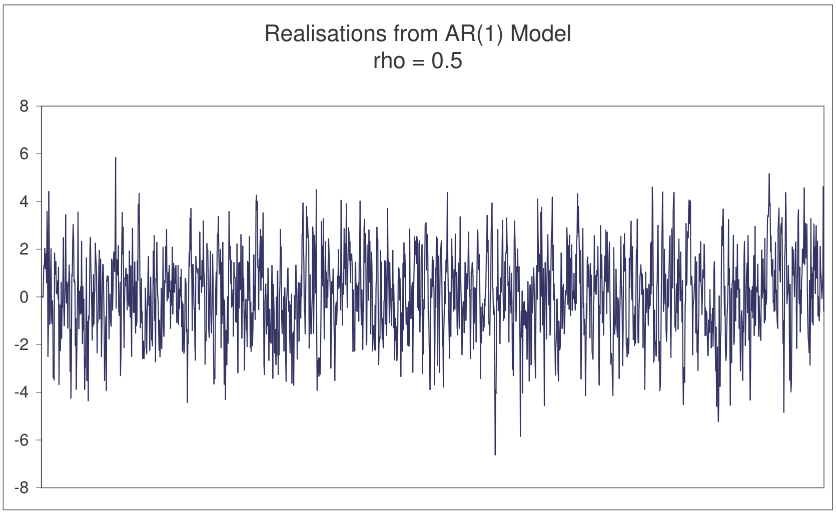
That is, the first order autocorrelation $\rho_1$ is given by the lag coefficient $\phi_1$.

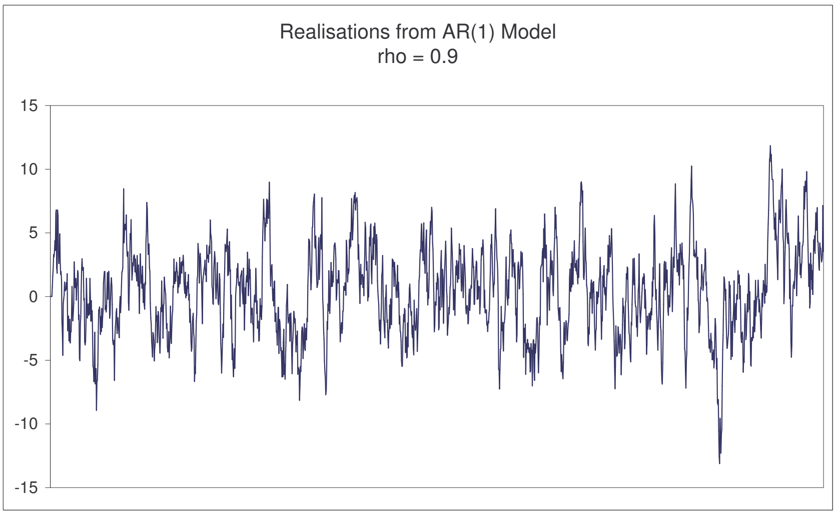
As the following slide demonstrates, mean reversion is the faster the lower the autocovariances are.

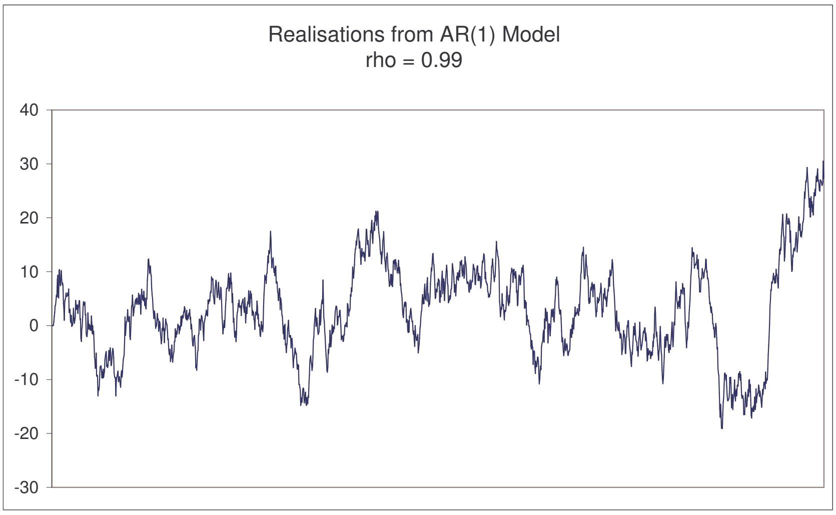
The fastest reverting process of all is the white noise process, which may be interpreted as an AR(1) process with $\phi_1 = 0$.

The larger the the autocovariance becomes, the longer it takes to revert to the mean and the further drifts the series away from its mean despite identical innovations $u_t$.

In the limit when $\phi_1 = 1$, the series is no longer stationary and there is no mean reversion.


Realisations from AR(1) Model
rho = 0.5
8
6
4
2
0
-2
-4
-6
-8


Realisations from AR(1) Model
rho = 0.9
15
10
5
0
-5
-10
-15


Realisations from AR(1) Model
rho = 0.99
40
30
20
10
0
-10
-20
-30

The AR(1) process may be generalized to a pth order autoregressive model by adding further lags as follows.

AR(p)-process:

(17)

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + u_t$$

where $u_t$ is again white noise with $\mu = 0$.

The AR(p)-process is stationary if the roots of the characteristic equation

$$m^p - \phi_1 m^{p-1} - \ldots - \phi_p = 0 \quad (18)$$

are inside the unit circle (have modulus less than one).

Example: The characteristic equation for the AR(1)-process is $m - \phi_1 = 0$ with root $m = \phi_1$, such that the unit root condition $|m| < 1$ is identical to the earlier stated stationarity condition $|\phi_1| < 1$ for AR(1)-processes.

The moving average model of order q is

MA(q)-process:

(19) $y_t = c + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \ldots + \theta_q u_{t-q} + u_t$

where again $u_t \sim WN(0, \sigma^2)$.

Its first and second moments are

(20) $E(y_t) = c$

(21) $V(y_t) = (1 + \theta_1^2 + \theta_2^2 + \ldots + \theta_q^2)\sigma^2$

(22) $\gamma_k = (\theta_k + \theta_{k+1}\theta_1 + \ldots + \theta_q\theta_{q-k})\sigma^2$

for $k \leq q$ and 0 otherwise.

This implies that in contrast to AR(p) models, MA(q) models are always covariance stationary without any restrictions on their parameters.

## Autoregressive Moving Average (ARMA) model

A covariance stationary process is an ARMA($p$,$q$) process of autoregressive order $p$ and moving average order $q$ if it can be written as

$$y_t = \phi_0 + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + u_t + \theta_1 u_{t-1} + \cdots + \theta_q u_{t-q} \tag{23}$$

For this process to be stationary the number of moving average coefficients $q$ must be finite and the roots of the same characteristic equation as for the AR(p) process,

$$m^p - \phi_1 m^{p-1} - \ldots - \phi_p = 0$$

must all lie inside the unit circle.

EViews displays the roots of the characteristic equation in the 'ARMA Diagnostics View', which is available from the output of any regression including AR or MA terms by selecting 'ARMA Structure' under 'Views'.

Example: (Alexander, PFE, Example II.5.1)

Is the ARMA(2,1) process below stationary?

$$y_t = 0.03 + 0.75y_{t-1} - 0.25y_{t-2} + u_t + 0.5u_{t-1} \tag{24}$$

The characteristic equation is

$$m^2 - 0.75m + 0.25 = 0$$

with roots

$$\begin{aligned} m &= \frac{0.75 \pm \sqrt{0.75^2 - 4 \cdot 0.25}}{2} \\ &= \frac{0.75 \pm i\sqrt{0.4375}}{2} \\ &= 0.375 \pm 0.3307i, \end{aligned}$$

where $i = \sqrt{-1}$ (the imaginary number).

The modulus of both these roots is

$$\sqrt{0.375^2 + 0.3307^2} = 0.5,$$

which is less than one in absolute value. The process is therefore stationary.

## Inversion and the Lag Operator

ARMA models may be succinctly expressed using the lag operator $L$:

$$Ly_t = y_{t-1},\ L^2 y_t = y_{t-2}, \ldots,\ L^p y_t = y_{t-p}$$

In terms of lag-polynomials

$$\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p \tag{25}$$

$$\theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q \tag{26}$$

the ARMA($p$,$q$) in (23) can be written shortly as

$$\phi(L) y_t = \phi_0 + \theta(L) u_t \tag{27}$$

or

$$y_t = \mu + \frac{\theta(L)}{\phi(L)} u_t, \tag{28}$$

where

$$\mu = \mathbb{E}[y_t] = \frac{\phi_0}{1 - \phi_1 - \cdots - \phi_p}. \tag{29}$$

It turns out that the stationarity condition for an ARMA process, that all roots of the characteristic equation be inside the unit circle, may be equivalently rephrased as the requirement that all roots of the polynomial

$$\phi(L) = 0 \tag{30}$$

are *outside* the unit circle (have modulus larger than one).

Example: (continued).

The roots of the lag polynomial

$$\Phi(L) = 1 - 0.75L + 0.25L^2 = 0$$

are

$$L = \frac{3 \pm \sqrt{3^2 - 4 \cdot 4}}{2} = 1.5 \pm \frac{\sqrt{7}}{2} i$$

with modulus

$$\sqrt{1.5^2 + 1.75} = 2 > 1.$$

Hence the process is stationary.

## Wold Decomposition

Theorem 3.1: (Wold) Any covariance stationary process $y_t$, $t = \ldots, -2, -1, 0, 1, 2, \ldots$ can be written as an infinite order MA-process (31)

$$y_t = \mu + u_t + a_1 u_{t-1} + \ldots = \mu + \sum_{h=0}^{\infty} a_h u_{t-h},$$

where $a_0 = 1$, $u_t \sim \mathtt{WN}(0, \sigma_u^2)$, and $\sum_{h=0}^{\infty} a_h^2 < \infty$.

In terms of the lag polynomial

$$a(L) = a_0 + a_1 L + a_2 L^2 + \cdots$$

equation (31) can be written in short as

(32) $$y_t = \mu + a(L) u_t.$$

For example, a stationary AR(p) process

$$\phi(L)y_t = \phi_0 + u_t \tag{33}$$

can be equivalently represented as an infinite order MA process of the form

$$y_t = \phi(L)^{-1}(\phi_0 + u_t), \tag{34}$$

where $\phi(L)^{-1}$ is an inifinite series in $L$.

<u>Example:</u> Inversion of an AR(1)-process
The lag polynomial of the AR(1)-process is $\phi(L) = 1 - \phi_1 L$ with inverse

$$\phi(L)^{-1} = \sum_{i=0}^{\infty} \phi_1^i L^i \text{ for } |\phi_1| < 1. \tag{35}$$

Hence,

$$\begin{aligned} y_t &= \sum_{i=0}^{\infty} \phi_1^i L^i(\phi_0 + u_t) \\ &= \frac{\phi_0}{1-\phi_1} + \sum_{i=0}^{\infty} \phi_1^i u_{t-i}. \end{aligned} \tag{36}$$

As a byproduct we have now proven that the AR(1) process is indeed stationary for $|\phi_1| < 1$, because then it may be represented as an MA process, which is always stationary.

Similiarly, a moving average process

$$y_t = c + \theta(L)u_t \tag{37}$$

may be written as an AR process of infinite order

$$\theta(L)^{-1}y_t = \theta(L)^{-1}c + u_t, \tag{38}$$

provided that the roots of $\theta(L) = 0$ lie outside the unit circle, which is equivalent to the roots of the characteristiq equation

$$m^q + \theta_1 m^{q-1} + \ldots + \theta_q = 0$$

being inside the unit circle. In that case the MA-process is called invertible. The same condition applies for invertibility of arbitrary ARMA(p,q) processes.

Example: (II.5.1 continued) The process (24)

$y_t = 0.03 + 0.75y_{t-1} - 0.25y_{t-2} + u_t + 0.5u_{t-1}$

is invertible because

$\theta(L) = 1 + 0.5L = 0$ when $L = -2 < -1$, and

$m + 0.5 = 0$ when $m = -0.5$, such that $|m| < 1$.

## Second Moments of General ARMA Processes

We shall in the following derive a strategy to find all autocorrelations for arbitrary ARMA processes. Consider the general ARMA process (23), where we assume without loss of generality that $\phi_0 = 0$. Multiplying with $y_{t-k}$ and taking expectations yields

(39)

$$\begin{aligned} & E(y_{t-k}, y_t) \\ = \; & \phi_1 E(y_{t-k}, y_{t-1}) + \cdots + \phi_p E(y_{t-k}, y_{t-p}) \\ & + E(y_{t-k}, u_t) + \theta_1 E(y_{t-k}, u_{t-1}) + \cdots + \theta_q E(y_{t-k}, u_{t-q}). \end{aligned}$$

This implies for $k > q$:

$$\gamma_k = \phi_1 \gamma_{k-1} + \ldots + \phi_p \gamma_{k-p}. \tag{40}$$

Dividing by the variance yields the so called Yule Walker equations, from which we may recursively determine the autocorrelations of any stationary ARMA process for $k > q$:

$$\rho_k = \phi_1 \rho_{k-1} + \ldots + \phi_p \rho_{k-p}. \tag{41}$$

Example: Autocorrelation function for AR(1)

We know from equation (16) that the first order autocorrelation of the AR(1) process

$$y_t = \phi_0 + \phi_1 y_{t-1} + u_t$$

is $\rho_1 = \phi_1$.

Hence, by recursively applying the Yule Walker equations with $p = 1$ we obtain

$$\rho_k = \phi_1^k, \tag{42}$$

such that the autocorrelation function levels off for increasing $k$, since $|\phi_1| < 1$.

For the general ARMA(p,q) process (23), we obtain for $k = 0, 1, \ldots, q$ from (39) a set of $q + 1$ linear equations, which we may solve to obtain $\gamma_0, \gamma_1, \ldots, \gamma_q$, and the autocorrelations $\rho_1, \ldots, \rho_q$ from dividing those by the variance $\gamma_0$.

Example: ACF for ARMA(1,1)

Consider the ARMA(1,1) model

$$y_t = \phi y_{t-1} + u_t + \theta u_{t-1}. \tag{43}$$

The equations (39) read for $k=0$ and $k=1$:

$$\gamma_0 = \phi\gamma_1 + \sigma^2 + \theta(\theta+\phi)\sigma^2, \tag{44}$$

$$\gamma_1 = \phi\gamma_0 + \theta\sigma^2. \tag{45}$$

In matrix form:

$$\begin{pmatrix} 1 & -\phi \\ -\phi & 1 \end{pmatrix} \begin{pmatrix} \gamma_0 \\ \gamma_1 \end{pmatrix} = \begin{pmatrix} (1+\theta(\theta+\phi))\sigma^2 \\ \theta\sigma^2 \end{pmatrix}, \tag{46}$$

such that

$$\begin{pmatrix} \gamma_0 \\ \gamma_1 \end{pmatrix} = \frac{1}{1-\phi^2} \begin{pmatrix} 1 & \phi \\ \phi & 1 \end{pmatrix} \begin{pmatrix} (1+\theta(\theta+\phi))\sigma^2 \\ \theta\sigma^2 \end{pmatrix}, \tag{47}$$

where we have used that

$$\begin{pmatrix} 1 & -\phi \\ -\phi & 1 \end{pmatrix}^{-1} = \frac{1}{1-\phi^2} \begin{pmatrix} 1 & \phi \\ \phi & 1 \end{pmatrix}. \tag{48}$$

The variance of the ARMA(1,1) process is therefore

$$\gamma_0 = \frac{1 + 2\theta\phi + \theta^2}{1 - \phi^2}\sigma^2. \tag{49}$$

Its first order covariance is

$$\gamma_1 = \frac{\phi + \theta^2\phi + \theta\phi^2 + \theta}{1 - \phi^2}\sigma^2, \tag{50}$$

and its first order autocorrelation is

$$\rho_1 = \frac{(1 + \phi\theta)(\phi + \theta)}{1 + 2\theta\phi + \theta^2}. \tag{51}$$

Higher order autocorrelations may be obtained by applying the Yule Walker equations

$$\rho_k = \phi\rho_{k-1}.$$

Note that these results hold also for ARMA(1,1) processes with $\phi_0 \neq 0$, since variance and covariances are unaffected by additive constants.

## Response to Shocks

When writing an ARMA(p,q)-process in MA($\infty$) form, that is,

$$y_t = \mu + \psi(L)u_t, \tag{52}$$

then the coefficients $\psi_k$ of the lag polynomial

$$\psi(L) = 1 + \psi_1 L + \psi_2 L^2 + \dots \tag{53}$$

measure the impact of a unit shock at time $t$ on the process at time $t+k$. The coefficient $\psi_k$ as a function of $k = 1, 2, \dots$ is therefore called the impulse response function of the process.

In order to find its values, recall from (28) that $\psi(L) = \theta(L)/\phi(L)$, such that

$$\begin{aligned} (54) \qquad & 1 + \theta_1 L + \ldots + \theta_q L^q \\ =&(1 - \phi_1 L - \ldots - \phi_p L^p) \\ & \times (1 + \psi_1 L + \psi_2 L^2 + \ldots) \end{aligned}$$

Factoring out the right hand side yields

$$\begin{aligned} (55) \quad & 1 + \theta_1 L + \ldots + \theta_q L^q \\ =&1 + (\psi_1 - \phi_1)L + (\psi_2 - \phi_1\psi_1 - \phi_2)L^2 \\ & + (\psi_3 - \phi_1\psi_2 - \phi_2\psi_1 - \phi_3)L^3 + \ldots \end{aligned}$$

such that by equating coefficients:

$$\begin{aligned} (56) \qquad \psi_1 =& \theta_1 + \phi_1 \\ \psi_2 =& \theta_2 + \phi_1\psi_1 + \phi_2 \\ \psi_3 =& \theta_3 + \phi_1\psi_2 + \phi_2\psi_1 + \phi_3 \\ & \ldots \end{aligned}$$

Example: (Alexander, PFE, Example II.5.2)

Consider again the stationary and invertible ARMA(2,1) process (24)

$$y_t = 0.03 + 0.75y_{t-1} - 0.25y_{t-2} + u_t + 0.5u_{t-1}.$$

The impulse response function is given by

$$\begin{aligned}\psi_1 =& 0.5 + 0.75 = 1.25,\\ \psi_2 =& 0.75 \cdot 1.25 - 0.25 = 0.6875\\ \psi_3 =& 0.75 \cdot 0.6875 - 0.25 \cdot 1.25 = 0.203125\\ \psi_4 =& 0.75 \cdot 0.203125 - 0.25 \cdot 0.6875 = -0.0195\\ & \ldots\end{aligned}$$

EViews displays the impulse response function under View/ARMA Structure. . . /Impulse Response.

## Forecasting with ARMA models

With estimated parameters $\widehat{\mu}$, $\widehat{\phi}_1, \ldots, \widehat{\phi}_p$ and $\widehat{\theta}_1, \ldots, \widehat{\theta}_q$, the optimal $s$-step-ahead forecast $\widehat{y}_{T+s}$ for an ARMA(p,q)-process at time $T$ is

(57)

$$\widehat{y}_{T+s} = \widehat{\mu} + \sum_{i=1}^{p} \widehat{\phi}_i(\widehat{y}_{T+s-i} - \widehat{\mu}) + \sum_{j=1}^{q} \widehat{\theta}_j u_{T+s-j},$$

where $u_{T+s-j}$ is set to zero for all future innovations, that is, whenever $s > j$.

These are easiest generated iteratively:

$$\widehat{y}_{T+1} = \widehat{\mu} + \sum_{i=1}^{p} \widehat{\phi}_i(y_{T+1-i} - \widehat{\mu}) + \sum_{j=1}^{q} \widehat{\theta}_j u_{T+1-j},$$

$$\widehat{y}_{T+2} = \widehat{\mu} + \sum_{i=1}^{p} \widehat{\phi}_i(\widehat{y}_{T+2-i} - \widehat{\mu}) + \sum_{j=2}^{q} \widehat{\theta}_j u_{T+2-j},$$

and so on.

These are called conditional forecasts, because they depend upon the recent realizations of $y_t$ and $u_t$, and are available from EViews (together with confidence intervals) from the 'Forecast' button.

## Unconditional Forecasting

The unconditional (long-run) forecast for an ARMA(p,q)-process is given by the estimated mean

$$\hat{\mu} = \frac{\hat{\phi}_0}{1 - \hat{\phi}_1 - \cdots - \hat{\phi}_p}. \tag{58}$$

$y_t$ is normally distributed with mean $\mu$ and variance $\gamma_0$, such that the usual confidence intervals for normally distributed random variables apply. In particular, a 95% confidence interval for $y_t$ is

$$CI_{95\%} = \left[\hat{\mu} \pm 1.96 \cdot \sqrt{\hat{\gamma}_0}\right]. \tag{59}$$

This is of practical interest when the series $y_t$ represents an asset which can be traded and one wishes to identify moments of over- or underpricing.

## Example.

Estimating an ARMA(1,1) model on the spread in Figure II.5.3 of Alexanders book (PFE) yields the output below:

Dependent Variable: SPREAD
Method: Least Squares
Date: 01/18/13 Time: 13:35
Sample (adjusted): 9/21/2005 6/08/2007
Included observations: 399 after adjustments
Convergence achieved after 9 iterations
MA Backcast: 9/20/2005

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| C | 65.34043 | 7.517512 | 8.691763 | 0.0000 |
| AR(1) | 0.867047 | 0.047842 | 18.12306 | 0.0000 |
| MA(1) | -0.599744 | 0.073494 | -8.160486 | 0.0000 |

| | | | |
|---|---|---|---|
| R-squared | 0.207908 | Mean dependent var | 64.68170 |
| Adjusted R-squared | 0.203907 | S.D. dependent var | 55.58968 |
| S.E. of regression | 49.59935 | Akaike info criterion | 10.65332 |
| Sum squared resid | 974197.7 | Schwarz criterion | 10.68332 |
| Log likelihood | -2122.338 | Hannan-Quinn criter. | 10.66520 |
| F-statistic | 51.97092 | Durbin-Watson stat | 1.921155 |
| Prob(F-statistic) | 0.000000 | | |

| | |
|---|---|
| Inverted AR Roots | .87 |
| Inverted MA Roots | .60 |

$\widehat{\mu} = 65.34$ and applying the variance formula for ARMA(1,1) processes (49) yields

$$\widehat{\gamma}_0 = \frac{1+2\cdot 0.867\cdot(-0.6)+(-0.6)^2}{1-0.867^2}\cdot 49.599^2 = 3\,168.2$$

$$\Rightarrow \quad CI_{95\%} = 65.34 \pm 1.96\cdot\sqrt{3\,168.2} = [-44.98; 175.66]$$

## Partial Autocorrelation

The partial autocorrelation function $\phi_{kk}$ of a time series $y_t$ at lag $k$ measures the correlation of $y_t$ and $y_{t-k}$ adjusted for the effects of $y_{t-1}, \ldots, y_{t-k+1}$. It is given by the $k$'th order autocorrelation function of the residuals from regressing $y_t$ upon the first $k-1$ lags:

(60)
$$\phi_{kk} = \mathbb{C}\mathrm{orr}[y_t - \hat{y}_t, y_{t-k} - \hat{y}_{t-k}],$$

where (assuming here for simplicity $\phi_0 = 0$)

(61)
$$\hat{y}_t = \hat{\phi}_{k1} y_{t-1} + \hat{\phi}_{k2} y_{t-2} + \cdots + \hat{\phi}_{k-1,k-1} y_{t-k+1}.$$

It may also be interpreted as the coefficient $\phi_{kk}$ in the regression

(62)
$$y_t = \phi_{k1} y_{t-1} + \phi_{k2} y_{t-2} + \cdots + \phi_{kk} y_{t-k} + u_t.$$

From this it follows immediately that in an AR($p$)-process $\phi_{kk} = 0$ for $k > p$, which is a help in discriminating AR-processes of different orders with otherwise similiar autocorrelation functions.

## Estimation of acf

Autocorrelation (and partial autocorrelation) functions are estimated by their empirical counterparts

$$\widehat{\gamma}_k = \frac{1}{T-k}\sum_{t=k+1}^{T}(y_t - \bar{y})(y_{t-k} - \bar{y}), \tag{63}$$

where

$$\bar{y} = \frac{1}{T}\sum_{t=1}^{T} y_t$$

is the sample mean.

Similarly

$$r_k = \widehat{\rho}_k = \frac{\widehat{\gamma}_k}{\widehat{\gamma}_0}. \tag{64}$$

## Statistical inference

If the model is well specified, the residuals must be white noise, thas is in particular uncorrelated.

It can be shown that if $\rho_k = 0$, then $\mathbb{E}[r_k] = 0$ and asymptotically

$$\mathbb{V}\text{ar}[r_k] \approx \frac{1}{T}. \tag{65}$$

Similarly, if $\phi_{kk} = 0$ then $\mathbb{E}[\hat{\phi}_{kk}] = 0$ and asymptotically

$$\mathbb{V}\text{ar}[\hat{\rho}_{kk}] \approx \frac{1}{T}. \tag{66}$$

In both cases the asymptotic distribution is normal.

Thus, testing

$$H_0 : \rho_k = 0, \tag{67}$$

can be tested with the test statistic

$$z = \sqrt{T} r_k, \tag{68}$$

which is asymptotically $N(0, 1)$ distributed under the null hypothesis (67).

The 'Portmanteau' statistics to test the hypothesis

$$H_0 : \rho_1 = \rho_2 = \cdots = \rho_m = 0 \tag{69}$$

is due to Box and Pierce (1970)

$$Q^*(m) = T \sum_{k=1}^{m} r_k^2, \tag{70}$$

$m = 1, 2, \ldots,$ which is (asymptotically) $\chi^2_m$-distributed under the null-hypothesis that all the first autocorrelations up to order $m$ are zero.

Mostly people use Ljung and Box (1978) modification that should follow more closely the $\chi^2_m$ distribution

$$Q(m) = T(T+2) \sum_{k=1}^{m} \frac{1}{T-k} r_k^2. \tag{71}$$

The latter is provided in EViews under 'View/ Residual Diagnostics/Correlogram–Q-statistics' for residuals, and under 'View/ Correlogram' for the series themselves.

On the basis of autocovariance functions one can preliminary infer the order of an ARMA-process

Theoretically:

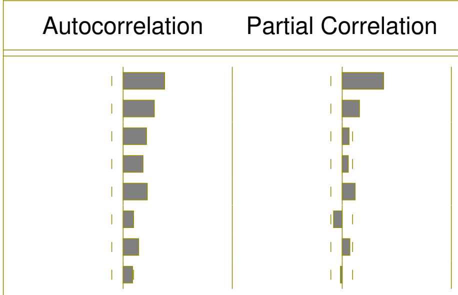
| | acf | pacf |
|---|---|---|
| AR(p) | Tails off | Cut off after p |
| MA(q) | Cut off after q | Tails off |
| ARMA(p,q) | Tails off | Tails off |

acf = autocorrelation function
pacf = partial autocorrelation function

## Example: (Figure II.5.3 continued)

Correlogram of Residuals

Date: 01/21/13 Time: 17:17
Sample: 9/20/2005 6/08/2007
Included observations: 400



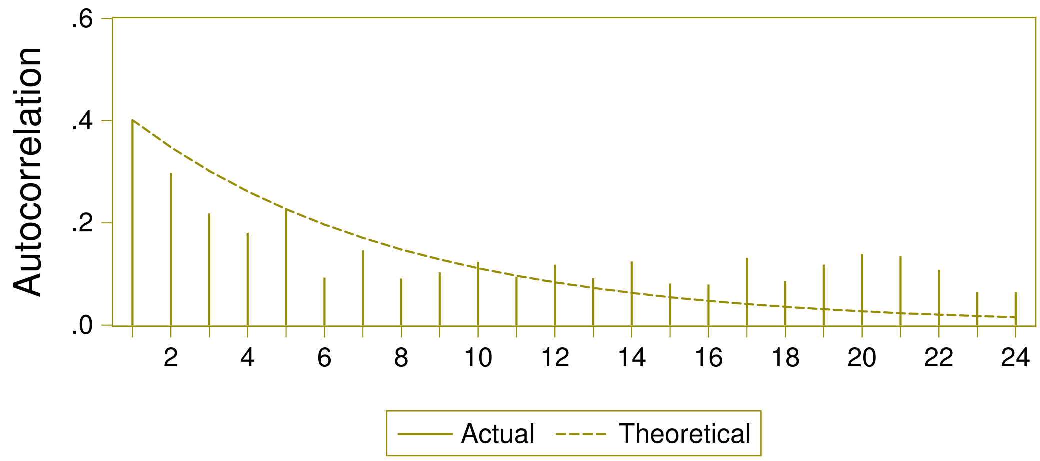

| | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | 0.401 | 0.401 | 64.783 | 0.000 |
| 2 | 0.299 | 0.165 | 100.93 | 0.000 |
| 3 | 0.222 | 0.068 | 120.83 | 0.000 |
| 4 | 0.190 | 0.059 | 135.41 | 0.000 |
| 5 | 0.235 | 0.131 | 157.81 | 0.000 |
| 6 | 0.103 | -0.074 | 162.13 | 0.000 |
| 7 | 0.150 | 0.074 | 171.38 | 0.000 |
| 8 | 0.097 | -0.011 | 175.26 | 0.000 |

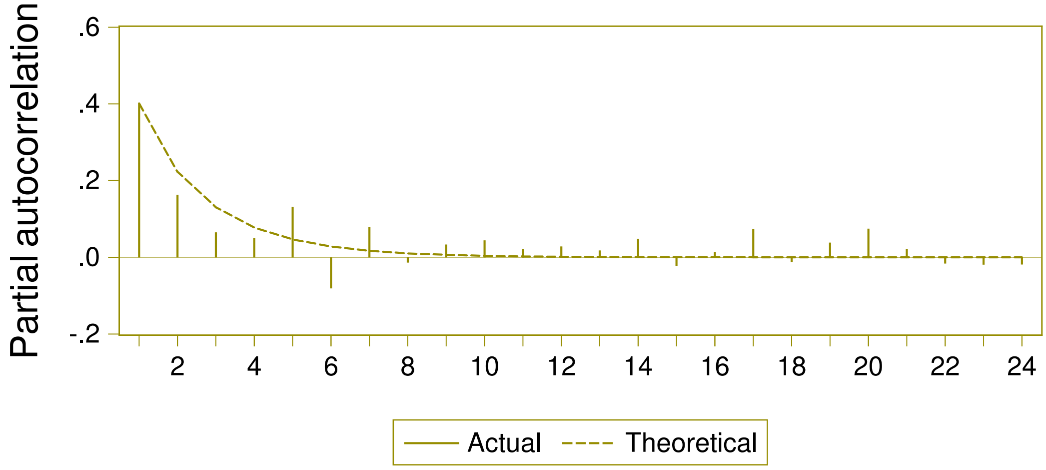
The ACF and PACF of the series suggest an ARMA$(p,q)$ model with $p$ no larger than two.

## Example: continued.

Neither a simple AR(1) model nor a simple MA(1) model are appropriate, because they have still significant autocorrelation in the residuals left (not shown).

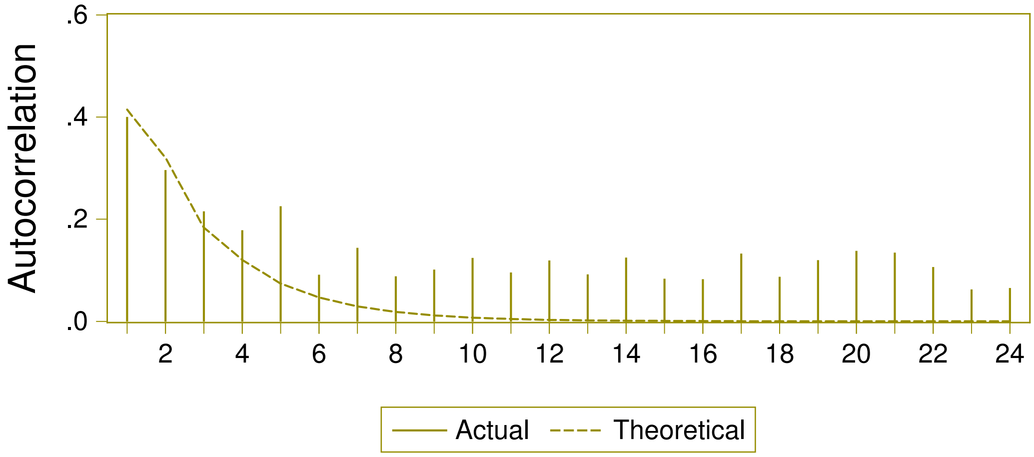
However, there is no autocorrelation left in the residuals of an ARMA(1,1) model, and it provides a reasonable fit of the ACF and PACF of $y_t$ (available in EViews under View/ARMA Structure.../Correlogram):

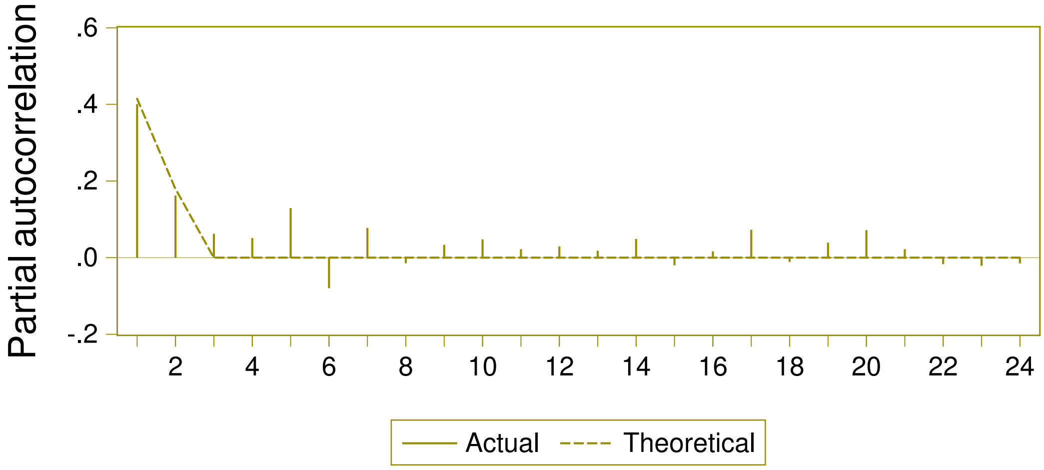


Example: continued.

Also the residuals or an AR(2) model are white noise (not shown), but the fit to the ACF and PACF of $y_t$ is considerably worse:



Example: continued.

An ARMA(2,1) has a slightly improved fit to the ACF and PACF of $y_t$ as compared to an ARMA(1,1) model (not shown), but the AR(2) term is deemed insignificant:

Dependent Variable: SPREAD
Method: Least Squares
Date: 01/21/13 Time: 18:00
Sample (adjusted): 9/22/2005 6/08/2007
Included observations: 398 after adjustments
Convergence achieved after 8 iterations
MA Backcast: 9/21/2005

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| C | 64.11695 | 8.201224 | 7.817974 | 0.0000 |
| AR(1) | 1.081246 | 0.104184 | 10.37822 | 0.0000 |
| AR(2) | -0.148846 | 0.076367 | -1.949092 | 0.0520 |
| MA(1) | -0.779206 | 0.087759 | -8.878954 | 0.0000 |

| | | | |
|---|---|---|---|
| R-squared | 0.215153 | Mean dependent var | 64.60553 |
| Adjusted R-squared | 0.209177 | S.D. dependent var | 55.63879 |
| S.E. of regression | 49.47860 | Akaike info criterion | 10.65096 |
| Sum squared resid | 964563.9 | Schwarz criterion | 10.69102 |
| Log likelihood | -2115.540 | Hannan-Quinn criter. | 10.66683 |
| F-statistic | 36.00285 | Durbin-Watson stat | 2.005595 |
| Prob(F-statistic) | 0.000000 | | |

| | | |
|---|---|---|
| Inverted AR Roots | .92 | .16 |
| Inverted MA Roots | .78 | |

We conclude that an ARMA(1,1) model is most appropriate for modelling the spread in Figure II.5.3.

Other popular tools for detecting the order of the model are Akaike's (1974) information criterion (AIC)

(72) $$\texttt{AIC}(p,q) = \log \hat{\sigma}_u^2 + 2(p+q)/T$$

or Schwarz's (1978) Bayesian information criterion (BIC)*

(75) $$\texttt{BIC}(p,q) = \log(\hat{\sigma}^2) + (p+q)\log(T)/T.$$

There are several other similar criteria, like Hannan and Quinn (HQ).

The best fitting model in terms of the chosen criterion is the one that minimizes the criterion.

The criteria may end up with different orders of the model!

*More generally these criteria are of the form

(73) $$\texttt{AIC}(m) = -2\ell(\hat{\boldsymbol{\theta}}_m) + 2m$$

and

(74) $$\texttt{BIC}(m) = -2\ell(\hat{\boldsymbol{\theta}}_m) + \log(T)m,$$

where $\hat{\boldsymbol{\theta}}_m$ is the MLE of $\boldsymbol{\theta}_m$, a parameter with $m$ components, $\ell(\hat{\theta}_m)$ is the value of the log-likelihood at $\hat{\theta}_m$.

Example: Figure II.5.3 continued.

| p | q | AIC | BIC |
|---|---|---|---|
| 0 | 0 | 10.87975 | 10.88972 |
| 1 | 0 | 10.70094 | 10.72093 |
| 0 | 1 | 10.75745 | 10.77740 |
| 2 | 0 | 10.67668 | 10.70673 |
| 1 | 1 | 10.65332 | 10.68332* |
| 0 | 2 | 10.71069 | 10.74062 |
| 1 | 2 | 10.65248 | 10.69247 |
| 2 | 1 | 10.65096 | 10.69102 |
| 2 | 2 | 10.64279* | 10.69287 |

* = minimum

The Schwarz criterion suggests ARMA(1,1), whereas AIC suggests ARMA(2,2) due to an even better fit.

However, the AR(1) term in the ARMA(2,2) model is deemed insignificant (not shown). Reestimating the model without the AR(1) term yields AIC=10.6454 and BIC=10.68547.

This is the best model if we prefer goodness of fit over parsimony. ARMA(1,1) is the best model if we prefer parsimony over goodness of fit.

## 3.2 Random Walks and Efficient Markets

According to the efficient market hypothesis all public information currently available is immediately incorporated into current prices. This means that any new information arriving tomorrow is independent of the price today. A price process with this property is called a random walk.

Formally we say that a process is a random walk (RW) if it is of the form

$$y_t = \mu + y_{t-1} + u_t, \tag{76}$$

where $\mu$ is the expected change of the process (*drift*) and $u_t \sim \text{i.i.d.}(0, \sigma_u^2)$. (i.i.d. means independent and identically distributed.)

Alternative forms of RW assume only that the $u_t$'s are independent (e.g. variances can change) or just that the $u_t$'s are uncorrelated (autocorrelations are zero).

## Martingales

It can be shown that properly discounted prices in arbitrage-free markets must be martingales, which in turn are defined in terms of so called conditional expectations

$$E_t(X) := E(X|\mathcal{I}_t), \tag{77}$$

that is, expected values conditional upon the information set $\mathcal{I}_t$ available at time $t$, which contains the realisations of all random variables known by time $t$.

$E_t(X)$ is calculated like an ordinary expectation, however replacing all random variables, the outcomes of which are known by time $t$, with these outcomes. It may be interpreted as the best forecast we can make for $X$ using all information available at time $t$.

Hence the ordinary (unconditional) expectation $E(X)$ may be thought of as a conditional expectation at time $t = -\infty$:

$$E(X) = E_{-\infty}(X). \tag{78}$$

A stochastic process $y_t$ is called a martingale if the best forecast of future realizations $y_{t+s}$ is the the realization at time $t$, that is,

$$E_t(y_{t+s}) = y_t. \tag{79}$$

The random walk (76) is a special case of a martingale if $\mu = 0$ since then

$$\begin{aligned} E_t(y_{t+s}) &= E_t(y_{t+s-1} + u_{t+s}) \\ &= E_t\left(y_t + \sum_{k=1}^{t+s} u_{t+k}\right) \\ &= y_t + E_t\left(\sum_{k=1}^{t+s} u_{t+k}\right) \\ &= y_t. \end{aligned}$$

The last equality follows from the fact that by independence of the innovations $u_t$ no information available before $t+k$ helps in forecasting $u_{t+k}$, such that

$$E_t(u_{t+k}) = E(u_{t+k}) = 0.$$

## The Law of Iterated Expectations

Calculations involving conditional expectations make frequently use of the law of iterated expectations, which states that optimal forecasts of random variables cannot be improved upon by producing optimal forecasts of future optimal forecasts, formally:

$$E_t(X) = E_t E_{t+s}(X) = E_{t+s} E_t(X), \qquad (80)$$

where $s \geq 0$, that is, the information set $\mathcal{I}_{t+s}$ at time point $t+s$ contains at least the information available at time point $t$ (formally: $\mathcal{I}_t \subset \mathcal{I}_{t+s}$). This implies in particular that

$$E(X) = E E_t(X) = E_t E(X) \text{ for all } t. \qquad (81)$$

Repeated application of (80) yields

$$E_t E_{t+1} E_{t+2} \cdots = E_t. \qquad (82)$$

## Example: Rational Expectation Hypothesis

Muth formulated in 1961 the rational expectation hypothesis, which says that in the aggregate market participants behave as if they knew the true probability distribution of an assets next periods price and use all available information at time $t$ in order to price the asset such that its current price $S_t$ equals the best available forecast of the price one period ahead,

$$S_t = E_t(S_{t+1}),$$

which is just the defining property of a martingale. The forecasting errors

$$\epsilon_{t+1} = S_{t+1} - E_t(S_{t+1})$$

are serially uncorrelated with zero mean since by the law of iterated expectations

$$E(\epsilon_{t+1}) = EE_t(\epsilon_{t+1}) = E(E_t(S_{t+1}) - E_t(S_{t+1})) = 0$$

and

$$\begin{aligned}\mathbb{C}\text{ov}(\epsilon_{t+1}, \epsilon_{t+s}) &= E(\epsilon_{t+1}\epsilon_{t+s}) = EE_{t+s-1}(\epsilon_{t+1}\epsilon_{t+s}) \\ &= E(\epsilon_{t+1}E_{t+s-1}\epsilon_{t+s}) = E(\epsilon_{t+1}\cdot 0) = 0.\end{aligned}$$

Discounted price changes are then also uncorrelated with zero mean since under rational expectations

$$S_{t+1} - S_t = S_{t+1} - E_t(S_{t+1}) = \epsilon_{t+1}.$$

## 3.3 Integrated Processes and Stocastic Trends

A times series process is said to be integrated of order 1 and denoted $I(1)$ if it is not stationary itself, but becomes stationary after applying the first difference operator

(83) $$\Delta := 1 - L,$$

that is, $\Delta y_t = y_t - y_{t-1}$ is stationary:

(84)
$$y_t \sim I(1) \quad \Leftrightarrow \quad y_t = \mu + y_{t-1} + u_t,\ u_t \sim I(0),$$

where $I(0)$ denotes stationary series.

A random walk is $I(1)$ with i.i.d. increments is a special case, but more general $I(1)$-processes could have autocorrelated differences with moving average components. If a process is $I(1)$, we call it a unit root process and say that it has a stochastic trend.

A process is integrated of order $d$ and denoted $I(d)$ if it is not stationary and $d$ is the minimum number of times it must be differenced to achieve stationarity. Integrated processes of order $d > 1$ are rare.

## ARIMA-model

As an example consider the process

$$\varphi(L)y_t = \theta(L)u_t. \tag{85}$$

If, say $d$, of the roots of the polynomial $\varphi(L)=0$ are on the unit circle and the rest outside the circle (the process has $d$ unit roots), then $\varphi(L)$ is a *nonstationary* autoregressive operator.

We can write then

$$\phi(L)(1-L)^d = \phi(L)\Delta^d = \varphi(L)$$

where $\phi(L)$ is a stationary autoregressive operator and

$$\phi(L)\Delta^d y_t = \theta(L)u_t \tag{86}$$

which is a stationary ARMA.

We say that $y_t$ follows and ARIMA($p$,$d$,$q$)-process.

A symptom of integrated processes is that the autocorrelations do not tend to die out.

## Example 3.5: Autocorrelations of Google (log) price series

Included observations: 285

| Autocorrelation | Partial AC | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| .\|******* | .\|******* | 1 | 0.972 | 0.972 | 272.32 | 0.000 |
| .\|******* | .\|. | 2 | 0.944 | -0.020 | 530.12 | 0.000 |
| .\|******* | *\|. | 3 | 0.912 | -0.098 | 771.35 | 0.000 |
| .\|****** | .\|. | 4 | 0.880 | -0.007 | 996.74 | 0.000 |
| .\|****** | .\|. | 5 | 0.850 | 0.022 | 1207.7 | 0.000 |
| .\|****** | .\|. | 6 | 0.819 | -0.023 | 1404.4 | 0.000 |
| .\|****** | .\|. | 7 | 0.790 | 0.005 | 1588.2 | 0.000 |
| .\|****** | .\|. | 8 | 0.763 | 0.013 | 1760.0 | 0.000 |
| .\|***** | .\|. | 9 | 0.737 | 0.010 | 1920.9 | 0.000 |
| .\|***** | .\|. | 10 | 0.716 | 0.072 | 2073.4 | 0.000 |
| .\|***** | .\|. | 11 | 0.698 | 0.040 | 2218.7 | 0.000 |
| .\|***** | *\|. | 12 | 0.676 | -0.088 | 2355.7 | 0.000 |

## Deterministic Trends

The stochastic trend of $I(1)$-pocesses given by (84) must be distinguished from a deterministic trend, given by

$$y_t = \alpha + \beta t + u_t, \quad u_t \sim \mathtt{i.i.d}(0, \sigma_u^2), \tag{87}$$

also called $I(0)$+trend process.

Neither (84) nor (87) is stationary, but the methods to make them stationary differ:

$I(1)$-pocesses must be differenced to become stationary, which is why they are also called difference-stationary.

A $I(0)$+trend process becomes stationary by taking the residuals from fitting a trend line by OLS to the original series, which is why it is also called trend-stationary.

Price series are generally difference-stationary, which implies that the proper transformation to render them stationary is taking first differences.

To see that taking deviations from a trend of a $I(1)$-pocesses does not make it stationary, iterate (84) to obtain

$$y_t = y_0 + \mu t + \sum_{i=1}^{t} u_i.$$

The term $\sum_{i=1}^{t} u_i$ is clearly non-stationary because its variance increases with $t$.

On the other hand, applying first differences to the trend-stationary process (87) (which should in fact be detrended by taking the residuals of an OLS trend), introduces severe negative autocorrelation:

$$\begin{aligned} y_t - y_{t-1} &= (\alpha + \beta t + u_t) - (\alpha + \beta(t-1) + u_{t-1}) \\ &= \beta + u_t - u_{t-1}, \end{aligned}$$

which is an MA(1) process with first-order autocorrelation coefficient -0.5.

## Testing for unit roots

Consider the general model

$$y_t = \alpha + \beta t + \phi y_{t-1} + u_t, \quad (88)$$

where $u_t$ is stationary.

If $|\phi| < 1$ then the (88) is trend stationary.

If $\phi = 1$ then $y_t$ is a unit root process (i.e., $I(1)$) with trend (and drift).

Thus, testing whether $y_t$ is a unit root process reduces to testing whether $\phi = 1$.

The ordinary OLS approach does not work!

One of the most popular tests is the Augmented Dickey-Fuller (ADF). Other tests are e.g. Phillips-Perron and KPSS-test.

Dickey-Fuller regression

$$\Delta y_t = \mu + \beta t + \gamma y_{t-1} + u_t, \tag{89}$$

where $\gamma = \phi - 1$.

The null hypothesis is: "$y_t \sim I(1)$", i.e.,

$$H_0 : \gamma = 0. \tag{90}$$

This is tested with the usual $t$-ratio.

$$t = \frac{\widehat{\gamma}}{\text{s.e.}(\hat{\gamma})}. \tag{91}$$

However, under the null hypothesis (90) the distribution is not the standard $t$-distribution.

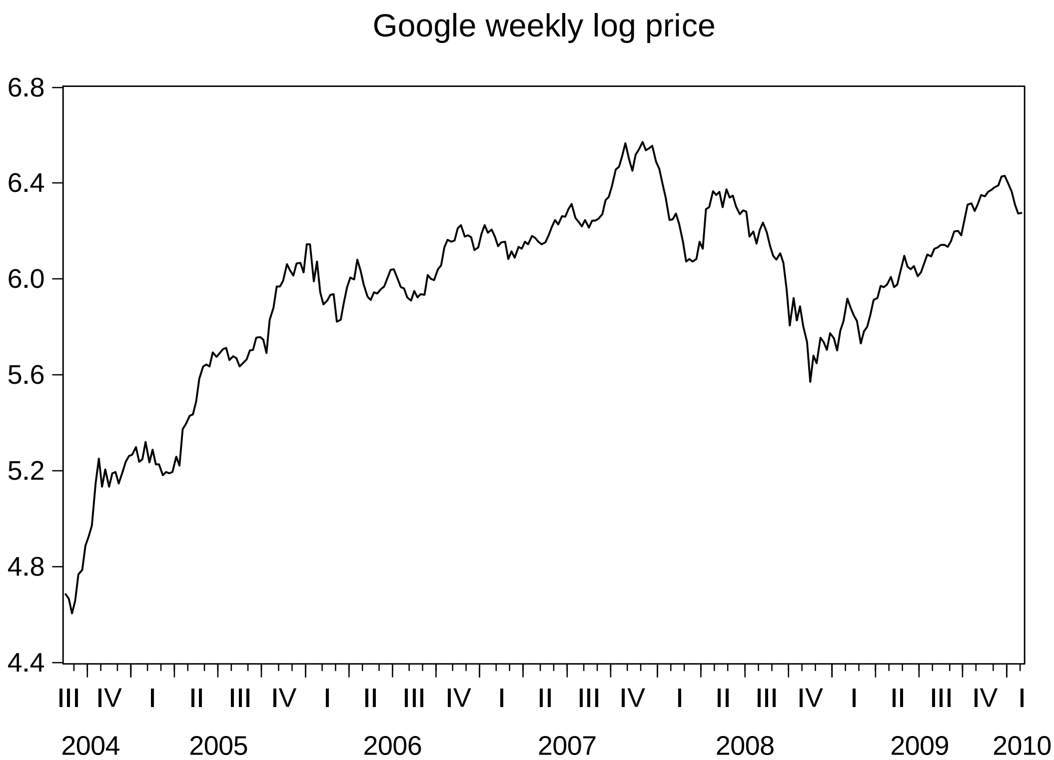
Distributions fractiles are tabulated under various assumptions (whether the trend is present ($\beta \neq 0$) and/or the drift ($\alpha$) is present. For example, including a constant but no trend:

| $n$ : | $\alpha = 1\%$ | $\alpha = 5\%$ | $\alpha = 10\%$ |
|---|---|---|---|
| 25 | -3.75 | -3.00 | -2.62 |
| 50 | -3.58 | -2.93 | -2.60 |
| 100 | -3.51 | -2.89 | -2.58 |
| 250 | -3.46 | -2.88 | -2.57 |
| 500 | -3.44 | -2.87 | -2.57 |
| $\infty$ | -3.43 | -2.86 | -2.57 |

$t$-statistics which are more negative than the critical values above lead to a rejection of a unit root in favor of a stationary series.

In practice also AR-terms are added into the regression to make the residual as white noise as possible. Doing so yields critical values different from the table above.

Example: Google weekly log prices.




The series does not appear to be mean-reverting, so we expect it to contain a unit root.

However, naively applying the unit-root test with default-settings in EViews rejects the unit root in favour of a stationary series:

Augmented Dickey-Fuller Unit Root Test on LPRICE

Null Hypothesis: LPRICE has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=15)

| | | t-Statistic | Prob.* |
|---|---|---|---|
| Augmented Dickey-Fuller test statistic | | -3.316975 | 0.0150 |
| Test critical values: | 1% level | -3.453153 | |
| | 5% level | -2.871474 | |
| | 10% level | -2.572135 | |

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LPRICE)
Method: Least Squares
Date: 01/25/13 Time: 11:46
Sample (adjusted): 8/23/2004 2/01/2010
Included observations: 285 after adjustments

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| LPRICE(-1) | -0.025118 | 0.007573 | -3.316975 | 0.0010 |
| C | 0.155012 | 0.045151 | 3.433193 | 0.0007 |

| | | | |
|---|---|---|---|
| R-squared | 0.037423 | Mean dependent var | 0.005580 |
| Adjusted R-squared | 0.034021 | S.D. dependent var | 0.051665 |
| S.E. of regression | 0.050778 | Akaike info criterion | -3.115696 |
| Sum squared resid | 0.729702 | Schwarz criterion | -3.090064 |
| Log likelihood | 445.9867 | Hannan-Quinn criter. | -3.105421 |
| F-statistic | 11.00233 | Durbin-Watson stat | 2.087380 |
| Prob(F-statistic) | 0.001029 | | |

This is because the default settings in EViews assume that the series contains no deterministic trend.

However, there is a clear upward trend in the series over the chosen time period. Allowing for a deterministic trend accepts the null hypothesis that there is at least one unit root:

Augmented Dickey-Fuller Unit Root Test on LPRICE

Null Hypothesis: LPRICE has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=15)

| | | t-Statistic | Prob.* |
|---|---|---|---|
| Augmented Dickey-Fuller test statistic | | -2.734332 | 0.2236 |
| Test critical values: | 1% level | -3.990585 | |
| | 5% level | -3.425671 | |
| | 10% level | -3.135994 | |

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LPRICE)
Method: Least Squares
Date: 01/25/13 Time: 11:48
Sample (adjusted): 8/23/2004 2/01/2010
Included observations: 285 after adjustments

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| LPRICE(-1) | -0.027529 | 0.010068 | -2.734332 | 0.0066 |
| C | 0.166826 | 0.055654 | 2.997561 | 0.0030 |
| @TREND(8/16/2004) | 1.77E-05 | 4.86E-05 | 0.364141 | 0.7160 |

| | | | |
|---|---|---|---|
| R-squared | 0.037875 | Mean dependent var | 0.005580 |
| Adjusted R-squared | 0.031051 | S.D. dependent var | 0.051665 |
| S.E. of regression | 0.050856 | Akaike info criterion | -3.109148 |
| Sum squared resid | 0.729359 | Schwarz criterion | -3.070701 |
| Log likelihood | 446.0536 | Hannan-Quinn criter. | -3.093736 |
| F-statistic | 5.550601 | Durbin-Watson stat | 2.083327 |
| Prob(F-statistic) | 0.004322 | | |

Generally it is a good idea to run unit root tests both with and without assuming a deterministic trend.

Applying a unit root test on the differenced series (that is, the log-returns), clearly rejects the unit root in favour of stationarity:

Augmented Dickey-Fuller Unit Root Test on D(LPRICE)

Null Hypothesis: D(LPRICE) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=15)

| | | t-Statistic | Prob.* |
|---|---|---|---|
| Augmented Dickey-Fuller test statistic | | -17.32300 | 0.0000 |
| Test critical values: | 1% level | -3.453234 | |
| | 5% level | -2.871510 | |
| | 10% level | -2.572154 | |

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LPRICE,2)
Method: Least Squares
Date: 01/25/13 Time: 11:49
Sample (adjusted): 8/30/2004 2/01/2010
Included observations: 284 after adjustments

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| D(LPRICE(-1)) | -1.030640 | 0.059495 | -17.32300 | 0.0000 |
| C | 0.005842 | 0.003092 | 1.889515 | 0.0598 |

| | | | |
|---|---|---|---|
| R-squared | 0.515536 | Mean dependent var | 7.99E-05 |
| Adjusted R-squared | 0.513818 | S.D. dependent var | 0.074291 |
| S.E. of regression | 0.051801 | Akaike info criterion | -3.075807 |
| Sum squared resid | 0.756695 | Schwarz criterion | -3.050110 |
| Log likelihood | 438.7646 | Hannan-Quinn criter. | -3.065505 |
| F-statistic | 300.0864 | Durbin-Watson stat | 1.990551 |
| Prob(F-statistic) | 0.000000 | | |

This implies that the log price series is $I(1)$.