5. **Event-Study Analysis** (Ch 4 in CLM)

The effect of an economic event on the value of a firm. Typical events are *firm-specific* events like earnings, investment, mergers and acquisitions, issues of new debt or equity, stock splits, etc. announcements, or *economy wide* events like inflation, interest rate, consumer confidence, trade deficient, etc. announcements. Also impacts of announcements in changes of regulatory environments or legal-liability cases are events that may affect the firm value.

Event studies have a long history, Dolley (1933)* investigated the impact of stock splits.

*Dolley, J. (1933). Characteristics and procedure of common stock split-ups. *Harvard Business Review*, 316–326. Other important papers are Brown, S. and J. Warner (1983) *Journal of Financial Economics*, 8, 205–258, and (1985) *Journal of Financial Economics*, 14, 3–31. Boehmer, E. J. Musumeci, and A. Poulsen (1991). *Journal of Financial Economics*, 30, 253–272.

5.1 Outline of an event study (CLM, pp. 151–152)

1. Event definition: The event of interest and the period over which the related security prices will be examined—*event window*.

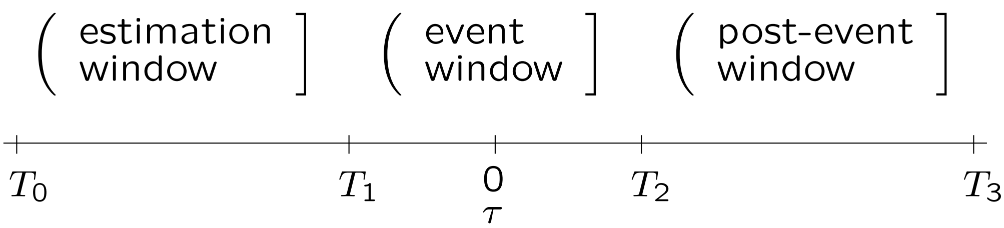
2. Selection criteria for inclusion of a given firm in the study. Availability of data, listing in particular stock exchange, membership in a specific industry, etc.

3. Normal and abnormal returns

$$\epsilon^*_{it} = R_{it} - \mathbb{E}[R_{it}|X_t],$$

   where ϵ^*_{it}, R_{it}, and $\mathbb{E}[R_{it}|X_t]$ are the abnormal, actual, and normal returns, respectively, and X_t is the conditioning information for normal performance. Two common choices for $\mathbb{E}[R_{it}|X_t]$ are the constant-mean-return ($\mathbb{E}[R_{it}|X_t] = \mathbb{E}[R_{it}] = \mu_i$)and the market model, with X_t the market return R_{mt}, so that $\mathbb{E}[R_{it}|X_t] = \alpha_i + \beta_i R_{mt}$.

4. Estimation procedure. The parameters of the normal performance are estimated using *estimation window*, which is set before the event window. In a daily data the estimation sample period is typically 120 or 250 trading days. Usually event period is not included.



5. Testing procedure. A (statistically) significant abnormal return indicates a response of the event on returns. Usually a version of t-test is employed.

6. Empirical results. The basic empirical results, and diagnostics should be presented (distribution statistics of the abnormal returns, and especially outliers should be checked)

7. Interpretation and conclusions. Information leaks, adjustment process (immediate, gradual).

Example. (CLM pp. 152–167)

1) Event definition: Information content of quarterly earnings announcements. Earnings surprise can be defined with respect to the market expectations (e.g. analysts mean prediction). Three categories: good news (exceed predictions at least 2.5%), no news (as expected), bad news (below expectations at least 2.5%). Event window ± 20 days around the announcement day. Thus the length of event window is 41 days.

2) Selection criteria: 30 firms in the Dow Jones Industrial Index over the five-year period from January 1988 to December 1993, total of 600 announcements.

3) Normal and abnormal returns. Market model returns.

4) Estimation: 250 days estimation window.

5.2 Models for Measuring Normal Performance

Statistical models: Constant-mean-model, market model, multifactor models

Economic models: CAPM family of models, APT family of models

Statistical Models

Conventional assumption:

(**A1**) *Let* $\mathbf{R}_t$ *be an* $N \times 1$ *vector of asset returns for calendar time period* t*.* $\mathbf{R}_t$ *is independently multivariate normally distributed with mean vector* μ *and covariance matrix* Ω *for all* t*.*

Constant-Mean-Return Model

(2) $$R_{it} = \mu_i + \xi_{it}$$

with $\mathbb{E}[\xi_{it}] = 0$ and $\mathbb{V}\mathrm{ar}[\xi_{it}] = \sigma^2_{\xi_i}$.

Brown and Warner (1980, 1985): this model often yields results similar to those of more sophisticated.

Market Model

(3) $$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it}$$

with $\mathbb{E}[\epsilon_{it}] = 0$ and $\mathbb{V}\text{ar}[\epsilon_{it}] = \sigma^2_{\epsilon_i}$, where R_{mt} is period-t market return (e.g. S&P500 in US markets).

Note. 1) If $\beta_i = 0$ one gets the constant-mean-model. 2) If $\beta_i = 1$ and $\alpha_i = 0$ such that $\epsilon^*_{it} = R_{it} - R_{mt}$, one obtains *market-adjusted-model*. In this case no estimation period is necessarily needed!

Warning. Imposing wrong restrictions may arise bias!

Other possibilities are different kinds of multi-index (multifactor) models

(4) $$R_{it} = \beta_{i,0} + \beta_{i1} I_{1,t} + \cdots + \beta_{ip} I_{p,t} + \eta_{it}$$

with $\mathbb{E}[\eta_{it}] = 0$ and $\mathbb{V}\text{ar}[\eta_{it}] = \sigma^2_{\eta_i}$, where $I_{j,t}$ are some market factors (e.g., industry returns), $j = 1, \ldots, p$.

Note. The market model and constant-mean-model are special cases of the multi-index model.

In practice, however, the gains from employing multifactor models for event studies are limited.

Sensitivity to Normal Return Model

Use of the market model reduces the variance of the abnormal return compared to the constant-mean-model, implying more powerful tests. This is because

$$\sigma^2_{\epsilon_i} = (1 - r^2_{im})\mathbb{V}\text{ar}[R_{it}] \tag{5}$$

where $r_{im} = \mathbb{C}\text{orr}(R_{it}, R_{mt})$.

Exercise. Verify the above formula.

For the constant mean model

$$\sigma^2_{\xi_i} = \mathbb{V}\text{ar}[R_{it} - \mu_i] = \mathbb{V}\text{ar}[R_{it}]. \tag{6}$$

Thus

$$\sigma^2_{\epsilon_i} = (1 - r^2_{im})\sigma^2_{\xi_i} \le \sigma^2_{\xi_i} \tag{7}$$

because $0 \le r^2_{im} \le 1$.

Example (CLM, p. 163).

Economic Models

Economic models restrict the parameters of statistical models to provide more constrained normal return models. Most important of these are CAPM and APM. No practical advantages relative to the unrestricted market model found in event studies. Thus employed rarely.

Measuring and Analyzing Abnormal Returns

Let τ denote the time index in the event study.
Event date: $\tau = 0$
Event window: $\tau = T_1 + 1$ to $\tau = T_2$.
Estimation window $\tau = T_0 + 1$ to $\tau = T_1$.
Post-event window: $\tau = T_2 + 1$ to $\tau = T_3$.
The window lengths: $L_1 = T_1 - T_0$, $L_2 = T_2 - T_1$ and $L_3 = T_3 - T_2$.

Estimation of the Market Model

The market model in matrix form is

$$\mathbf{R}_i = \mathbf{X}_i\theta_i + \epsilon_i \tag{8}$$

where $\mathbf{R}_i = (R_{i,T_0+1}, \ldots, R_{i,T_1})'$: $(L_1 \times 1)$,
$\mathbf{X}_i = (\mathbf{1}, \mathbf{R}_m)$: $(L_1 \times 2)$ with $\mathbf{1} = (1, \ldots, 1)'$: $(L_1 \times 1)$,
and $\mathbf{R}_m = (R_{m,T_{0+1}}, \ldots, R_{m,T_1})'$: $(L_1 \times 1)$,
$\theta_i = (\alpha_i, \beta_i)'$: (2×1),
$\epsilon_i = (\epsilon_{i,T_0+1}, \ldots, \epsilon_{i,T_1})'$, and the prime denotes the transpose.

OLS estimators

$$\hat{\theta}_i = (\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i'\mathbf{R}_i$$

$$\hat{\sigma}^2_{\epsilon_i} = \frac{1}{L_1 - 2}\hat{\epsilon}_i'\hat{\epsilon}_i$$

$$\hat{\epsilon}_i = \mathbf{R}_i - \mathbf{X}_i\hat{\theta}_i \tag{9}$$

$$\mathbb{V}\text{ar}[\hat{\theta}_i] = (\mathbf{X}_i'\mathbf{X}_i)^{-1}\sigma^2_{\epsilon_i}$$

$$\widehat{\mathbb{V}\text{ar}[\hat{\theta}_i]} = (\mathbf{X}_i'\mathbf{X}_i)^{-1}\hat{\sigma}^2_{\epsilon_i}$$

5.3 Regression based Event Study*

Consider first only a single event day, that is, $L_2 = 1$. If an event affects the price of stock i by imposing a return effect δ_i on the event date, we can model the case by introducing a dummy variable $D_\tau = 1$ for $\tau = 0$ (the event day), and $D_\tau = 0$ otherwise. Then

$$R_{i,\tau} = \alpha_i + \delta_i D_\tau + \beta_i R_{m,\tau} + \epsilon_{i,\tau}. \tag{10}$$

Then the abnormal returns are

$$\epsilon^*_{i,\tau} = R_{i,\tau} - (\alpha_i + \beta_i R_{m,\tau}) = \epsilon_{i,\tau} + \delta_i D_\tau \tag{11}$$

with

$$\mathbb{E}[\epsilon^*_{i,\tau}|R_{m,\tau}] = \delta_i D_\tau \tag{12}$$

Basically the no-event-effect null hypothesis then is

$$H_0 : \delta_i = 0, \tag{13}$$

which can be tested with a traditional t-test.

If $L_2 > 1$, we simply introduce L_2 dummy variables $D_{i1}, \ldots, D_{iL_2}$ for the return effects $\delta_{i1}, \ldots, \delta_{iL_2}$ on days $T_1+1, T_1+2, \ldots, T_2$. The no-event-effect null hypothesis then is

$$H_0 : \delta_{i1} + \ldots + \delta_{iL_2} = 0, \tag{14}$$

which can be tested with a traditional F-test for linear restrictions.

*Pynnönen, Seppo (2005). On regression based event study. Acta Wasaensia No. 143, 327-354

5.4 Statistical Properties of Abnormal Returns

Usually return data is too noisy to allow inference of event effects based upon a single stock, which necessitates aggregation of abnormal returns across securities.

Additionally, uncertainty about when the event information has entered the market often requires aggregation of abnormal returns over time.

In order to do that, we need the distributional properties of the abnormal returns. The estimated abnormal returns in the event window are

$$\hat{\epsilon}_i^* = \mathbf{R}_i^* - \hat{\alpha}_i \mathbf{1} - \hat{\beta}_i \mathbf{R}_m^* = \mathbf{R}_i^* - \mathbf{X}_i^* \hat{\theta}_i, \tag{15}$$

where $\mathbf{R}_i^* = (R_{i,T_1+1}, \ldots, R_{i,T_2})'$: $(L_2 \times 1)$, $\mathbf{R}_m^* = (R_{m,T_1+1}, \ldots, R_{m,T_2})'$: $(L_2 \times 1)$, and $\mathbf{X}_i^* = (\mathbf{1}, \mathbf{R}_m^*)$: $(L_2 \times 2)$. $\hat{\theta}_i$ was obtained in the estimation period as $\hat{\theta}_i = (\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i'\mathbf{R}_i$.

Now $\mathbb{E}[\mathbf{R}_i|\mathbf{X}_i^*] = \mathbf{X}_i^*\widehat{\theta}_i$ and by unbiasedness of OLS estimators and because $\widehat{\theta}_i$ is independent of $\mathbf{X}_i^*$ by assumption (**A1**), we get (under the null hypothesis of no event effect)

$$
\begin{aligned}
(16)\quad \mathbb{E}[\widehat{\epsilon}_i^*|\mathbf{X}_i^*] &= \mathbb{E}[\mathbf{R}_i^* - \mathbf{X}_i^*\widehat{\theta}_i|\mathbf{X}_i^*] = 0 \\
(17)\qquad\qquad \mathbf{V}_i &= \mathbb{E}[\widehat{\epsilon}_i^*\widehat{\epsilon}_i^{*'}|\mathbf{X}_i^*] \\
&= \mathbf{I}\sigma_{\epsilon_i}^2 + \mathbf{X}_i^*(\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i^{*'}\sigma_{\epsilon_i}^2,
\end{aligned}
$$

where $\mathbf{I}$ is the $L_2 \times L_2$ identity matrix.

Under the null hypothesis of no event-effect (and assumption **A1**)

$$\widehat{\epsilon}_i \sim N(\mathbf{0}, \mathbf{V}_i).$$

Note.

1) $\mathbf{V}_i$ is estimated (denoted as $\widehat{V}_i$) by replacing $\sigma_{\epsilon_i}^2$ with the estimator $\widehat{\sigma}_{\epsilon_i}^2$, defined in(9).
2) It can be shown that the abnormal returns are asymptotically independent, that is, $\mathbf{V}_i \to \mathbf{I}\sigma_{\epsilon_i}^2$ for infinitely long estimation windows ($L_1 \to \infty$). In finite estimation windows, however, $\mathbf{I}\sigma_{\epsilon_i}^2$ will always underestimate the true variance of abnormal returns.

5.5 Aggregation over Time

Let $T_1 < \tau_1 \leq \tau_2 \leq T_2$. Cumulative Abnormal Return, $\text{CAR}(\tau_1, \tau_2)$ is defined as

$$\widehat{\text{CAR}}_i(\tau_1, \tau_2) = \sum_{j=T_1+\tau_1}^{T_1+\tau_2} \hat{\epsilon}^*_{ij} = \gamma' \hat{\epsilon}^*_i, \tag{18}$$

where γ is an L_2-vector with ones in positions $\tau_1 - T_1$ to $\tau_2 - T_1$ and zeros elsewhere.

Then under the null hypothesis of no event impact

$$\widehat{\text{CAR}}_i(\tau_1, \tau_2) \sim N\left(0, \sigma_i^2(\tau_1, \tau_2)\right) \tag{19}$$

where

$$\sigma_i^2(\tau_1, \tau_2) = \mathbb{V}\text{ar}[\widehat{\text{CAR}}_i(\tau_1, \tau_2)] = \gamma' \mathbf{V}_i \gamma. \tag{20}$$

with $\mathbf{V}_i$ defined in (17). In other words, to obtain the variance of the cumulative abnormal returns within the estimation window, we must sum up all elements of $\mathbf{V}_i$ and not just its diagonal elements, as we would if the $\hat{\epsilon}^*_{ij}$ were independent (and is the case for $L_1 \to \infty$).

Exercise. Show that the variance is of this form.

A useable test statistic for the null hypothesis of no event impact for security i is then the Standardized Cumulative Abnormal Return

$$\widehat{\text{SCAR}}(\tau_1,\tau_2) = \frac{\widehat{\text{CAR}}_i(\tau_1,\tau_2)}{\hat{\sigma}_i(\tau_1,\tau_2)} \tag{21}$$

with

$$\hat{\sigma}_i(\tau_1,\tau_2) = \sqrt{\gamma'\widehat{V}_i\gamma}. \tag{22}$$

Under the null hypothesis of no-event-effect, the standardized CAR has a t-distribution with L_1-2 degrees of freedom (for large L_1 approximately $N(0,1)$).

<u>Note.</u> Using the standardized SCAR defined in (21) and the F-test for assessing the linear restriction (14) in the regression approach will always lead to the same conclusion, since both the t-test based upon (21) and the F-test to assess (14) produce the same p-values.

5.6 Aggregation across Securities

Average abnormal return:

$$\overline{\epsilon}^* = \frac{1}{N} \sum_{i=1}^{N} \widehat{\epsilon}_i^* \tag{23}$$

with variance-covariance matrix

$$\mathbb{V}\text{ar}[\overline{\epsilon}^*] = \mathbf{V} = \frac{1}{N^2} \sum_{i=1}^{N} \mathbf{V}_i \tag{24}$$

provided that the abnormal returns are uncorrelated across securities, that is, there must not be any overlap in the event windows of any two securities. The case of overlapping event windows (clustered events) will be discussed later.

Averaging security CARs yields

(25)

$$\overline{\text{CAR}}(\tau_1, \tau_2) = \gamma'\bar{\epsilon}^* = \frac{1}{N}\sum_{i=1}^{N} \widehat{\text{CAR}}_i(\tau_1, \tau_2)$$

and

$$\mathbb{V}\text{ar}[\overline{\text{CAR}}(\tau_1, \tau_2)] = \bar{\sigma}^2(\tau_1, \tau_2) = \gamma'\mathbf{V}\gamma \tag{26}$$

or equivalently

(27)

$$\bar{\sigma}^2(\tau_1, \tau_2) = \frac{1}{N^2}\sum_{i=1}^{N} \sigma_i^2(\tau_1, \tau_2) = \frac{\sigma_A^2(\tau_1, \tau_2)}{N},$$

where

$$\sigma_A^2(\tau_1, \tau_2) = \frac{1}{N}\sum_{i=1}^{N} \sigma_i^2(\tau_1, \tau_2) \tag{28}$$

is the average variance over the N securities, that is, the average of the variances $\sigma_i^2(\tau_1, \tau_2)$ defined in (20).

Inference about the cumulative abnormal returns (under the null hypothesis of no effect) can be based on

(29) $$\overline{\mathrm{CAR}}(\tau_1, \tau_2) \sim N\left(0, \bar{\sigma}^2(\tau_1, \tau_2)\right)$$

which gives a test statistic for testing the null hypotesis of no event impact

(30) $$J_1 = \frac{\overline{\mathrm{CAR}}(\tau_1, \tau_2)}{\sqrt{\hat{\bar{\sigma}}^2(\tau_1, \tau_2)}} \sim AN(0, 1),$$

where

(31)
$$\hat{\bar{\sigma}}^2(\tau_1, \tau_2) = \frac{1}{N^2}\sum_{i=1}^{N} \hat{\sigma}_i^2(\tau_1, \tau_2) = \frac{\hat{\sigma}_A^2(\tau_1, \tau_2)}{N},$$

$\hat{\sigma}_i(\tau_1, \tau_2)$ has been defined in (22), and $J_1 \sim AN(0, 1)$ means that the distribution of J_1 is asymptotically normal.

A second alternative is to average the standardized cumulative abnormal returns (21) across securities in order to obtain

$$\text{(32)} \quad \overline{\mathsf{SCAR}}(\tau_1, \tau_2) = \frac{1}{N} \sum_{i=1}^{N} \widehat{\mathsf{SCAR}}_i(\tau_1, \tau_2).$$

with variance

$$\text{(33)} \quad \mathbb{V}\text{ar}\left[\overline{\mathsf{SCAR}}(\tau_1, \tau_2)\right] = \frac{L_1 - 2}{N(L_1 - 4)}.$$

assuming cross-sectionally independent events.

This yields the test statistics*

(34)

$$J_2 = \left(\frac{N(L_1 - 4)}{L_1 - 2}\right)^{\frac{1}{2}} \overline{\mathsf{SCAR}}(\tau_1, \tau_2) \sim AN(0, 1).$$

under the null hypothesis of no event effect.

Usually J_2 is more powerful than J_1 (detecting abnormal returns more easily) and should therefore be preferred in empirical work.

*Patell (1976), Journal of Accounting and Research 14, 246–276.

5.7 Isolating the Mean Effect

So far the null hypothesis has been that the event has no impact on the behavior of the return whatsoever. Both a mean effect and a variance effect violate this hypothesis.

If we are interested in the mean effect only, the analysis must be expanded to allow for changing variances. A popular way to do this is to estimate the cross-sectional variance of the (standardized) cumulative abnormal returns within the event window in order to obtain for their cross-sectional averages

(35)

$$\widehat{\mathbb{V}\mathrm{ar}}\left[\overline{\mathrm{CAR}}(\tau_1,\tau_2)\right]=\frac{1}{N^2}\sum_{i=1}^{N}\left(\mathrm{CAR}_i(\tau_1,\tau_2)-\overline{\mathrm{CAR}}(\tau_1,\tau_2)\right)^2$$

and (Boehmer et al., 1991)*
(36)

$$\widehat{\mathbb{V}\mathrm{ar}}\left[\overline{\mathrm{SCAR}}(\tau_1,\tau_2)\right]=\frac{1}{N^2}\sum_{i=1}^{N}\left(\mathrm{SCAR}_i(\tau_1,\tau_2)-\overline{\mathrm{SCAR}}(\tau_1,\tau_2)\right)^2.$$

Exercise. Find a rationale for these variance estimators. Discuss assumptions behind the validity of these estimators

*Boehmer E., Musumeci J. and Poulsen A. (1991). Journal of Financial Economics 30, 253-272.

Replacing $\widehat{\bar{\sigma}}^2(\tau_1, \tau_2)$ with (35) in (30) and using the cross-sectional estimator (36) instead of (33) in (34) yields the new test statistics

(37)

$$J_1' = \frac{N \cdot \overline{\text{CAR}}(\tau_1, \tau_2)}{\sqrt{\sum_{i=1}^{N} \left(\text{CAR}_i(\tau_1, \tau_2) - \overline{\text{CAR}}(\tau_1, \tau_2)\right)^2}}$$

and

(38)

$$J_2' = \frac{N \cdot \overline{\text{SCAR}}(\tau_1, \tau_2)}{\sqrt{\sum_{i=1}^{N} \left(\text{SCAR}_i(\tau_1, \tau_2) - \overline{\text{SCAR}}(\tau_1, \tau_2)\right)^2}},$$

both of which are asymptotically standard normally distributed under the null hypothesis of no event effect upon the return level (ignoring changes in variance).

J_1' is easier to calculate than J_2', because it circumvents the calculation of $\widehat{V}_i$, but it is less powerful. Also the assumption of equal variances in the cross section needed in the derivation of (35) and (36) is more likely to hold for the standardized CAR, which is why J_2' should be preferred over J_1' in empirical work.

5.8 Inferences with Clustering

The basic assumption in the aggregation over securities is that individual securities are *uncorrelated in the cross section*. This is the case if the event windows over different securities do not overlap in calendar time.

If they do, the correlation should be taken into account.

The easiest way is to aggregate the individual securities with overlapping event windows to equal weighted *portfolios* and then apply the above standard event study analysis. That is, apply the t-test for the standardized CAR defined in (21) or the equivalent F-test in a dummy regression in order to test the null hypothesis (14) on the portfolio, as if it was a single security.

This approach, however, lacks power as compared to an analysis without aggregation into an artificial portfolio of abnormal returns, which we shall turn to on the next slide.

*Correcting for Cross-Sectional Correlation**

When the abnormal returns are correlated in the cross-section, the variance of the average standardized CAR (33) becomes

(39)

$$\mathbb{V}\mathrm{ar}\left[\overline{\mathrm{SCAR}}(\tau_1,\tau_2)\right] = \frac{L_1-2}{N(L_1-4)}\cdot(1+(N-1)\bar{\rho}),$$

where $\bar{\rho}$ denotes the average cross-sectional correlation between the assets abnormal returns.

This yields the modified Patell-statistics

(40)

$$J_2^* = \overline{\mathrm{SCAR}}(\tau_1,\tau_2)\Bigg/\sqrt{\frac{L_1-2}{N(L_1-4)}\cdot(1+(N-1)\bar{r})}$$

with $\bar{r}$ denoting the average cross-sectional correlation coefficient of abnormal returns in the estimation period. J_2^* is again asymptotically standard normally distributed under the null hypothesis of neither a mean nor a variance effect.

*Kolari and Pynnönen (2010), Review of Financial Studies 23, 3996–4025

If we wish to test for a mean effect only, also the cross-sectional variance estimators (35) and (36) need to be scaled with $(1+(N-1)\bar{\rho})$.

Additionally, the variance-covariance matrix of the average abnormal returns (24) becomes

(41) $$\mathbb{V}\text{ar}[\bar{\epsilon}^*] = \mathbf{V}/(1-\bar{\rho}),$$

such that the new test statistics for establishing a mean effect ignoring variance effects becomes

(42) $$J_2'^* = J_2' \cdot \sqrt{\frac{1-\bar{r}}{(1+(N-1)\bar{r})}}$$

with J_2' defined in (38).

Again, this statistic is asymptotically standard normally distributed under the null hypothesis of no event effect upon the return level.

5.9 Nonparametric Tests

The advantage of nonparametric approach is that it is free of specific assumptions conserning the return distribution. Common nonparametric tests are the sign test and the rank test.

The Sign Test

The sign test is based on the sign of the abnormal return.

Assumptions:
(1) Independence: abnormal returns are independent accross securities,
(2) Symmetry: positive and negative returns are equally likely under the null hypothesis of no event effect.

Let $p = P(\text{CAR}_i \geq 0)$, then if the research hypothesis is that there is a positive return effect of the event the statitical null and alternative hypotheses are

$$H_0 : p \leq 0.5 \text{ versus } H_1 : p > 0.5 \tag{43}$$

Let N be the total number of assets and let N^+ be the number of assets with positive cumulative abnormal returns within the event window. Then a statistic for testing the null hypotesis H_0 of no event effect can be formulated as

$$J_3 = \left[\frac{N^+}{N} - 0.5\right]\frac{N^{\frac{1}{2}}}{0.5} \sim AN(0,1). \tag{44}$$

Large absolute values of J_3 imply rejection of H_0.

Exercise. Can you derive a small sample test for the null hypothesis? Using the Central Limit Theorem, try to justify the asymptotic distribution result of J_3. *Hint*: Define random variables Y_i such that $Y_i = 1$, if the $\text{CAR}_i > 0$ and $Y_i = 0$ otherwise. Then $N^+ = \sum_{i=1}^{N} Y_i$.

A Rank Test

A weakness of the the sign test is that it may not be well defined if the (abnormal) return distribution is skewed, i.e. if $P[\epsilon^*_{it} \geq 0|H_0] \neq P[\epsilon^*_{it} < 0|H_0]$.

The ank test is a possible choice which allows for nonsymmetry.

Consider only the case for testing the null hypothesis that the event day abnormal return is zero, that is, the event window becomes our estimation period and we wish to test whether the event had an impact on day $\tau = 0$.

The rank test is as follows:

Consider a sample of L_2 abnormal returns for each of N securities. Order the returns from smallest to largest, and let $K_{i,\tau} = \text{rank}(\hat{\epsilon}^*_{i,\tau})$ be the rank number (i.e. $K_{i,\tau}$ ranges from 1 to L_2).

Under the null hypothesis of no event impact the abnormal return should be an arbitrary random value, and consequently obtain an arbitrary rank position from 1 to L_2. That is each observation should take each rank value equally likely, i.e., with probability $1/L_2$. Consequently the expected value of $K_{i,\tau}$ at each time point τ and for each security i under the null hypothesis is

(45)

$$\mu_K = \mathbb{E}[K_{i,\tau}] = \sum_{j=1}^{L_2} jP(K_{i,\tau} = j) = \frac{1}{L_1}\sum_{j=1}^{L_2} j = \frac{1}{2}(L_2 + 1).$$

and variance

$$\text{(46)} \quad \mathbb{V}\text{ar}[K_{i,\tau}] = \sum_{j=1}^{L_2} (j - \mu_K)^2 P(K_{i,\tau} = j).$$

A test statistic for testing the event day ($\tau = 0$) effect, suggested by Corrado (1989)†, is

$$\text{(47)} \qquad J_4 = \frac{\frac{1}{N}\sum_{i=1}^{N}\left(K_{i,0} - \frac{L_2+1}{2}\right)}{s(L_2)},$$

where

(48)

$$s(L_2) = \sqrt{\frac{1}{L_2}\sum_{\tau=T_1+1}^{T_2}\left(\frac{1}{N}\sum_{i=1}^{N}\left(K_{i,\tau} - \frac{L_2+1}{2}\right)\right)^2}$$

Under the null hypothesis $J_4 \sim AN(0,1)$.

Typically nonparametric tests are used in conjuction with parametric tests.

†Corrado, C. (1989). A nonparametric test for abnormal security price performance. *Journal of Financial Economics*, 23, 385–395.

5.10 Cross-Sectional Models

Here the interest is in the magnitude of association between abnormal return and characteristics specific to the observed event.

Let
$\mathbf{y}$: $N \times 1$ vector of CARs
$\mathbf{X}$: $N \times K$ matrix of $K-1$ characteristics
(The first colum is a vector of ones for the intercept term).

Then a cross-sectional (linear) model to explain the magnitudes of CARs is

$$\mathbf{y} = \mathbf{X}\theta + \eta, \tag{49}$$

where β is a $K \times 1$ coefficient vector and η is an $N \times 1$ disturbance vector.

OLS estimators

$$\widehat{\theta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} \tag{50}$$

which is consistent (i.e., $\text{plim}\, \widehat{\theta} = \theta$) if $\mathbb{E}[\mathbf{X}'\eta] = \mathbf{0}$ (i.e., residual are not correlated with the explanatory variables).

$$\mathbb{V}\text{ar}[\widehat{\theta}] = (\mathbf{X}'\mathbf{X})^{-1}\sigma_\eta^2 \tag{51}$$

Replacing σ_η^2 by its consistent estimator

$$\widehat{\sigma}_\eta^2 = \frac{1}{N-K}\widehat{\eta}'\widehat{\eta} \tag{52}$$

where $\widehat{\eta} = \mathbf{y} - \mathbf{X}\widehat{\eta}$, makes possible o calculate standard errors of the regression coefficients and construct t-test to make inference on θ-coefficients.

In financial markets homoscedasticity is a questionable assumption. This is why it is usually suggested to use White's heteroscedasticity-consistent standard errors of θ-estimates. These are obtained as square roots from the main diagonal of

(53)

$$\widehat{\mathbb{V}\text{ar}}[\widehat{\theta}] = \frac{1}{N}(\mathbf{X}'\mathbf{X})^{-1}\left[\sum_{i=1}^{N}\mathbf{x}_i\mathbf{x}_i'\widehat{\eta}_i^2\right](\mathbf{X}'\mathbf{X})^{-1}$$

These are usually available in most econometric packages (e.g. in EViews by choosing an appropriate option).

Newey and West (1987) have proposed a more general estimator that is consistent of both heteroscedasticity and autocorrelation. Again this is an option e.g. in EViews and many other (econometric) packages. Note, however, that this may be used only for time series regression. Not for cros-sectional regression!

For discussion on studies applying cross-sectional models in conjunction of event studies see CLM p. 174.

5.11 Power of Tests

The goodness of a statistical test is its ability to detect false null hypothesis. This is called the power of the test, and is technically measured by power function, which depends on the parameter values under the H_1 (in the case of abnormal returns, δ)

$$\pi_\alpha(\delta) = P_\delta[\text{reject } H_0], \tag{54}$$

where α denotes the size of the test (i.e., the significance level which usually is 1% or 5%), and $P_\delta[\cdot]$ denotes the probability as a function of δ.

Thus the power function gives the probability to reject H_0 on different values of the tested parameter (δ).

Example. Consider the J_1 test and test the event day abnormal return ($\tau_1 = -1$ and $\tau_2 = 0$). Furthermore, assume for simplicity that the market model parameters are known with $\sigma_A^2(\tau_1, \tau_2) = 0.0016$. Then the power depends on the sample size N, the level of significance δ and the magnitude of the (average) abnormal return δ. Fix $\alpha = 0.05$. The two-sided test, i.e.,

$$H_0 : \delta = 0 \text{ vs } H_1 : \delta \neq 0$$

has the power function

$$\pi_{.05}(\delta) = P_\delta[J_1 < -z_{.025}] + P_\delta[J_1 > z_{0.25}].$$

The distribution of J_1 depends on δ such that

$$\mathbb{E}[J_1] = \frac{\delta\sqrt{N}}{\sigma_A(\tau_1, \tau_2)} \equiv \mu_\delta.$$

Thus

$$J_1 \sim N(\mu_\delta, 1).$$

Note that $J_1 - \mu_\delta \sim N(0, 1)$.
The power function is then

$$\begin{aligned} \pi_{.05}(\delta) &= P[J_1 < -z_{.025}] + P[J_1 > z_{.025}] \\ &= P[J_1 - \mu_\delta < -z_{.025} - \mu_\delta] + P[J_1 - \mu_\delta > z_{.025} - \mu_\delta] \\ &= \Phi(-z_{.025} - \mu_\delta) + (1 - \Phi(z_{.025} - \mu_\delta)), \end{aligned}$$

where $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the standardized normal distribution, $N(0, 1)$.

Below are graphs of the power function for sample sizes $N = 10, 20, and 50$

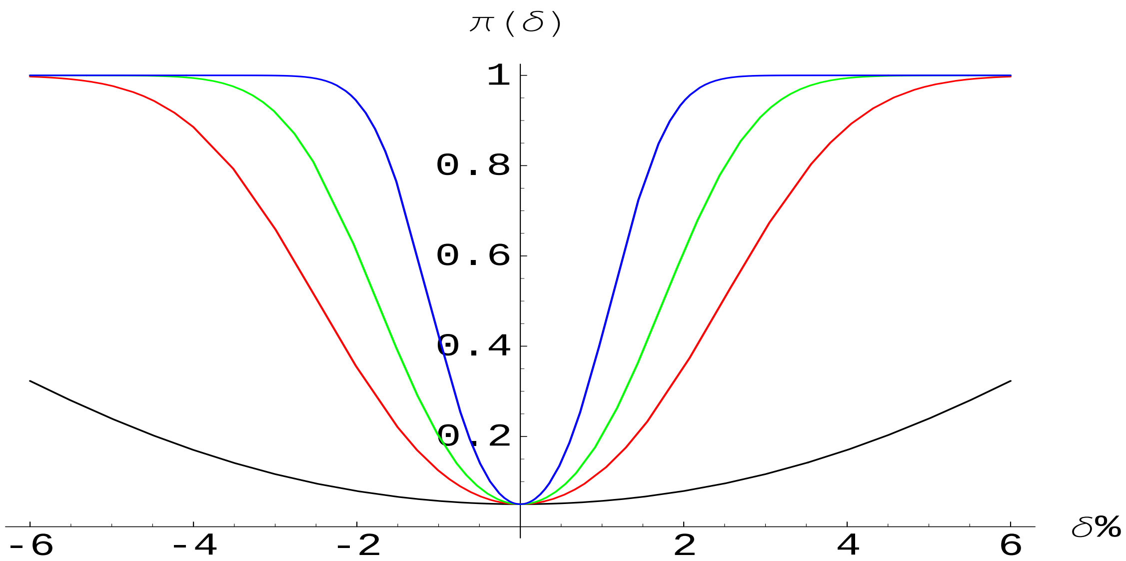


Figure. Power function of the J_1 test at the 5% significance level for sample sizes 1, 10, 20 and 50.

We observe that the smaller the effect is the larger the sample size must be in order for the test statistic to detect it. Especially for $N = 1$ (individual stocks) the effect must be relatively high before it can be statistically identified.

The important factor affecting the power is the parameter $\mu_\delta = \delta\sqrt{N}/\sigma_A$, which is a kind of signal-to-noise ratio, where δ is the amount of signal and $\sigma_A/\sqrt{N}$ is the noise component, which decreases as a function of the sample size (number of events).

5.12 Further Issues

Role of the Sampling Interval

The interval between adjacent observations constitute the sampling interval (minutes, hour, day, week, month). If the event time is known accurately a shorter sampling interval is expected lead higher ability to identify the event effect (power of the test increases).[‡]

Use of intraday data may involve some complications due to thin trading, autocorrelation, etc. So the benefits of very short interval is unclear.

[‡]For an empirical analysis/example, see Morse D. (1984). An econometric analysis of the choice of daily versus monthly returns in tests of information content. *Journal of Accounting Research*, 22, 605–623.

Inferences with Event-Date Uncertainty

Sometimes the exact event date may be difficult to identify. Usually the uncertainty is of the whether the event information published e.g. in newspapers was available to the markets already a day before.

A practical way to accommodate this uncertainty is to expand the event window to two days, the event day 0 and next day +1. This, however, reduces the power of the test (extra noise is incorporated to the testing).

Possible Biases

Nonsynchronous and thin trading: Actual time between e.g. daily returns (based on closing prices) is not exactly one day long but irregular, which is a potential source of bias to the variance and correlation estimates.