2.6 Cointegration

Motivation and Definition

Consider the *unbiased forward rate hypothesis* in the foreign exchange market, according to which the forward exchange rate of a currency should equal today's expectation of the spot rate one period ahead, that is,

$$f_t = \mathsf{E}_t s_{t+1},$$

where f_t and s_t denote the logarithm of the forward and the spot exchange rate respectively. Now rational expectations require that the forecasting errors

$$\epsilon_t := s_{t+1} - \mathsf{E}_t s_{t+1} = s_{t+1} - f_t$$

are serially uncorrelated with zero mean, in particular ϵ_t should be stationary. This appears to be quite a special relationship, because both f_t and s_t are $I(1)$ variables, and for most cases, linear combinations of $I(1)$ variables are $I(1)$ variables themselves *.

*Recall that, if x_t and y_t are $I(1)$, and z_t is stationary then for any $a, b(\neq 0)$ (*i*) $ax_t + bz_t \sim I(1)$ (*ii*) usually $ax_t + by_t \sim I(1)$.

The preceding example illustrates the concept of cointegration introduced by Engle and Granger (1987). They consider a set of economic variables in equilibrium when

$$\beta_1 x_{1t} + \beta_2 x_{2t} + \ldots + \beta_n x_{nt} = 0$$

or

$$\beta x_t = 0$$

where $\beta = (\beta_1, \ldots, \beta_n)$ and $x_t = (x_{1t}, \ldots, x_{nt})'$. The time series of deviations from the long run equilibrium relationship $\beta x_t = 0$

$$e_t := \beta x_t$$

is called the equilibrium error. Clearly, if the equilibrium relationship has any meaning, the equilibrium error process must be stationary.

Now the processes $\{x_{1t}\}, \{x_{2t}\} \ldots, \{x_{nt}\}$ are said to be cointegrated, if they are integrated but the equilibrium error e_t is stationary.

The general definition of cointegration is:
The components of $x_t = (x_{1t}, \ldots, x_{nt})'$ are said to be <u>cointegrated of order d, b</u>, denoted by $x_t \sim CI(d, b)$, if

1. All components of x_t are integrated of the same order d.
2. There exists a vector $\beta = (\beta_1, \ldots, \beta_n) \neq 0$ such that $\beta x_t \sim I(d-b)$, where $b > 0$.

<u>Note:</u> The most common case is $x_t \sim CI(1, 1)$.

<u>Example:</u> *Unbiased forward rate hypothesis*
$x_t := (s_{t+1}, f_t)'$ is cointegrated of order 1,1 with cointegrating vector $\beta = (1, -1)$ since:

1. $s_{t+1}, f_t \sim I(1)$, and
2. $(1, -1)(s_{t+1}, f_t)' = \epsilon_t \sim I(0)$.

<u>Note:</u> If β is a cointegrating vector, then $\lambda\beta$ is also a cointegrating vector for any $\lambda \neq 0$. Usually the cointegrating vector is normalized such that one of the components of β is equal to one.

Cointegrating regressions

Consider regressing a univariate time series $y = (y_1, \ldots, y_T)'$ upon a possibly stochastic regressor $x = (x_1, \ldots, x_T)'$, that is,

$$y = x\beta + \epsilon, \qquad \epsilon = (\epsilon_1, \ldots, \epsilon_T)',$$

where nothing is said yet about the statistical properties of y, x and ϵ, except that ϵ is a residual, that is $\mathrm{E}(\epsilon) = 0$, if stationary. Innocent estimation of β by OLS yields

$$\begin{aligned}\widehat{\beta} &= (x'x)^{-1}x'y = (x'x)^{-1}x'(x\beta + \epsilon)\\ &= \beta + (x'x)^{-1}x'\epsilon = \beta + \left(\sum_{t=1}^{T} x_t^2\right)^{-1} \cdot \left(\sum_{t=1}^{T} x_t\epsilon_t\right)\\ &= \beta + \frac{\frac{1}{T}\sum_{t=1}^{T} x_t\epsilon_t}{\frac{1}{T}\sum_{t=1}^{T} x_t^2}.\end{aligned}$$

Note that, assuming ergodicity and stationarity, the numerator of the fraction estimates the covariance between x_t and ϵ_t, whereas the denominator estimates the second moment of x_t. Whether $\widehat{\beta}$ is a consistent estimator of β depends upon the statistical properties of x_t and ϵ_t. We consider 4 cases.

Case 1
$x_1 = \ldots = x_T =$const., ϵ_t stationary with $\mathsf{E}(\epsilon_t) = 0$. This is the linear statistical model, the most simple case of OLS. Since x_t is non-stochastic, we have

$$\widehat{\beta} = \beta + \frac{\frac{1}{T}\sum_{t=1}^{T} x_t\epsilon_t}{\frac{1}{T}\sum_{t=1}^{T} x_t^2} = \beta + \frac{x_t\frac{1}{T}\sum_{t=1}^{T}\epsilon_t}{\frac{1}{T}(Tx_t^2)}$$
$$\overset{T\to\infty}{\longrightarrow} \beta + \frac{\mathsf{E}(\epsilon_t)}{x_t} = \beta.$$

So $\widehat{\beta}$ is a consistent, unbiased estimator of β.

Case 2
Both x_t and ϵ_t are stationary with $\mathsf{E}(\epsilon_t) = 0$, then:

$$\widehat{\beta} = \beta + \frac{\frac{1}{T}\sum_{t=1}^{T} x_t\epsilon_t}{\frac{1}{T}\sum_{t=1}^{T} x_t^2} \overset{T\to\infty}{\longrightarrow} \beta + \frac{\mathsf{cov}(x_t,\epsilon_t)}{\mathsf{E}(x_t^2)}.$$

We note that $\widehat{\beta}$ is still a consistent, though biased, estimator of β, unless x_t and ϵ_t are uncorrelated, which is always the case when x_t is weakly exogeneous with respect to β.

Case 3
x_t and y_t are cointegrated with $\epsilon_t = y_t - \beta x_t$ stationary:

$$\widehat{\beta} = \beta + \frac{\frac{1}{T}\sum_{t=1}^{T} x_t\epsilon_t}{\frac{1}{T}\sum_{t=1}^{T} x_t^2} \overset{T\to\infty}{\longrightarrow} \beta \quad \text{since} \quad \frac{1}{T}\sum_{t=1}^{T} x_t^2 \to \infty.$$

So $\widehat{\beta}$ is a consistent, unbiased estimator of β no matter whether x_t is correlated with ϵ_t or not. It can even be shown that convergence of $\widehat{\beta}$ to β is faster than in the stationary case, which is why $\widehat{\beta}$ is then called a superconsistent estimator of β.

Case 4

x_t and y_t are integrated, but not cointegrated, such that $\epsilon_t = y_t - \beta x_t$ contains a unit root. Then

$$\widehat{\beta} = \beta + \frac{\frac{1}{T}\sum_{t=1}^{T} x_t \epsilon_t}{\frac{1}{T}\sum_{t=1}^{T} x_t^2} \quad \text{does not converge,}$$

because both the numerator and the denominator approach infinity for increasing sample size T. So $\widehat{\beta}$ is not consistent and any regression line between x_t and y_t is spurious!

Even worse, it can be shown that, applying usual t and F statistics, that $\widehat{\beta}$ tends to appear statistically different from zero, even though there is no relationship between x_t and y_t whatsoever! The main lesson to learn from this is that it is dangerous to regress non-stationary variables upon each other, unless having checked before that they are cointegrated!

Cointegration and Common Trends

Consider an arbitrary unit root (or difference stationary) process

$$(1-L)y_t = \mu + \Psi(L)\epsilon_t.$$

Then the Beveridge-Nelson decomposition asserts that we may decompose any $I(1)$ process as above into a random walk (with or without drift for μ equal or unequal zero) and a stationary component (not necessarily white noise), where the random walk component is referred to as the stochastic trend of the process.

Let's apply this to two $I(1)$ processes y_t and z_t, and check what is required for the resulting vector $x_t = (y_t, z_t)'$ to be cointegrated:

$$y_t = \mu_{yt} + e_{yt}, \quad z_t = \mu_{zt} + e_{zt} \qquad \text{with}$$

μ_{it} = random walk (stochastic trend) in variable i,
e_{it} = stationary component of variable i.

In order for $\{y_t\}, \{z_t\} \sim CI(1,1)$ we need to find nonzero values of β_1, β_2 such that

$$\begin{aligned}\beta_1 y_t + \beta_2 z_t &= \beta_1(\mu_{yt} + e_{yt}) + \beta_2(\mu_{zt} + e_{zt}) \\ &= (\beta_1 \mu_{yt} + \beta_2 \mu_{zt}) + (\beta_1 e_{yt} + \beta_2 e_{zt})\end{aligned}$$

is stationary, which requires

$$\beta_1 \mu_{yt} + \beta_2 \mu_{zt} = 0 \quad \text{or} \quad \mu_{yt} = \frac{\beta_2}{\beta_1} \mu_{zt}.$$

We note that the parameters of the cointegrating vector must be such that they eliminate the stochastic trend from the linear combination of the $I(1)$ processes, which implies that both stochastic trends are identical up to a constant. This insight by Stock and Watson (1988) is easily generalized to the case of n variables by considering the vector representation

$$x_t = \mu_t + e_t,$$

where $x_t = (x_{1t}, \ldots, x_{nt})'$ is the vector of $I(1)$ processes, $\mu_t = (\mu_{1t}, \ldots, \mu_{nt})'$ is the vecot of stochastic trends and e_t is an $(n \times 1)$ vector of stationary components. Then cointegration will occur whenever the trend in one variabel can be expressed as a linear combination of the trends in the other variables, because then

$$\beta \mu_t = \beta_1 \mu_{1t} + \ldots + \beta_n \mu_{nt} = 0$$

which implies

$$\beta x_t = \beta \mu_t + \beta e_t = \beta e_t,$$

such that $\beta x_t = \beta e_t$ is stationary.

Cointegration and Error Correction

Consider two $I(1)$ variables x_1 and x_2, for which the equilibrium relationship $x_1 = \beta x_2$ holds. Now suppose that the equilibrium is currently disturbed, $x_{1,t} > \beta x_{2,t}$, say. In that case there are three possibilities to restore equilibium:

1. a decrease in x_1 and/or an increase in x_2,
2. an increase in x_1 but a larger increase in x_2,
3. a decrease in x_2 but a larger decrease in x_1.

Such a dynamic may be modelled in an error correction model as follows:

$$\Delta x_{1,t} = -\ \alpha_1(x_{1,t-1} - \beta x_{2,t-1}) + \epsilon_{1,t},\ \alpha_1 > 0$$
$$\Delta x_{2,t} = \quad \alpha_2(x_{1,t-1} - \beta x_{2,t-1}) + \epsilon_{2,t},\ \alpha_2 > 0$$

where $\epsilon_{1,t}$ and $\epsilon_{2,t}$ are (possibly correlated) white noise processes and α_1 and α_2 may be interpreted as speed of adjustment parameters to the equilibium. Note that validity of the error correction model above requires $x_1, x_2 \sim CI(1,1)$ with cointegrating vector $(1, -\beta)$, since both $\Delta x_{i,t}$ and $\epsilon_{i,t}$ are assumed to be stationary!

Nothing about this cointegration requirement changes if we introduce lagged changes into the model:

$$\begin{aligned}
\Delta x_{1,t} &= a_{10} - \alpha_1(x_{1,t-1} - \beta x_{2,t-1}) \\
&\quad + \sum_{i=1}^{p} a_{11}(i)\Delta x_{1,t-i} + \sum_{i=1}^{p} a_{12}(i)\Delta x_{2,t-i} + \epsilon_{1,t}, \\
\Delta x_{2,t} &= a_{20} + \alpha_2(x_{1,t-1} - \beta x_{2,t-1}) \\
&\quad + \sum_{i=1}^{p} a_{21}(i)\Delta x_{1,t-i} + \sum_{i=1}^{p} a_{22}(i)\Delta x_{2,t-i} + \epsilon_{2,t}.
\end{aligned}$$

This is because $\epsilon_{i,t}$ and all terms involving $\Delta x_{1,t}$ and $\Delta x_{2,t}$ are stationary.

The result, that an error-correction representation implies cointegrated variables, may be generalized to n variables as follows. Formally the $I(1)$ vector $x_t = (x_{1t}, \ldots, x_{nt})'$ is said to have an error-correction representation if it may be expressed as

$$\Delta x_t = \pi_0 + \pi x_{t-1} + \sum_{i=1}^{p} \pi_i \Delta x_{t-i} + \epsilon_t$$

where π_0 is a $(n \times 1)$ vector of intercept terms, π is a $(n \times n)$ matrix not equal to zero, π_i are $(n \times n)$ coefficient matrices and ϵ_t is a $(n \times 1)$ vector of possibly correlated white noise.

Then the stationarity of $\Delta x_{t-i},\ i = 0, 1, \ldots, p$ and ϵ_t implies that

$$\pi x_{t-1} = \Delta x_t - \pi_0 - \sum_{i=1}^{p} \pi_i \Delta x_{t-i} - \epsilon_t$$

is stationary with the rows of π as cointegrating vectors!

It can also be shown that any cointegration relationship implies the existence of an error-correction model. The equivalence of cointegration and error-correction is summarized in Granger's representation theorem:

Let x_t be a difference stationary vector process. Then $x_t \sim C(1,1)$ if and only if there exists an error-correction representation of x_t:

$$\Delta x_t = \pi_0 + \pi x_{t-1} + \sum_{i=1}^{p} \pi_i \Delta x_{t-i} + \epsilon_t, \quad \pi \neq 0$$

such that $\pi x_t \sim I(0)$.

Note that the $(n \times n)$ matrix π in the error-correction representation may be decomposed into two $(n \times r)$ matrices α and β as $\pi = \alpha\beta'$, where β' contains the cointegrating (row) vectors, α contains the (column) vectors of speed of adjustment parameters to the respective equilibria, and $r \leq n$ is the rank of π.

<u>Example:</u> For our two-component error correction model we had

$$\Delta x_t = \begin{pmatrix} \Delta x_{1,t} \\ \Delta x_{2,t} \end{pmatrix} = \begin{pmatrix} -\alpha_1 & \alpha_1\beta \\ \alpha_2 & -\alpha 2\beta \end{pmatrix} \begin{pmatrix} x_{1,t-1} \\ x_{2,t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{pmatrix}$$
$$= \pi x_{t-1} + \epsilon_t$$
$$\text{with } \pi = \begin{pmatrix} -\alpha_1 & \alpha_1\beta \\ \alpha_2 & -\alpha 2\beta \end{pmatrix} = \begin{pmatrix} -\alpha_1 \\ \alpha_2 \end{pmatrix} \begin{pmatrix} 1 & -\beta \end{pmatrix}$$

and $\text{rank}(\pi) = 1$, since the second row is $-\frac{\alpha_2}{\alpha_1}$ times the first row and the second column is $-\beta$ times the first column, so there is only 1 linearly independent vector involved.

Error Correction and VAR

Consider again a multivariate difference stationary series $y_t = (y_{1t}, \ldots, y_{nt})'$. It has been mentioned earlier, that modelling Δy_t in a vector autoregression model is inappropriate if y_t is cointegrated. In order to see this important point, assume that y_t follows a VAR(p) in levels:

$$y_t = \mu + \sum_{i_1}^{p} \Phi_i y_{t-i} + \epsilon_t, \quad \epsilon_t \sim \text{NID}(0, \Sigma).$$

We shall now show that it is always possible to rewrite the VAR in levels as a vector error correction model for the first differences. For that purpose, introduce $\pi := \sum_{i=1}^{p} \Phi_i - \mathbf{I}_m$, such that

$$\Delta y_t = \mu + \pi y_{t-1} + \sum_{i=1}^{p} \Phi_i (y_{t-i} - y_{t-1}) + \epsilon_t.$$

Now, note that

$$y_{t-1} - y_{t-i} = (y_{t-1} - y_{t-2}) + (y_{t-2} - y_{t-3}) + \ldots + (y_{t-i+1} - y_{t-i})$$
$$= \sum_{j=1}^{i-1} \Delta y_{t-j}$$

such that

$$\sum_{i=1}^{p} \Phi_i (y_{t-i} - y_{t-1}) = -\sum_{i=1}^{p} \Phi_i (y_{t-1} - y_{t-i}) = -\sum_{i=1}^{p} \Phi_i \sum_{j=1}^{i-1} \Delta y_{t-j}$$

$$= -\Phi_2 \Delta y_{t-1} - \Phi_3(\Delta y_{t-1} + \Delta y_{t-2}) - \ldots - \Phi_p \sum_{j=1}^{p-1} \Delta y_{t-j}$$

$$= \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} \quad \text{where } \Gamma_i = -\sum_{j=i+1}^{p} \Phi_j.$$

Therefore,

$$\Delta y_t = \mu + \pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \epsilon_t.$$

Comparing this with an ordinary VAR in differences,

$$\Delta y_t = \mu + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \epsilon_t$$

we notice that such a VAR in differences is misspecified (by leaving out the explanatory variable y_{t-1}) whenever $\pi \neq 0$, which is exactly what is required for y_t being cointegrated. Intuitively, for cointegrated series, the term πy_{t-1} is needed in order to model how far the system is out of equilibrium.

Exogeneity in cointegrated systems

Consider again a two variate cointegrated vector $x = (x_1, x_2)'$, where this time the speed of adjustment parameter for the first component α_1 is zero, such that Δx_2 has to do all of the adjustment to equilibrium:

$$\begin{aligned}
\Delta x_{1,t} &= a_{10} + \sum_{i=1}^{p} a_{11}(i)\Delta x_{1,t-i} + \sum_{i=1}^{p} a_{12}(i)\Delta x_{2,t-i} + \epsilon_{1,t}, \\
\Delta x_{2,t} &= a_{20} + \sum_{i=1}^{p} a_{21}(i)\Delta x_{1,t-i} + \sum_{i=1}^{p} a_{22}(i)\Delta x_{2,t-i} + \epsilon_{2,t} \\
&\quad + \alpha_2(x_{1,t-1} - \beta x_{2,t-1}).
\end{aligned}$$

We note that in such a case the marginal distribtution of x_1 contains no information about the paramters of interest, α_2 and β. Therefore, x_1 is weakly exogeneous with respect to these paramters, such that estimation of α_2 and β can be done based upon the equation for Δx_2 alone without reference to the specific model for Δx_1.

Also, *it is necessary to reinterpret Granger causality in a cointegrated system.* Note that x_2 does not Granger cause x_1 if both all terms proportional to $\Delta x_{2,t-i}$ vanish and x_1 does not respond to deviations from long-run equilibrium (because this involves a term proportional to $x_{2,t-1}$). So block exogeneity of x_1 requires both $a_{12}(i) = 0$ for all $i = 1, \ldots, p$ and $\alpha_1 = 0$, that is weak exogeneity of x_1. Therefore, cointegration between two $I(1)$ processes implies always Granger causality in at least one direction!

Cointegration and Rank

For notational convenience, consider the simple error correction model

$$\Delta y_t = \pi y_{t-1} + \epsilon_t, \quad \epsilon_t \sim \text{NID}(0, \Sigma)$$

where $y_t = (y_{1t}, \ldots, y_{nt})'$ as before.

We shall show in the following that we can use the rank of π in order to determine whether y_t is cointegrated. More precisely, *the number of cointegrating relationships, or cointegrating vectors, is given by the rank of* π. There are 3 cases.

1. rank$(\pi) = 0$ which implies $\pi = 0$.
   Therefore the model reduces to $\Delta y_t = \epsilon_t$, that is all $y_{it} \sim I(1)$ since $\Delta y_t = \epsilon_t \sim I(0)$, and there is no linear combination of the y_{ti}'s which is stationary because all vectors β with the property $\beta y_t \sim I(0)$ have zero entries everywhere. So all components of y_t are unit root processes and y_t is not cointegrated.

2. rank(π) $= r$ with $1 \leq r < n$.
   Consider first the case rank(π) $= 1$, that is, there is only one linearly independent row in π, which implies that all rows of π can be written as scalar multiples of the first. Thus, each of the $\{\Delta y_{it}\}$ sequences can be written as

   $$\Delta y_{it} = \frac{\pi_{ij}}{\pi_{1j}}(\pi_{11}y_{1,t-1}+\pi_{12}y_{2,t-1}+\ldots+\pi_{1n}y_{n,t-1})+\epsilon_{it}.$$

   Hence, the linear combination

   $$(\pi_{11}y_{1,t-1}+\pi_{12}y_{2,t-1}+\ldots+\pi_{1n}y_{n,t-1}) = \frac{\pi_{1j}}{\pi_{ij}}(\Delta y_{it}-\epsilon_{it})$$

   is stationary, since both Δy_{it} and ϵ_{it} are stationary. So each row of π may be regarded as cointegrating vector of the same cointegrating relationship.

   Similarly, if rank(π) $= r$, each row may be written as a linear combination of r linearly independent combinations of the $\{y_{it}\}$ sequences that are stationary. That is, there are r cointegrating relationships (cointegrating vectors).

3. rank(π) $= n$ $\quad\Rightarrow\quad$ the inverse matrix π^{-1} exists. Premultiplying the error correction model with π^{-1} yields then

   $$\pi^{-1}\Delta y_t = y_{t-1} + \pi^{-1}\epsilon_t$$

   such that all components of y_t are stationary, since both $\pi^{-1}\Delta y_t$ and $\pi^{-1}\epsilon_t$ are stationary. In particular, y_t is not cointegrated.

Johansen's Cointegration tests

Recall from introductory courses in matrix algebra that the rank of a matrix equals the number of its nonzero eigenvalues, also called characteristic roots. Johansen's (1988) test procedure exploits this relationship for identifying the number of cointegrating relations between non-stationary variables by testing for the number of significantly nonzero eigenvalues of the $(m \times m)$ matrix π in

$$\Delta x_t = \pi_0 + \pi x_{t-1} + \sum_{i=1}^{p} \pi_i \Delta x_{t-i} + \epsilon_t.$$

Specifically, the Johansen cointegration test statistics are

1. $\lambda_{\text{trace}}(r) = -T \sum_{i=1}^{m} \log(1 - \hat{\lambda}_i)$, and
2. $\lambda_{\text{max}}(r, r+1) = -T \log(1 - \hat{\lambda}_{r+1})$,

referred to as *trace statistics* and *maximum eigenvalue statistics*, where T is the number of usable observations and $\hat{\lambda}_i$ are the estimated characteristic roots obtained from the estimated π matrix in decreasing order.

The first test statistic

$$\lambda_{\text{trace}}(r) = -T \sum_{i=1}^{m} \log(1 - \hat{\lambda}_i)$$

tests the null hypothesis of less or equal to r distinct cointegrating vectors against the alternative of m cointegrating relations, that is a stationary VAR in levels. Note that λ_{trace} equals zero when all $\lambda_i = 0$. The further the estimated characteristic roots are from zero, the more negative is $\log(1-\hat{\lambda}_i)$ and the larger is λ_{trace}.

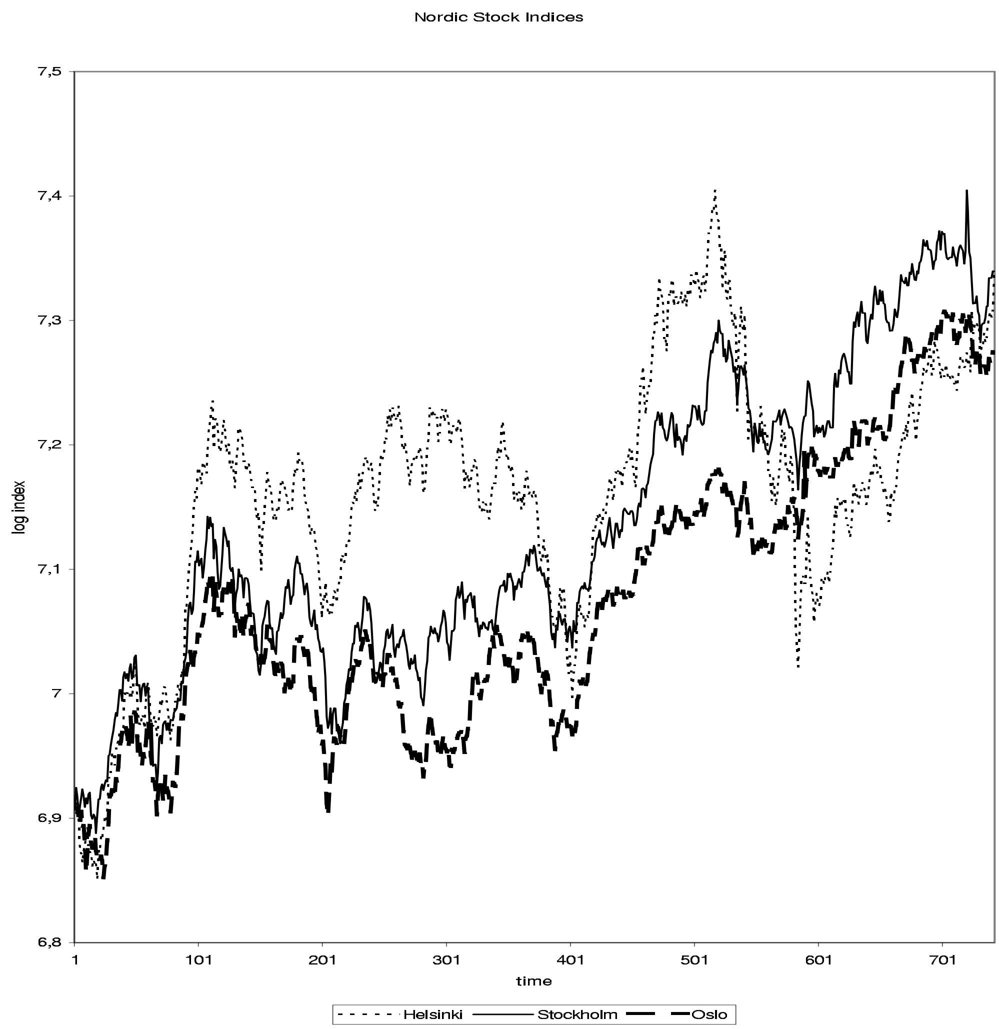
The second test statistic

$$\begin{aligned}\lambda_{\max}(r, r+1) &= -T \log(1 - \hat{\lambda}_{r+1}) \\ &= \lambda_{\text{trace}}(r) - \lambda_{\text{trace}}(r+1)\end{aligned}$$

tests the null of r cointegrating vectors against the alternative of $r+1$ cointegrating vectors. Again $\lambda_{\max}$ will be small if $\hat{\lambda}_{r+1}$ is small.

Critical values of both the λ_{trace} and $\lambda_{\max}$ statistics are obtained numerically via Monte Carlo simulations.

Example. Consider Helsinki, Oslo and Stockholm stock indexes (see figure).



We observe that especially Stockholm and Oslo seem to follow each other rather closely, so the series might well be cointegrated.

In the first step we test the integration of the three stock index shares price series. (EViews output). ADF with five lags.

```
--------------------
Stock       ADF Test
Exchange    Statistic
--------------------
Finland      -2.50
Norway       -0.96
Sweden       -1.24
--------------------
  First differences
--------------------
Finland      -12.0
Norway       -10.8
Sweden       -11.1
====================

    1%   Critical Value*   -3.4418
    5%   Critical Value    -2.8658
    10%  Critical Value    -2.5691

*MacKinnon critical values for rejection
of hypothesis of a unit root.
```

None of the test statistics for the indexes is significant. Hence all (logarithmic) stock indexes are integrated, and checking the first differences reveals that each index is integrated of order one.

Next we test whether the series are cointegrated using Johansen's likelihood ratio tests.

```
Included observations: 736 after adjusting for endpoints
Test assumption: Linear deterministic trend
Series: LFIN LNOR LSWE
Lags interval (in first differences): 1 to 5

Unrestricted Cointegration Rank Test (Trace)
=================================================================
Hypothesized             Trace     5 Percent      1 Percent
No. of CE(s) Eigenvalue Statistic  Critical Value Critical Value
-----------------------------------------------------------------
None **      0.046420    43.04799      29.68          35.65
At most 1    0.010303    8.064672      15.41          20.04
At most 2    0.000601    0.442435       3.76           6.65
=================================================================
*(**) denotes rejection of the hypothesis at 5%(1%) level
Trace test indicates 1 cointegrating equation(s)
at both 5% and 1% levels

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)
=================================================================
Hypothesized            Max-Eigen   5 Percent      1 Percent
No. of CE(s) Eigenvalue Statistic  Critical Value Critical Value
-----------------------------------------------------------------
None **      0.046420    34.98331      20.97          25.52
At most 1    0.010303    7.622238      14.07          18.63
At most 2    0.000601    0.442435       3.76           6.65
=================================================================
*(**) denotes rejection of the hypothesis at 5%(1%) level
Max-eigenvalue test indicates 1 cointegrating equation(s)
at both 5% and 1% levels
```

Both test results suggest that there is one cointegrating vector.

Note: Whether or not significant cointegration relations are found depends crucially upon the assumptions regarding the possible presence of intercepts and deterministic trends in the variables and/or the cointegration equation. Refer to Seppo Pynnönen's lecture notes or the EViews manual for further details.

Below are $\widehat{\beta}$ and $\widehat{\alpha}$ with the cointegrating coefficient for Sweden scaled to one.

```
==============================================================
1 Cointegrating Equation(s):          Log likelihood  7371.934
==============================================================
Normalized cointegrating coefficient (std.err. in parentheses)

          LSWE       LNOR     LFIN
        1.000000 -1.052674  -0.032285
                  (0.04941)  (0.05263)

Adjustment coefficients (std.err. in parentheses)
        D(LSWE)  -0.025430
                  (0.01477)
        D(LNOR)   0.031677
                  (0.01278)
        D(LFIN)  -0.056941
                  (0.01838)
==============================================================
```

Looking at the estimates of the cointegrating vector $\widehat{\beta}$, it seems that the Finnish series is not statistically significant in the cointegration relation while the cointegration relation beween Sweden and Norway could well be $\beta = (1, -1)'$. This suggests testing the restriction $\beta = \lambda(1, -1, 0)'$ which can be done using likelihood ratio tests.