Analysis of Financial Time Series: Exercise Sheet 3

1. Consider the TARCH(1,1) model $u_t|\mathcal{F}_{t-1} \sim N(0, h_t)$ with

$$h_t = \omega + \alpha u_{t-1}^2 + \phi u_{t-1}^2 d_{t-1} + \delta h_{t-1},$$

where $d_t = 1$, if $u_t < 0$ and zero otherwise.

a) Show that

$$\sigma_u^2 := \text{var}(u_t) = \frac{\omega}{1 - (\alpha + \delta + \phi/2)}$$

Hint: The fact that $z_t := u_t/\sqrt{h_t}|\mathcal{F}_{t-1} \sim N(0,1)$ *implies that* $E_t(z_{t+1}^2) = 1$ *and* $E_t(d_{t+1}) = 1/2$*, such that* $E_t(z_{t+1}^2 d_{t+1}) = 1/2$*.*

b) Show that the k-step ahead forecast of the variance is

$$\mathrm{E}_t(u_{t+k}^2) = \sigma_u^2 + (\alpha + \delta + \phi/2)^{k-1}(h_{t+1} - \sigma_u^2),$$

which implies that long horizon forecasts converge to σ_u^2.
Hint: Use the law of iterated expectations.

2. a) Show that $\mathrm{E}(|X|) = \int_{-\infty}^{\infty} |x| f(x)\, dx = \sqrt{2/\pi}$ for $X \sim N(0,1)$.
Hint: Split up the integral into two integrals with limits $0, \pm\infty$ *and apply the symmetry property of the integrand.*

Consider now the EGARCH(1,1) model $z_t := u_t/\sqrt{h_t}|\mathcal{F}_{t-1} \sim N(0, h_t)$ with $\log h_t = \omega + \delta \log h_{t-1} + \alpha\, |z_{t-1}| + \phi z_{t-1}$.

b) Use your result from a) to derive the k-step ahead forecast for the logarithm of the variance $\mathrm{E}_t(\log h_{t+k})$ and show that it converges to $\frac{\omega + \sqrt{2/\pi}\alpha}{1-\delta}$ for $k \to \infty$.

3. Consider the Nordic EViews file from the course webpage, which contains stock index series from Denmark, Finland, Norway and Sweden.

a) Generate and plot for one of these indexes the logreturn series `ret=100*log(index(0)/index(-1))`, its frequency distribution with sample statistics, and the correlogram.

b) If the return series is not white noise, find the best fitting AR, MA, or ARMA model with lag length up to 1, print the estimation output and residual correlogram, and give the reasons for your choice.

c) If there is autocorrelation in the squared residuals, find the best fitting GARCH model with lag length up to 1, print the estimation output and correlogram of squared residuals, and give the reasons for your choice. Consider also asymmetric GARCH. Is the model correctly specified?

4. Let Υ_k denote the k'th cross autocovariance matrix of the jointly covariance stationary residual series ϵ_i and ϵ_j with elements

$$\gamma_{ij}(k) = \mathrm{E}(\epsilon_{i,t-k} \cdot \epsilon_{j,t}).$$

Show that $\Upsilon_k = \Upsilon'_{-k}$.