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Wavelet Multiresolution Analysis of Financial Time Series

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Tiivistelmä

Tämän tutkimuksen tarkoitus on kehittää uusia aallokkeita hyödyntäviä sovelluksia rahoituksen ja taloustieteiden alalle. Aallokkeet ja aallokemuunnos ovat sukua siniaalloille ja Fourier-muunnokselle. Aallokemuunnoksen suurin etu verrattuna Fourier-muunnokseen on sen kyky säilyttää informaatiota myös ajan suhteen ja soveltuu siten paremmin rahoituksen alan tutkimukseen.

Tutkimus sisältää viisi eri sovellusta. Osa sovelluksista on kokonaan uusia ja osa aikaisempien tutkimusten laajennuksia. Kokonaan uusia tutkimuskohteita ovat contagion-ilmiön tutkiminen aallokekoherenssin avulla ja optioista johdettujen valuuttakurssien jakaumien momenttien tutkiminen aalloke-ristikorrelaation avulla. Aikaisempien tutkimusten laajennuksia ovat kansainvälisten osakemarkkinoiden välisten yhteyksien tutkiminen aallokekorrelaation avulla, valuuttakurssien kausaliteetin analysointi aallokkeiden avulla ja aallokeverkkojen toimivuus ennustamisessa.

Tutkimus osoittaa miten aallokemenetelmillä saadaan uutta tietoa rahoituksen ja taloustieteiden tutkimuskohteista. Aallokkeet jakavat tutkittavat rahoituksen aikasarjat eri aikaskaaloihin, joita voidaan tutkia erikseen. Tämä jakaminen tapahtuu tavalla joka sopii erinomaisesti rahoituksen ja taloustieteiden aikasarjoihin. Kun jaamme näitä prosesseja aallokkeiden avulla eri aikaskaaloihin, jaamme samalla prosesseja niiden luonnollisiin rakennusosiin. Tutkimuksen tulokset auttavat ymmärtämään aikaisempaa yksityiskohtaisemmin rahoituksen aikasarjojen ja sitä kautta markkinoiden käyttäytymistä. Siten tutkimuksen tuloksilla on myös suoraa hyötyä markkinoilla toimiville sijoittajille.

Asiasanat

korrelaatio, koherenssi, aallokkeet, kansainväliset markkinat, multiresoluutioanalyysi, ennustaminen

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Abstract

The contribution of this thesis is to develop new applications for economics and finance that are based on wavelet methods. Wavelet methods are closely related to Fourier methods. However the main advantage of wavelet methods is the ability to conserve time information and is therefore is more suitable for financial research than Fourier methods.

In the following, five different applications of the wavelet methods are presented. These five applications aim to extend wavelet methods to new research areas in finance. Some of them are novel applications, while some of them are extensions of previous research. Novel applications study contagion phenomenon with wavelet coherence and the moments of exchange rate distributions implied by exchange rate options with wavelet cross-correlation. Extensions of previous research include the studies of overall linkages of major equity markets using wavelet correlation and wavelet cross-correlation, the causality between exchange rates with wavelets and an application of wavelet networks to financial forecasting.

Overall this thesis show that many issues previously dealt in economic and financial time series analysis may gain new insight with wavelet analysis by separating processes on different time scales and repeating the traditional analysis on these separate scales. The characteristics of wavelet methods fit perfectly to the features of financial time series. Economic and financial processes constitute inherently from multiple processes on different time scales. When economic and financial time series are decomposed to their wavelet components, they are concurrently decomposed to their natural building blocks. The results of this thesis give new information from financial time series and improve our understanding of financial markets. Therefore practitioners in markets benefit directly from the results of this thesis.

Keywords

correlation, coherence, wavelets, international markets, multiresolution analysis, forecasting

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As a 25 year old student I decided that my dissertation would be finished by the time I reach 28. At the age of 31 I am still finishing my dissertation. Things have not gone according to a plan. But life never goes according to plans; otherwise it would be quite boring to live. Now that this work is finally coming to an end, I have many people to thank.

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Prioritizing things is not an easy task. My thesis is finished at the age 31. It could be 28 if I had not been blessed with three beautiful children. To say that one could die for someone, and truly mean it, is an astonishing feeling. I have had the best moments of my life with Minttu, Mitja and Milja. Later on Raisa joined my family. They have taken up most of my time during the last couple of years and I have been forced to make compromises in many other areas of my life. But I do not regret any of these compromises.

During my thesis work, life has shown me its unpredictable nature. Things have turned upside-down many times. There were periods of time when I did not have a clue where I would be in a couple of months. But I have had something exceptional with me through turbulent times. Love from my wife, Marina, and she is the reason I am here now. Therefore I would like to dedicate this dissertation to my lovely wife Marina.

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1 INTRODUCTION

A natural concept in financial time series is the notion of multiscale features. That is, an observed time series may contain several structures, each occurring on a different time scale. Wavelet techniques possess an inherent ability to decompose this kind of time series into several sub-series which may be associated with a particular time scale. Processes at these different time-scales, which otherwise could not be distinguished, can be separated using wavelet methods and then subsequently analyzed with ordinary time series methods. Wavelet methods present a lens to the researcher, which can be used to zoom in on the details and draw an overall picture of a time series in the same time. In a way one could say that with wavelet methods we are able to see both the forest and the trees. Gençay et al. (2002a) argue that wavelet methods provide insight into the dynamics of economic/financial time series beyond that of standard time series methodologies. Also wavelets work naturally in the area of non-stationary time series, unlike Fourier methods which are crippled by the necessity of stationarity.

There are several examples of wavelet methods which have a lot of potential in economics and finance. The maximal overlap discrete wavelet transform (MODWT) (Percival & Walden 2000) is one, which is a modification of the ordinary discrete wavelet transform. This transform loses orthogonality but acquires attributes suitable for economic research like smoothness and possibility to analyze non-dyadic processes (processes that are not multiples of two). There are many applications of the MODWT which are surveyed more closely in the following chapters. Central to this thesis are the estimators of wavelet variance, wavelet correlation and wavelet cross-correlation. Another potential method is wavelet coherence analysis (Grinsted et al. 2004), which allows correlation analysis in the state space. A third group of methods are wavelet networks and their applications of forecasting an economic or financial time series. This thesis focuses on the three examples mentioned above and analyzes their possibilities in detail.

Traditionally financial analysis has almost exclusively used the time domain in econometric modeling. Although wavelet literature has rapidly expanded in other disciplines, the potential for using wavelets in economics has been long overlooked. Some pioneering work has been made, but these papers have not been widely cited and have in fact, been largely ignored. The connection to Fourier analysis may have diminished the interest in wavelet analysis, because Fourier spectral analysis largely failed in the area of economics research. The problem of Fourier analysis is that the time information is lost completely. The assumption of "natural" periods and stationarity that are inherent in the Fourier methods are also problematic. The variation in frequencies and "non-natural" periods are an inherent part of an economic time series. Therefore the Fourier methods do not work here. But the analysis of these kinds of processes is the strength of wavelet analysis; the key is its ability to separate the dynamics in a time series over a variety of different time horizons.

In recent years the interest for wavelet methods has increased in economics and finance. This recent interest has focused on multiple research areas in economics and finance like exploratory analysis, density estimation, analysis of local inhomogeneities, time scale decomposition of relationships and forecasting (Crowley 2005). Behind all these possible applications is the capability of wavelets to decompose processes on different time scales, but still preserve time localization. In some sense, wavelet analysis picks up the best of both worlds, introducing an intelligent compromise between time and frequency analysis. It provides an efficient way to localize changes across time scales while maintaining the entropy conservation. This locality property and the ability to stationarize data make wavelets a suitable tool for analyzing economic and financial stochastic nonstationary processes. Therefore, these new methods bring fresh thinking to financial and economic analysis. By decomposing a time series on different scales, one may expect to obtain a better understanding of the data generating process as well as dynamic market mechanisms behind the time series. Investigation methods applied to a financial time series over the last decades can now be implemented to multiple time series presenting different scales (frequencies) of the original time series. Therefore efficient discretization of the time-frequency space allows isolation of many interesting structures and features of economic and financial time series which are not visible in the ordinary time-space analysis or in the ordinary Fourier analysis.

The contribution of this thesis is to present new applications in economics and finance where wavelets demonstrate significant potential. In the following chapters, five different applications of the wavelet methods are presented. These five applications aim to extend wavelet methods to new research areas in finance. Some of them are novel applications (chapters 3 and 5), while some of them are extensions of previous research (chapters 2, 4 and 6). The next chapter studies the overall linkages of major equity markets using wavelet correlation and wavelet cross-correlation. Markets consist of agents working in different time horizons. Therefore, it would be natural that the dynamics of the interrelations between markets consist of scales that possibly behave differently. Indications of this kind of structure have been verified by previous studies (see for example Schleicher 2002). The third chapter uses wavelet coherence and wavelet correlation methods to analyze contagion, during the last 25 years. The contagion refers to phenomenon where interrelations between markets strengthen after some crisis. The contagion phenomenon is inherently transient in nature. Therefore, wavelets have just the right characteristics to analyze its existence. The fourth chapter studies the linkages between exchange rates. The sample consists of European exchange rates and the focus is on causality between the series on different time scales. This scale-dependent causality is of great value to the participants in those markets since markets accumulate of investors working on many different time horizons (like institutional investors and day traders). The fifth chapter applies wavelet methods in analyzing the moments of exchange rate distributions implied by exchange rate options. This novel approach introduces wavelets to a totally new and specific area of financial research. The sixth chapter utilizes wavelets to financial forecasting. A wavelet network method is compared to a basic linear forecast method and a random walk model.

Overall these chapters show that many issues previously dealt in economic and financial time series analysis may gain new insight with wavelet analysis by separating processes on different time scales and repeating the traditional analysis on these separate scales. The characteristics of wavelet methods fit perfectly to the features of financial time series. Economic and financial processes constitute inherently from multiple processes on different time scales. When economic and financial time series are decomposed to their wavelet components, they are concurrently decomposed to their natural building blocks.

1.1 Historical survey

Although this thesis is about wavelet analysis, a thorough presentation of Fourier analysis is provided as well. There are two reasons for this. First of all, the Fourier methods are an alternative (and a competitor) for the wavelet methods, so comparing these methods is natural. Secondly, although the wavelet methods are different, they are based on Fourier analysis (Mallat 1999). This chapter begins with a historical survey of Fourier methods, continues with a historical survey of wavelet methods and concludes with a thorough literature review of wavelet methods in financial analysis and the presentation of contribution.

1.1.1 Fourier theory

Usually the origins of Fourier theory are attributed to Joseph Fourier, who presented a paper to the Paris Academy in 1807, where he argued that an arbitrary 2π -periodic function can be represented as an infinite series of sines and cosines. (Jaffard, Meyer & Ryan 2001)

$$f(x) = a_0 + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx)$$

The seeds of Fourier theory were planted over 50 years earlier by d'Alembert, D. Bernoulli, Euler and Lagrange (Dym & McKean 1972). d'Alembert (1747) studied the oscillations of a violin string which can be obtained from a differential equation of the form

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2},$$

where $u = u(t,x)$ presents the displacement of the string, as a function of the time t and place x. The solution to the differential equation above is

$$u(t,x) = \frac{1}{2} f(x+t) + \frac{1}{2} f(x-t).$$

Euler proposed in 1748 that the solution could be presented as a series, where

$$f(x) = \sum_{n=1}^{\infty} \hat{f}(n) \sin n\pi x,$$

such that

$$u(t,x) = \sum_{n=1}^{\infty} \hat{f}(n) \cos n\pi t \cdot \sin n\pi x.$$

The formula for calculating the coefficients,

$$\hat{f}(n) = 2\int_0^1 f(x) \sin n\pi x dx,$$

was introduced by Euler in year 1777 (Dym & McKean 1972).

In his paper Théorie analytique de la chaleur (published 1822), discussing the problems of heat flow

$$\frac{\partial u}{\partial t} = \frac{1}{2} \frac{\partial^2 u}{\partial x^2}$$

and presented to the Académie des Sciences in 1811, Fourier tried to prove that any piecewise smooth function *f* can be expanded into a trigonometric sum. Paul Du Bois-Reymond constructed in 1873 a continuous, 2π -periodic function of the real variable *x* whose Fourier series diverged at a given point. The whole of 19th century went into the study of the challenging questions of the convergence of the Fourier series, attracting the greatest mathematicians of that time such as Poisson, Dirichlet and Riemann. (Jaffard et al. 2001)

Lebesgue, in his dissertation "Intégrale, longueur, aire" in 1902 presented that the proper setting of Fourier series turned out to be the class of “Lebesgue measurable” functions of period 2π , say, with

$$\|f\|^2 \equiv \int_0^1 |f(x)|^2 \, dx < \infty .$$

What Fourier had found was a new functional space of square-integrable functions, denoted $L_2[0,2\pi]$. The result that bins all together is the theorem of Riesz-Fischer (Viaclovsky 2003): For square-Lebesgue-integrable functions, the Fourier coefficients

$$\hat{f}(n) = \int_0^{2\pi} f(x) e^{-2\pi inx} dx, \quad n \in \mathbb{Z}$$

provide a one to one map of the function space onto the space of sequences $\hat{f}(n)$, $(n = ..., -1, 0, 1, 2, ...)$ with

$$\|\hat{f}\|^2 \equiv \sum_{n=-\infty}^{\infty} |\hat{f}(n)|^2 < \infty .$$

This map preserves geometry $\|f\| = \|\hat{f}\|$ and the associated Fourier series

$$f(x) = \sum_{n=-\infty}^{\infty} \hat{f}(n) e^{2\pi inx}$$

converges in the sense that

$$\lim_{n\to\infty} \int_0^{2\pi} \left| f(x) - \sum_{|k|\le n} \hat{f}(k) e^{2\pi ikx} \right| dx = 0 .$$

A parallel development was carried out for the Fourier integral (or the Fourier transform)

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i\omega x} dx$$

for nonperiodic functions *f*, decaying sufficiently rapidly at ±∞ (Dym & McKean 1972). This progress ended in the theorem of Plancherel in 1910 (Weisstein 2006): If *f* is Lebesgue measurable and if

$$\|f\|^2 \equiv \int_{-\infty}^{\infty} |f(x)|^2 \, dx < \infty$$

then $\left\|\hat{f}\right\| = \|f\|$ and f itself may be recovered from $\hat{f}$ via the inverse Fourier integral (or transform)

$$f(x) = \int_{-\infty}^{\infty} \hat{f}(\omega) e^{2\pi i \omega x} dx.$$

After all one can say that the significance of Fourier theory in different applications has been huge. The 20th century was a very active time in applying Fourier theory to different applications. For example the Fast Fourier transform (FFT) introduced by Cooley and Tukey (1965), has been said to be the most important numerical algorithm of our lifetime.

1.1.2 Wavelet theory

Although solid progress in the field was first made in the beginning of the 1980's, the seeds of wavelet theory were planted already in the beginning of 20th century by Alfred Haar. In 1909 he found an orthogonal system of functions defined on $[0,1]$, that form a series converging uniformly to a continuous function *f* on. What Haar found was the simplest basis of the family of wavelet bases. Formation of the Haar basis begins with the function h such that $h(x) = 1$ for $x \in [0, \frac{1}{2})$, $h(x) = -1$ for $x \in [\frac{1}{2}, 1)$ and $h(x) = 0$ for $x \notin [0,1)$. The basis functions are then formed according to the rule

$$h_n(x) = 2^{j/2} h\left(2^j x - k\right),$$

where $n = 2^j + k \geq 1$, $j \geq 0$ and $0 \leq k < 2^j$. Adding the function $h_0(x) = 1$ on $[0,1)$, the sequence $h_0, h_1, h_2, ..., h_n, ...$ is an orthonormal basis for $L^2[0,1]$ (Jaffard et al. 2001). $S_n(f)(x) = \langle f, h_0 \rangle h_0(x) + \ldots + \langle f, h_n \rangle h_n(x)$, where $\langle \cdot, \cdot \rangle$ is the ordinary inner product of the functions, are then approximations of a continuous function by step functions whose values are the mean values of $f(x)$ in the appropriate intervals.

The simplest case of the Haar wavelet basis is problematic. There is a lack of coherence because we are approximating a continuous function with discontinuous functions. Estimating a function *f*, that is $C^n, n = 1,2,3,\ldots$ on the interval $[0,1]$, with the Haar basis does not work.

Faber and Schauder (1914) investigated this problem in the early 20th century (see Jaffard et al. 2001 for discussion). Their improved basis consists of continuous polygonal lines, i.e. "triangles". They set $\Delta_0(x) = x$ and $\Delta_{-1}(x) = 1$. The remaining functions are defined as

$$\Delta_n(x) = \Delta(2^j x - k), \quad n = 2^j + k, \quad j \geq 0, \quad 0 \leq k \leq 2^j,$$

where

$$\Delta(x) = \begin{cases} 2x, & 0 \leq x \leq \frac{1}{2} \\ 2(1-x), & \frac{1}{2} \leq x \leq 1 \\ 0, & x \notin [0,1] \end{cases}$$

Then the sequence $\Delta_{-1}, \Delta_0, \Delta_1 \ldots, \Delta_n, \ldots$ is a Schauder basis for the Banach space E of continuous functions on $[0,1]$ so that every continuous function on $[0,1]$ may be written as

$$f(x) = a + bx + \sum_{n=1}^{\infty} \alpha_n \Delta_n(x),$$

with uniform convergence and unique coefficients. The Schauder basis is superior to the Fourier basis for studying local regularity properties. For example, the Schauder basis can be used to study the multifractal structure of the Brownian motion. (Jaffard et al. 2001)

With the Fourier basis it is difficult to localize the energy of a function (where energy can be defined as an integral of square function). The spatial distribution of the function's energy remains "hidden". In the 1930s Littlewood and Paley (1937) discovered a way to manipulate the Fourier series so that energy localization can be revealed. They formed dyadic blocks to decompose the series and then applied Fourier series to those blocks. The connection to wavelets was made by Antoni Zygmund and his group at the University of Chicago (see Altmann (1996) for discussion). Their work is based on a sequence of operators $\Delta_j, j \in \mathbb{Z}$ that constitute a bank of band-pass filters, oriented on frequency intervals covering approximately one octave. A band-pass filter is a filter that passes through a certain interval of frequencies. Dyadic wavelets analyze the function in octave intervals. (Altmann 1996)

In the year 1927 Philip Franklin, using the usual Gram-Schmidt procedure, created a new orthonormal wavelet basis from the Schauder basis (Franklin 1927). This sequence (f_n) is called the Franklin basis and satisfies

$$\int_0^1 f_n(x)\,dx = \int_0^1 x f_n(x)\,dx = 0 \quad \text{for} \quad n \geq 1.$$

The advantage of the Franklin basis over the Haar and the Schauder basis is that it may be used to decompose any function *f* in $L^2[0,1]$. The Franklin basis works in both regular and irregular situations. However the problem with the Franklin basis is its complex algorithmic structure; Franklin wavelets are not derived from a fixed wavelet function by integer translations and dyadic (multiples of two) dilations. (Jaffard et al. 2001)

In the 1930s Lusin introduced Hardy spaces, which can be identified as closed subspaces of $L^p(\mathbb{R})$ and today they are important today in signal processing. Guido Weiss and Ronald Coifman were the first to interpret Lusin's theory in terms of atoms and atomic decomposition, which is one of the cornerstones of wavelet theory. Atoms are the simplest elements of the function space and the objective is to find the atoms and the "assembly rules" that allow one to reconstruct all the elements of the function space using these atoms. Marcinkiewicz showed in 1938 that the simplest atomic decomposition for the spaces $L^p[0,1], 1 < p < \infty$, is given by the Haar system. (see Altmann (1996) for discussion)

One approach to atomic decompositions is given by Calderón's identity. It is based on the function ψ belonging to $L^2(\mathbb{R}^n)$. Its Fourier transform $\hat{\psi}(\omega)$ is subject to the condition that

$$\int_0^\infty \left|\hat{\psi}(t\omega)\right|^2 \frac{dt}{t} = 1$$

for almost all $\omega \in \mathbb{R}^n$. Let Q_t denote the operator defined as the convolution with ψ_t, where $\psi_t(x) = t^{-n}\psi\left(\frac{x}{t}\right)$. Similarly define Q_t^* as convolution with $\tilde{\psi}_t$, where $\tilde{\psi}_t = t^{-n}\bar{\psi}\left(-\frac{x}{t}\right)$ and $\bar{\psi}$ is conjugate of ψ. Then the Calderón's identity is a decomposition of the identity operator, written symbolically as (Jaffard et al. 2001)

$$I = \int_0^\infty Q_t Q_t^* \frac{dt}{t}.$$

Grossman and Morlet rediscovered this identity in 1980, 20 years after the work of Calderón. However they had a different interpretation for the identity which they relate to the coherent states of quantum mechanics (Jaffard et al. 2001). They came up with the notion of analyzing wavelets

$$\psi_{(a,b)}(x) = a^{-n/2}\psi\left(\frac{x-b}{a}\right), \quad a > 0, \quad b \in \mathbb{R}^n,$$

that work as an orthonormal basis for the function space. Grossman and Morlet were also the first ones to define the wavelet coefficients as the inner product of a function and the analyzing wavelet (following the notation of Jaffard et al. (2001))

$$W(a,b) = \int f(x)\bar{\psi}_{(a,b)}(x)dx$$

with the synthesis function

$$f(x) = \int_0^\infty \int_{\mathbb{R}^n} W(a,b)\bar{\psi}_{(a,b)}(x)\,db\frac{da}{a^{n+1}}.$$

Wavelets can be defined in multiple ways. The first definition of a wavelet comes from Grossman and Morlet and is quite broad.

A wavelet is a function ψ in $L^2(\mathbb{R})$ whose Fourier transform $\hat{\psi}(\omega)$ satisfies the condition $\int_0^\infty \left|\hat{\psi}(t\omega)\right|^2 \frac{dt}{t} = 1$ almost everywhere.

The second definition of a wavelet is adapted to the Littlewood-Paley-Stein theory. A wavelet is a function ψ in $L^2(\mathbb{R}^n)$ whose Fourier transform $\hat{\psi}(\omega)$ satisfies the condition $\sum_{-\infty}^{\infty}\left|\hat{\psi}\left(2^{-j}\omega\right)\right|^2 = 1$ almost everywhere. If ψ is a wavelet in this sense, then $\sqrt{\log 2}\psi$ satisfies the Grossmann-Morlet condition (Jaffard et al. 2001).

The third definition relates to the work of Haar and Strömberg. A wavelet is a function ψ in $L^2(\mathbb{R})$ such that $2^{j/2}\psi\left(2^j x - k\right), j,k \in \mathbb{Z}$, is an orthonormal basis for $L^2(\mathbb{R})$. It can be shown that such a wavelet satisfies the second condition.

In the beginning many different theories threaded together to form the wavelet theory. The work of Stéphane Mallat and Yves Meyer in the 1980s gave the wavelet theory a new start and a journey towards mainstream science. In 1985 Mallat discovered similarities between the following objects (Mallat 1989):

1. the quadrature mirror filters, which were invented by Croisier, Esteban and Galand for the digital telephone;
2. the pyramid algorithms of Burt and Adelson, which are used in the context of numerical image processing;
3. the orthonormal wavelet bases discovered by Strömberg and Meyer.

Mallat succeeded in unifying different aspects of wavelet theory when he came up with the concept of a "multiresolution analysis". This analysis also gives an elegant way of constructing wavelets.

Using Mallat's discovery, Ingrid Daubechies (1988) continued Haar's work. She constructed a family of orthonormal basis of the form $2^{j/2}\psi_r\left(2^j x-k\right), \quad j,k\in\mathbb{Z}$, with the following properties:

– The support of ψ_r is the interval $\left[0,2r+1\right], \quad r\in\mathbb{Z}$.

– $\int\limits_{-\infty}^{\infty} x^n\psi_r\left(x\right)dx=0,$ for $0\le n\le r$.

– ψ_r has γr continuous derivatives, where the constant γ is about 1/5.

When $r=0$, this reduces to the Haar system. The Daubechies wavelets are very suitable for applied work because they have a preassigned degree of smoothness and compact support. They are more efficient in signal compression than the Haar wavelet. Synthesis using Daubechies's wavelets also gives better results than the Haar wavelet. The problem of the Haar wavelet is that a regular function is approximated by functions which have strong discontinuities. This problem is prevented by the smoothness of the Daubechies wavelets.

Due to the history behind the wavelet theory, applications of wavelets emerged in economics and finance much later than in engineering. Most of the theory of wavelets was done in the context of deterministic functions, not stochastic processes, which are central in economics and finance. The statistical theory for wavelets emerged in the mid 1990s and only today wavelets are on the verge of entering mainstream econometrics (Schleicher 2002).

1.2 Wavelets in finance

Wavelets have achieved an impressive popularity in natural sciences, especially in earth sciences (see for example Labat (2005) and Labat et al. (2005)). Wavelet methods have been applied in engineering for nearly two decades now, but still the first applications of wavelets in economics and finance emerged only ten years ago, despite its' suitability for this discipline. In the following, the literature review of wavelets in finance is presented. This review is separated into three different sections. The first section focuses on the decomposition applications of wavelet methods. The second section limits the applications to interdependence studies with wavelets. The last section is a collection of studies with wavelets that do not fit into either of the two earlier sections.

1.2.1 Wavelet as a decomposition tool

One of the fundamental advantages of wavelet analysis is the capability to decompose time series into different components. This aspect has also been widely applied in recent research. Capobianco (2004) applies wavelet methods to the multiresolution analysis of high frequency Nikkei stock index data. He applies the matching pursuit algorithm of Mallat and Zhang (1993) and argues that it suits excellently to financial data. Capobianco shows how the wavelet matching pursuit algorithm can be used to uncover hidden periodic components. Crowley and Lee (2005) analyze the frequency components of European business cycles with wavelet multiresolution analysis. They use a real GDP as a proxy for the business activity of European countries. The maximal overlap discrete wavelet transform is used for the analysis. They find significant differences between the countries, where the degree of integration varies significantly. Other wavelet related findings are that most of the energy in these economic time series can be found in longer term fluctuations. Also, they find indications that recessions are a result of a simultaneous dip in growth cycles at all frequencies. Gençay et al. (2001a) investigate the scaling properties of foreign exchange rates using wavelet methods. They use the maximal overlap discrete wavelet transform estimator of the wavelet variance to decompose variance of the process and find that foreign exchange rate volatilities are described by different scaling laws on different horizons. Similar wavelet-multiscale studies are also in Gençay et al. (2001b), Gençay et al. (2003), Gençay & Selçuk (2004), Gençay et al. (2005) and Gençay & Fan (2009).

Gençay et al. (2001b) use wavelets to construct a method for seasonality extraction from a time series. Their method emphasizes many advantages of wavelet methods. It is simple, free of model selection parameters, translationally invariant, is associated with a zero-phase filter and is circular. Ordinary discrete transform filters are not zero-phase. Gençay et al. however use the maximal overlap discrete wavelet transform, which has zero-phase filters. Gençay et al. (2005) use somewhat similar ideas to propose a new approach for estimating the systematic risk of an asset. They find that the estimations of CAPM might be flawed because of the multiscale nature of risk and return. Gencay et al. (2003) decompose a given time series on a scale-by-scale basis. On each scale, the wavelet variance of the market return and the wavelet covariance between the market return and a portfolio are calculated to obtain an estimate of a portfolio's beta. This reveals that the estimations of the CAPM are more relevant in the medium and long run than on to the short time horizons. Gencay et al. (2004) propose a simple yet powerful method to analyze the relationship between a stock market return and volatility on multiple time scales using wavelet decomposition. The results show that the leverage effect is weak at high frequencies but becomes prominent at lower frequencies. Also the positive correlation between the current volatility and future returns becomes dominant on the timescales of one day and higher, providing evidence that risk and return are positively correlated. Vuorenmaa (2005, 2006) analyzes stock market volatility using the maximal overlap discrete wavelet transform. He finds that the global scaling laws and long memory of stock's volatility may not be time-invariant.

1.2.2 Wavelets and interdependence between variables

Wavelets have been widely used to study interdependence of economic and financial time series. The studies presented in the following have also decomposition aspects but their main aspect is in the interdependence of processes. In & Kim (2006c, 2007), In et al. (2008) and Kim & In (2005, 2006, 2007) have conducted many studies in finance using the wavelet variance, wavelet correlation and cross-correlation. Kim & In (2005) study the relationship between stock markets and inflation using the MODWT estimator of the wavelet correlation. They conclude that there is a positive relationship between stock returns and inflation on a scale of one month and on a scale of 128 months, and a negative relationship between these scales. Furthermore they stress how the wavelet based scale analysis is of utmost importance in the economics studies since their results solve many puzzles around the Fisher hypothesis previously noted in literature. In et al. (2008) study the performance of US mutual funds using wavelet multiscaling methods and the Jensen's alpha. The results reveal that none of the funds are dominant over all time-scales. In & Kim (2006c) study the relationship between stock and futures markets with the MODWT based estimator of wavelet cross-correlation. There is a feedback relationship between the stock and the futures markets on every scale. The results also reveal that correlation increases as time scale increases. In & Kim (2007) examine how well the Fama-French factor model works on different time scales. They conclude that the SMB (small capital business minus big capital business) and the HML (high book-to-market minus low book-to-market) share much of the information with alternative investment opportunities in the long run but not in the short run. A similar study is Kim & In (2007) examining the relationship between stock prices and bond yields in the G7 countries. The key results include indications that the correlation between changes in stock prices and bond yields can differ from one country to another and can also depend on the time scale. Therefore the importance of scale-dimension is verified again. Kim & In (2006) find that correlation between industry returns and inflation does not vary along with the scale. Furthermore they find indications that industry returns can be used as a hedge against inflation, depending on the particular industry.

Gençay et al. (2001a) also include a study which analyzes the dependencies between foreign exchange markets. Findings include an increase of correlation from intra-day scale towards the daily timescale and the stabilization of correlation for longer time scales. Dalkir (2004) studies the causality relationship between money and output on different timescales using wavelets. He finds scale dependent changes in the direction of causality between money and income and so emphasizes the importance of scale-dimension in causality studies. Fernandez (2005) studies the return spillovers in major stock markets on different time scales. Her conclusions are mainly that G7 countries significantly affect global markets and the reverse reaction is much weaker. Lee (2004) conducts somewhat similar study with the discrete wavelet transform based multiresolution analysis. He finds indications of volatility and return spillovers from the developed markets to the emerging markets on multiple scales. In the relationships between economic variables, Gallegati (2008) studies the relationship between stock market returns and economic activity. He applies the maximum overlap discrete wavelet transform to the Dow Jones Industrial Average stock price index and to the industrial production index for the US. Use of wavelet variance, wavelet correlation and cross-correlations are applied to analyze the association as well as the lead/lag relationship between stock prices and industrial production on different time scales. His results show that stock market returns lead economic activity at lower frequencies. This lead also increases along with the scale.

The work of Crowley and Lee (2005) was already mentioned in the previous section. They also study interdependencies inside the euro zone using wavelet methods. The results reveal significant differences between European countries in the degree of integration. Some countries like Germany, France and Belgium have strong correlations with the euro zone aggregate. On the other hand Finland, Ireland, Sweden and the UK have much lower correlation with the euro zone aggregate. Shrestha & Tan (2005) empirically analyze the long-run and short-run relationships among real interest rates in G-7 countries. A wavelet transform based analysis reveals the existence of both short-run and long-run relationships. They do not find evidence for strict interest rate parity. Gallegati & Gallegati (2005) study the industrial production index of G-7 countries using multi-scaling approach based on the MODWT estimator of wavelet variance and correlation. Lee (2004) investigates the international transmission mechanism of stock market movements via wavelet analysis. Using a daily data of stock indices, he finds a strong evidence for price as well as volatility spillover effects from the developed stock market to the emerging market, but not vice versa.

1.2.3 Other topics with wavelets in finance

Decomposition and interdependence applications have two most extensively studied areas of wavelets in finance. Wavelets, however, can be applied in many kinds of situation in financial research. Gencay et al. (2001b) propose a simple wavelet multiscale method for extracting intraday seasonalities from a high frequency data. These seasonalities cause distortions in the estimation of volatility models and are also a dominant source for the underlying misspecifications of these volatility models. Their methodology is simple and efficient in preventing the estimation errors mentioned above. Gencay & Fan (2009) develop a wavelet approach to test the presence of a unit root in a stochastic process and applying it to financial time series. Their conclusions are similar to Gencay et al. (2001b).

Ramsey and Zhang (1997) use wavelets or more generally waveforms to analyze foreign exchange data. Their method is based on the matching pursuit algorithm introduced by Mallat and Zhang (1993). The results reveal that waveform dictionaries are most efficient with non-stationary data. Since economic variables tend to fall into this category, again proof for the importance of wavelet methods in economics and finance is found. Jensen (2000) applies wavelet methods cunningly to develop an alternative maximum likelihood estimator of the differencing parameter d of fractionally integrated processes. He shows how the wavelet transform of these kinds of processes have a sparse covariance matrix that can be approximated at high precision with a diagonal matrix. Therefore the calculation of the likelihood function is of an order smaller than calculations with the exact MLE methods. Furthermore he demonstrates how the wavelet-MLE method is superior compared to other semi parameter estimation methods. Tkacz

(2000) applies the method of Jensen (1999) to interest rates in the U.S. and Canada and find that rates are mean-reverting in the very long run, with the fractional order of integration increasing with the term to maturity.

Conway and Frame (2000) use wavelets for spectral analysis of New Zealand output gaps. They use wavelets to compare different spectral estimation methods and find substantial differences in their low frequency components. Therefore they question the reliability of low frequency results of previous spectral estimation studies. Murtagh et al. (2004) extend the applications of wavelet methods to forecasting. They apply the maximal overlap discrete wavelet transform to decompose the time series and then forecast these different scale crystals separately. The results indicate that the multiresolution approaches outperform the traditional approaches in modeling and forecasting. Renaud et al. (2003) investigate very similar ideas. They also divide original time series to multiresolution crystals and then forecast these crystals separately. These forecasts are then combined to achieve an aggregate forecast for the original time series. In simulation studies the method works very well and competes with up-to-date methods.

Neuman and Greiber (2004) use wavelets as one of the applied filters to study the importance of money for inflation in the euro area. They use wavelets to study the relation between money and inflation in the frequency domain. The results show that the relation between money and inflation appears to rest on relatively long-lasting cycles of monetary growth. Short to medium-term fluctuations of money growth with cycles of up to about 8 years were found to be insignificant for inflation. Atkins and Sun (2003) use wavelets to uncover the Fisher effect between nominal interest rates and inflation. They eliminate long memory using the discrete wavelet transform and then estimate the standard Fisher equation regression in the wavelet domain. This method is then applied to study the Fisher effect to conclude that it cannot be identified on a short time scale. The degree of fit of the regression increases towards longer time scales.

Whitcher & Jensen (2000) propose a nonstationary class of stochastic volatility models that feature time-varying parameters and use them to analyze the long-memory behavior of a time series. In their estimation of the long-memory parameter they use a log linear relationship between the local variance of maximum overlap discrete wavelet transform's coefficients and their scaling parameter to produce a semi parametric OLS estimator. Nekhili et al. (2002) compare the empirical distributions of exchange rates with well-known continuous-time processes at different frequencies. Using wavelets they find that there is not a distribution that suits both the low and high frequency data of exchange rates. Leong & Huang (2006) propose a new way to detect spurious regression using wavelet covariance and correlation. They achieve an efficient and simple method which is able to detect the spurious relationship in a bivariate time series more directly than ordinary methods. Antoniou & Vorlow (2003) demonstrate how a wavelet semi-parametric approach can provide useful insight on the structure and behavior of stock index prices, returns and volatility.

1.3 Framework and contribution

1.3.1 Framework

This section presents the contribution of this thesis in detail. The aim of this thesis is to extend the applications of wavelet methods in finance. The next chapter studies correlation of the returns of major world stock indices. The non-decimated discrete wavelet transform is implemented to quantify international volatility linkages between markets. This transform decomposes volatility on a scale by scale basis and gives information of correlation at certain time scales. The following chapter succeeds the previous chapter although the focus is somewhat different. A thorough examination of contagion among the major world markets during the last 25 years is carried out. The analysis uses a novel way to study contagion with the help of wavelet methods. The comparison is made between correlations at different time scales using wavelet coherence and the MODWT estimator of wavelet correlation. The fourth chapter extends the interrelation studies to European exchange rates. Lead-lag relations of major European currencies are studied using wavelet cross-correlation. The estimators of wavelet cross-correlation are constructed using the maximal overlap discrete wavelet transform. The fifth chapter provides a novel wavelet analysis on the cross-dynamics of exchange rate expectations. Over-the-counter currency options on the euro, the Japanese yen, and the British pound vis-à-vis the U.S. dollar are used to extract the expected probability density functions of future exchange rates and recent wavelet cross-correlation techniques are applied to analyze linkages in these expectations. The last chapter examines the predictability of return and volatility series with different time scales and examines the benefit of using a non-linear predictor, namely a wavelet network, in financial framework. The time series used is a daily currency rate between the Japanese yen and the US dollar, which is forecasted two weeks ahead using only present and previous values of the time series and its low-pass filtered transformations.

1.3.2 Contribution and results

The results of the following chapters introduce many new results and open up new frontiers. Wavelet methods play an important role in many of these new results. In some of the results wavelets play a vital part in detecting them. Two main aspects are behind the effectiveness of wavelets in finance. One is the intelligent compromise between the time dimension and the frequency dimension that helps wavelets to avoid the obstacles that have plagued spatial or frequency analysis. Another is the multiscale structure that is a natural part of financial processes. Investors work on many different timescales. And with wavelets we can separate these different time scales.

The knowledge of time scale dynamics between financial markets is important to the participants of the market in their investment planning. Investors should take into account also their investment horizon when they make risk management and portfolio allocation decisions based on correlation structure between markets. Although correlation between returns and volatility are extensively studied subjects in the literature, there is very little research of timescale dynamics of correlation. Some time scale research has been made with intraday dynamics, but usually the longest timescale in these studies is the daily time scale. With wavelet methods we have an easy way to study this new dimension. The results of the next chapter, which focuses on the correlations of stock indices, show that linkages between stock index returns have rich time scale dependant structure. The correlations are weakest at the shortest scales and strengthen with increasing scale. Thus diversification in portfolio management should be most efficient on a short time scale. Even more strongly this scale-dependency is seen with volatilities. Therefore one can argue that this rich structure in the dynamics between the studied indices needs wavelet methods to be revealed.

The third chapter applies wavelets to study contagion. Clear signs of contagion among the major markets are found. Contagion exists also around crises, where its existence has previously been under debate. The results show that short time scale correlation increases during these major crises. At the same time long time scale correlations remain approximately at the same level, indicating contagion. The inclusion of a multiresolution analysis, i.e. different time scales, proves out to be very important. Maybe even vital as correlations change quickly as a function of scale and many changes are seen only on certain time scales. Overall the results indicate that contagion has been a major factor between markets many times in the last 25 years. This has not changed since almost the strongest signs of contagion can be seen during the ongoing financial crisis. Also an overall increase of interdependence is found. As a result of these two aspects, markets have become very highly correlated in the 20th century.

The fourth chapter shows how wavelet cross-correlation allows very fine analysis of lead-lag relations between financial time series. The maximal overlap discrete wavelet transform based estimator of cross-correlation gives good insight to time-scale dependant dynamics of exchange rates. The results indicate that Euro and the Swiss franc lead the British pound on larger scales. On a one month and longer time scales the lag of the Pound is obvious. Including scale-dimension, a more complete picture of the interrelations can be drawn. The importance of this dimension cannot be stressed enough, because market participants naturally have different time horizons in their investment plans. This way, the wavelet methods are just the right solution for them because they can pick up the time-scale from the wavelet analysis that interests them most and make decisions according to this time-scale.

The most specific application is in chapter five, where option implied exchange rate expectations are studied using wavelets. Significant lead-lag relationships between the expected probability densities of major exchange rates are found regardless of time scales. At higher frequencies, the expected volatility of the JPY/USD exchange rate is found to affect the expected volatility of the EUR/USD and GBP/USD exchange rates. However, at lower frequencies, there is also a significant feedback effect from the GBP/USD volatility expectations to the JPY/USD volatility expectations. The higher-order moments of option-implied exchange rate distributions indicate that the market expectations of the JPY/USD exchange rate are unrelated to the developments of the European currencies while moments of the expected EUR/USD and GBP/USD densities are strongly linked with each other. This analysis suggest that the dynamic structure of the relations between exchange rate expectations varies over different time scales. In general, empirical findings suggest that the dynamic structure of exchange rate expectations may vary considerably over different time-scales. Therefore, it is a situation again where important information would have been missed without wavelet based multiresolution analysis.

The last chapter introduces a somewhat different application of wavelet methods. A wavelet network is used to forecast financial time series. The results, however, are not so triumphant. On the contrary some criticism is presented over the practicality of wavelets in forecasting. At least for time series used, there is not any nonlinear structure in the forecast that the wavelet network could capture. The fit to the training data is always better with the wavelet network, but the fit to the testing data is always better with the linear model. This suggests that the wavelet network only adapts to the noise of the training data and this makes the testing data forecast worse. Forecasts of both the wavelet network model and the linear model are somewhat better than the random walk model, suggesting that there is predictability in the series. However the improvements are quite modest so in this sense conclusions are still quite close to the conclusions of Meese and Rogoff (1983). The predictability does not improve on longer forecasting horizons. On the contrary there is a larger difference between the studied models and the random walk model when we are dealing with shorter forecast horizons. This is inconsistent with the recent results that forecasts improve when time horizon increases (Chinn & Meese 1995) and somewhat supports the findings of Carriero et al. (2009). Using even longer forecast horizon might change the picture.

Excluding the last chapter the results of these separate studies show clearly the importance of scale-dimension in economic and financial research. Throughout these chapters there are instances, where wavelet analysis has the key role in the contribution of the study. Financial processes form when multiple agents working on different time horizons participate in the markets. With wavelets we can at least approximately decompose this process into sub-processes presenting contributions of different agents. Therefore we are decomposing the processes to their natural components. Thus, we achieve better understanding of the dynamics of the financial and economics processes.

2 CORRELATION STRUCTURE OF EQUITY MARKETS*

Linkages of major world stock indices are studied. Wavelet methods are used to form a scale dependent correlation of returns and cross-correlation of volatility, because these are the most important statistics for investors. The results indicate that the linkages between the indices vary along with time scale. The correlations between returns and volatilities are weakest at the shortest scales and increase with time horizon. The cross-correlation structure between volatilities is different at shorter time scales than on longer time scales suggesting nonlinear linkages between the markets.

2.1 Introduction

Linkages between equity markets have been widely studied. These linkages are important to investors. Knowledge of return and volatility linkages between markets give an investor tools for efficient diversification and portfolio management as well as risk management. In this chapter, the time-scale dependant correlation of returns and cross-correlation of volatilities between major world indices is analyzed. Analysis is made using wavelet multiresolution techniques. This gives us decomposition on a scale by scale basis and therefore allows us to make time scale dependent analysis of major stock market linkages.

In the following only the previous research important to this work is discussed. The research of international equity market linkages and integration is a much broader subject. For a good survey of the research area, see Kearney and Lucey (2004)

The research of correlation and cross-correlation between major world equity markets has long traditions in financial research. Lin et al. (1994) study the correlation of the volatility of New York and Tokyo markets. They find that information revealed during the trading hours of one market has a global impact on the returns of the other market. Hamao et al. (1990) end up to similar conclusions in the study of New York, London and Tokyo markets. Lin et al. argue, however, that the studied markets are more efficient than suggested by the results of Hamao et al. Ramchand & Susmel (1997) examine the relation between

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correlation and variance in conditional time and state varying frameworks using switching ARCH techniques. They find that correlations between the U.S. and other world markets are on average 2 to 3.5 times higher when the U.S. market is in a high variance state as compared to a low variance regime. Andersen et al. (2001) study daily equity return volatility and correlation obtained from high-frequency intraday transaction prices on individual stocks in the Dow Jones Industrial Average. For correlations between stock return volatility they found significant co-movements, reducing the benefits of portfolio diversification when the market is most volatile.

Longin & Solnik (2001) question the previous studies between correlation and volatility of international equity markets. They argue that correlation is not related to market volatility per se but to the market trend and that correlation between markets increase in bear markets. Differences in these conclusions might be a result of different types of data. For example Wongsman (2006) argues that many types of linkages might be missed if too low frequency data is used. Ball & Torous (2000) examine correlations across a number of international stock market indices using filtering methods to extract stochastic correlation from returns data. Their results indicate that the estimated correlation structure is dynamically changing over time. Their findings also include that stochastic correlation tends to increase in response to higher volatility. Similar conclusions were also made earlier by Longin and Solnik (1995) and Bekaert and Harvey (1995). Kearney (2000) studies the volatility of monthly data on stock market returns, interest rates, exchange rates, inflation and industrial production for Britain, France, Germany, Japan and the US. His data spans from July 1973 to December 1994. Results demonstrate that world equity market volatility is caused mostly by volatility in Japanese/US markets and transmitted to European markets. He also found that low inflation tends to be associated with high stock market volatility.

The volatility linkages of Far-East markets are also widely studied. Hu et al. (1997), Wei et al. (1995), Ng (2000) and Gallo & Otranto (2008) find a rich structure between the linkages of Far-East markets. These papers suggest that linkages are much more complex than merely the flow from the US to other markets. Similar conclusions are also made by Miyakoshi (2003). Cifarelli and Paladino (2005) investigate the high frequency behavior of the US, British and German stock markets using symmetric and asymmetric GARCH models. Their main conclusion is that the volatility transmission across countries is mostly accounted for by stock market exuberance. Baele (2005) studies the magnitude and time-varying nature of volatility spillovers from the aggregate European and US markets to 13 local European equity markets. Evidence is found in both markets for increasing spillover intensity throughout the 1980s and 1990s.

Furthermore, evidence is also found of contagion between the US market and local European equity markets during periods of high world market volatility are found. Morana and Belratti (2008) study the comovements in international stock markets. They form monthly realized moments for stock market returns for the US, the UK, Germany and Japan to assess the linkages between these markets. Results include progressive integration of the four stock markets, leading to increasing comovements in prices, returns, volatilities and correlations. Koulakiotis et al. (2009) study spillover effects inside Europe. They divide the euro area to three different regions and study the transmissions of volatility inside these regions. They find evidence for the same kind of complex linkages as found by the other studies in different market regions. They conclude that it is not always the case of the largest market in one region being the driving factor.

The analysis of this paper extends the previous work of correlation studies by decomposing correlation and cross-correlation on different time scales. Previous research has mainly studied temporal correlation and possibly its time variations. The innovation of this study is the addition of a new dimension to the research. Instead of time changes, focus is on the changes in scale-dimension (frequency-dimension). This is achieved using wavelet correlation and wavelet cross-correlation. Wavelet correlation is a recent method in financial time series analysis. Gallegati & Gallegati (2005) apply the wavelet correlation to the analysis of the industrial production indices of G-7 countries. Kim & In (2005) analyze the relationship between stock returns and inflation using wavelet correlation. Results indicate that there is a positive relationship between stock returns on the shortest and longest time scales, while a negative relationship is shown on the intermediate scales. In & Kim (2006b) study the correlation between the stock and futures markets with wavelet correlation methods. They find that the wavelet correlation between two markets varies over different investment horizons but remains very high. In & Brown (2007) use similar wavelet correlation analysis in international swap markets. Again they conclude that correlation between swap markets varies over time but remains very high, especially between the dollar and the euro. Furthermore they note that correlations with the yen market are lower implying that the yen market remains relatively less integrated with other major swap markets. Additional studies with wavelet correlation are Razdan (2004) on the study of strongly correlated financial time series, Simonsen (2003) on the study of the Nordic electricity spot market and Conlon et al. (2008) on the study of hedge funds. These studies show the potential of wavelet correlation and wavelet cross-correlation within financial research and which are extended in this thesis.

2.2 Wavelet correlation

The analyzed return series is calculated as a difference of the logarithmic price series. This study applies the generalized autoregressive conditional heteroscedasticity (GARCH) model to calculate the conditional volatility series. Conditional volatility of a time series implies explicit dependence on a past sequence of observations. GARCH model is a technique that can be used to model the serial dependence of volatility. The following model is used:

$$\begin{aligned} y_t &= C + \varepsilon_t \\ \sigma_t^2 &= A_1 + A_2\sigma_{t-1}^2 + A_3\varepsilon_{t-1}^2, \end{aligned} \tag{1}$$

where y_t is the time series and σ_t^2 the conditional variance of the innovations ε_t. So a constant time series is assumed, where volatility depends on the previous value of volatility and the square of the previous innovation. The model above is called the constant mean GARCH(1,1) model.

2.2.1 *Maximal overlap discrete wavelet transform*

The Maximal Overlap Discrete Wavelet Transform (MODWT) (Percival & Walden 2000) is similar to the Discrete Wavelet Transform (DWT) in that high-pass and low-pass filters are applied to the input signal at each level. However, in the MODWT, the output signal is not subsampled (not decimated). Instead, the filters are upsampled at each level.

Suppose we are given a signal $s[n]$ of length *N* where $N = 2^J$ for some integer *J*. Let $h_1[n]$ and $g_1[n]$ be a low-pass filter and a high-pass filter defined by an orthogonal wavelet. At the first level of MODWT, the input signal $s[n]$ is convolved with $h_1[n]$ to obtain approximation coefficients $a_1[n]$, and with $g_1[n]$ to obtain detail coefficients $d_1[n]$:

$$a_1[n] = h_1[n] * s[n] = \sum_k h_1[n-k]s[k] \tag{2}$$

$$d_1[n] = g_1[n] * s[n] = \sum_k g_1[n-k]s[k]. \tag{3}$$

Without subsampling, $a_1[n]$ and $d_1[n]$ are of length *N* instead of $N/2$ as in the DWT. At the next level of the MODWT, $a_1[n]$ is filtered using the same scheme, but with modified filters $h_2[n]$ and $g_2[n]$ obtained by dyadic upsampling $h_1[n]$

and $g_1[n]$. This process is continued recursively. For $j = 1,2,\ldots,J_0 - 1$, where $J_0 \le J$, define

$$a_{j+1}[n] = h_{j+1}[n] * a_j[n] = \sum_k h_{j+1}[n-k]\, a_j[k] \tag{4}$$

$$d_{j+1}[n] = g_{j+1}[n] * a_j[n] = \sum_k g_{j+1}[n-k]\, a_j[k], \tag{5}$$

where $h_{j+1}[n] = U\left(h_j[n]\right)$ and $g_{j+1}[n] = U\left(g_j[n]\right)$. Here U is the upsampling operator that inserts a zero between every adjacent pair of elements of the time series. The output of the MODWT is then the detail coefficients $d_1[n], d_2[n], d_3[n], \ldots, d_{J_0}[n]$ and the approximation coefficients $a_{J_0}[n]$.

2.2.2 *MODWT estimator for the wavelet correlation*

In this section, an estimator for wavelet correlation is constructed using the MODWT. This estimator was introduced by Percival (1995), Whitcher (1998) and Whitcher et al. (2000). An estimator for wavelet cross-correlation is a natural extension of the estimator of wavelet correlation and has similar properties.

The MODWT coefficients indicate changes on a particular scale. Thus, applying the MODWT to a stochastic time series produces a scale-by-scale decomposition. The basic idea of wavelet variance is to substitute the notion of variability over certain scales for the global measure of variability estimated by sample variance (Percival & Walden 2000). Same applies to wavelet covariance. The wavelet covariance decomposes sample covariance into different time scales. In other words, wavelet covariance on a particular time scale indicates the contribution of covariance between two stochastic variables from that scale. The wavelet covariance at scale $\lambda_j \equiv 2^{j-1}$ can be expressed as (Gencay et al. 2002a)

$$\mathrm{cov}_{XY}\left(\lambda_j\right) \equiv \frac{1}{\tilde{N}} \sum_{t=L_j-1}^{N-1} d_{j,t}^{X} d_{j,t}^{Y}, \tag{6}$$

where $d_{j,t}^{l}$ are the MODWT wavelet coefficients of variables l on a scale λ_j. $\tilde{N}_j = N - L_j + 1$ is the number of coefficients unaffected by the boundary, and $L_j = \left(2^j - 1\right)\left(L-1\right) + 1$ is the length of the scale λ_j wavelet filter.

An estimator of the wavelet covariance can be constructed by simply including the MODWT wavelet coefficients affected by the boundary and renormalizing.

This covariance is, however, to some degree biased. Because covariance is dependent on the magnitude of the variation of time series. It is natural to introduce the concept of wavelet correlation.

The wavelet correlation is simply made up of the wavelet covariance for $\{X_t, Y_t\}$ and the wavelet variance for $\{X_t\}$ and $\{Y_t\}$. The MODWT estimator of the wavelet correlation can be expressed as

$$\rho_{XY}\left(\lambda_j\right) \equiv \frac{\operatorname{cov}_{XY}\left(\lambda_j\right)}{\sqrt{\nu_X\left(\lambda_j\right)\nu_Y\left(\lambda_j\right)}}, \tag{7}$$

where $\nu_l\left(\lambda_j\right) \equiv \frac{1}{\tilde{N}}\sum_{t=L_j-1}^{N-1}\left[d_{j,t}^l\right]^2, \quad l = X, Y$ is the wavelet variance of stochastic process (Percival, 1995).

Confidence intervals

Calculation of confidence intervals is based on Whitcher et al. (1999, 2000). The random interval

$$\left[\tanh\left\{h\left[\rho_{XY}\left(\lambda_j\right)\right] - \frac{\Phi^{-1}(1-p)}{\sqrt{N_j - 3}}\right\}, \tanh\left\{h\left[\rho_{XY}\left(\lambda_j\right)\right] + \frac{\Phi^{-1}(1-p)}{\sqrt{N_j - 3}}\right\}\right] \tag{8}$$

captures the true wavelet correlation and provides an approximate $100(1-2p)\%$ confidence interval. The function $h(p) \equiv \tanh^{-1}(\rho)$ defines the Fisher's z-transformation. N_j is the number of wavelet coefficients associated with a certain scale computed via the DWT, not the MODWT. This is because the Fisher's z-transformation assumes uncorrelated observations and the DWT is known to approximately decorrelate a wide range of power-law processes.

2.3 Empirical analysis

2.3.1 Empirical Data

The sample data consists of daily returns and conditional volatilities of major world stock indices. The indices included are DAX 30 (Germany), FTSE 100 (Great Britain), S&P 500 Composite (US) and Nikkei 225 (Japan). The sample period spans from May 10, 1988 to January 31, 2007, including 4891 values. The conditional volatilities are calculated using the GARCH model in equation (1). The volatility and return series are presented in Figure 1. Summary statistics for the series are in tables 1 and 2. The mean of the return series are almost the same for SP500, DAX30 and FTSE100 while the mean of Nikkei is slightly negative. The standard deviation of Nikkei and DAX30 are somewhat larger than SP500 and FTSE100. Nikkei and DAX30 have been more volatile than SP500 and FTSE100 in the past, although the standard deviation for these volatility series has also been larger.

Table 1. Descriptive statistics for the return data of the studied indices. Mean, median and standard deviation are presented as percentages.

RETURNS	*SP500*	*NIKKEI*	*DAX30*	*FTSE100*
Mean (%)	0.035 %	-0.010 %	0.030 %	0.025 %
Median (%)	0.021 %	0.000 %	0.035 %	0.008 %
Standard Deviation (%)	0.964 %	1.377 %	1.376 %	0.979 %
Kurtosis	4.412	3.890	6.236	3.373
Skewness	-0.146	0.152	-0.433	-0.129
Minimum	-0.071	-0.072	-0.137	-0.059
Maximum	0.056	0.124	0.076	0.059
Count	4891	4891	4891	4891

Table 2. Descriptive statistics for the conditional volatility data of the studied indices. Mean, median and standard deviation are presented as percentages.

VOLATILITY	*SP500*	*NIKKEI*	*DAX30*	*FTSE100*
Mean (%)	0.0091	0.0132	0.0127	0.0091
Median (%)	0.0081	0.0124	0.0109	0.0081
Standard Deviation (%)	0.0034	0.0049	0.0055	0.0035
Kurtosis	2.0711	2.2037	4.9024	5.9183
Skewness	1.3838	1.1695	2.0459	2.1411
Minimum	0.0046	0.0053	0.0065	0.0051
Maximum	0.0236	0.0444	0.0460	0.0300
Count	4891	4891	4891	4891

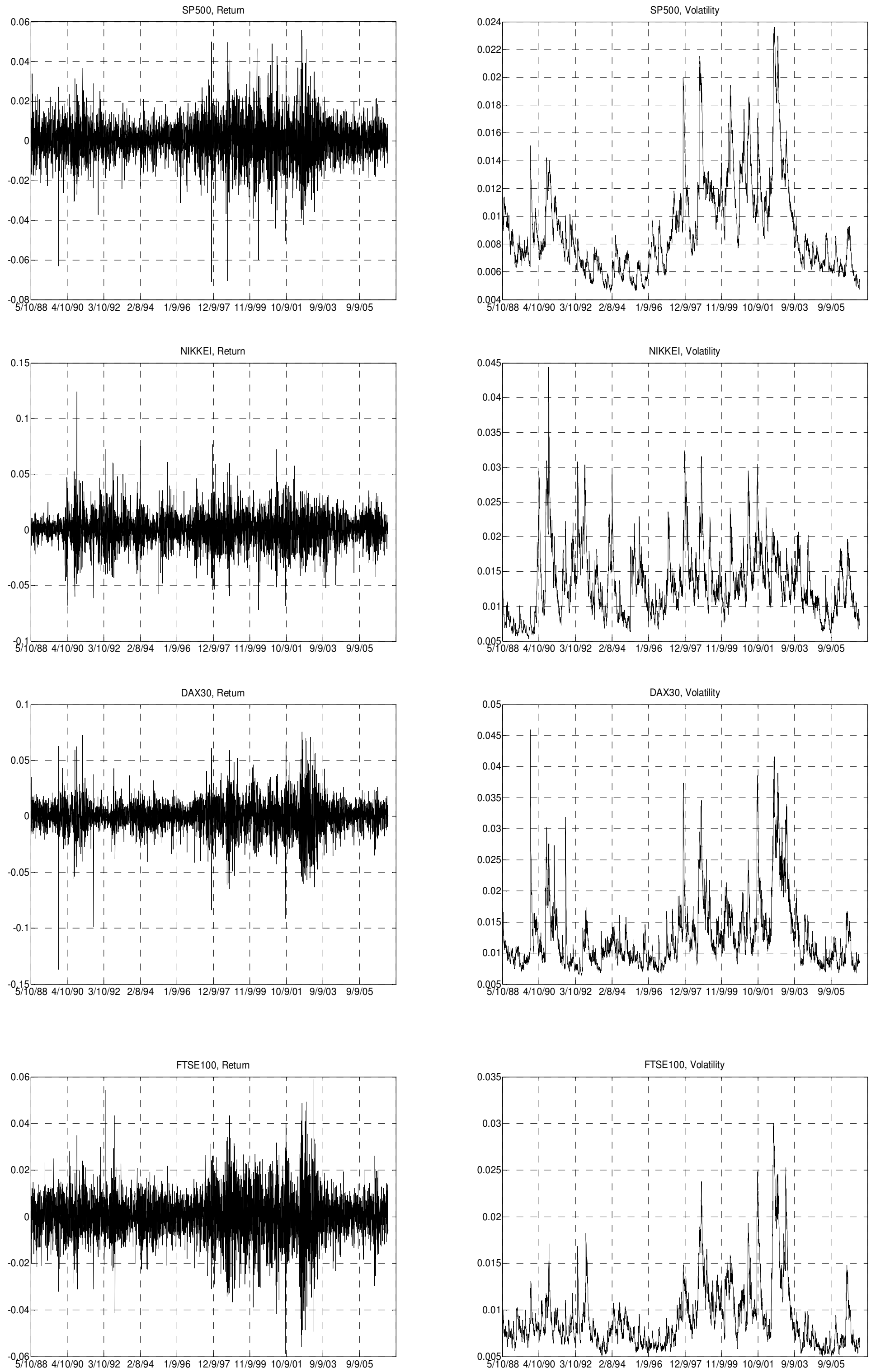

Figure 1. Return and conditional volatility series for studied indices. The sample period spans from May 10, 1988 to January 31, 2007, spanning 4891 values.

The MODWT estimator for wavelet correlation is calculated from the return series and the estimator of wavelet cross-correlation from the volatility series. Multiresolution analysis with nine scales is performed. The first scale represents 1-2 day averages and the ninth scale represents 256-512 day averages. 95% confidence intervals are used to analyze statistical significance. After experimenting with a few different wavelet filters, the Daubechies least asymmetric wavelet filter of level 8 (LA8) was utilized in the MODWT. This filter is favored mostly in literature (Percival & Walden, 2000). The decomposition low-pass and high-pass filters of LA8 are presented in figure 2.

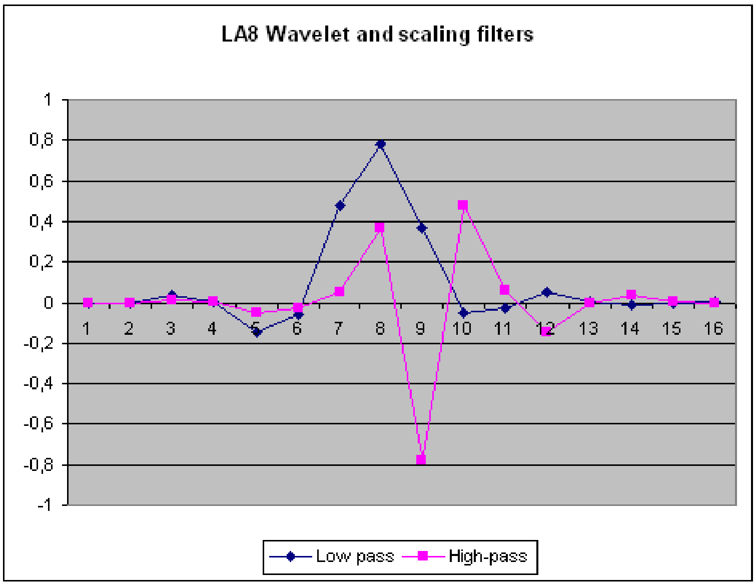

Figure 2. Daubechies least asymmetric wavelet (High-pass) and scaling (Low-pass) filters of level 8. High-pass filter is used to extract detail information from time series and low-pass filter to extract low-level approximation. In the next phase a modified high-pass filter is used to extract details of another time scale.

2.3.2 Empirical results

Figure 3 demonstrates the wavelet correlations for the returns of the four index series. The correlations increase from shorter time scales to longer time scales. Previous research has argued that correlations are stronger on the intra-day time scales and become weaker when moving towards the daily time-scale (for an example see Wongsman 2006). Now the wavelet correlations indicate that correlations increase from the daily time scale onwards. Thus the daily timescale appears to have the lowest correlations on the scale dimension.

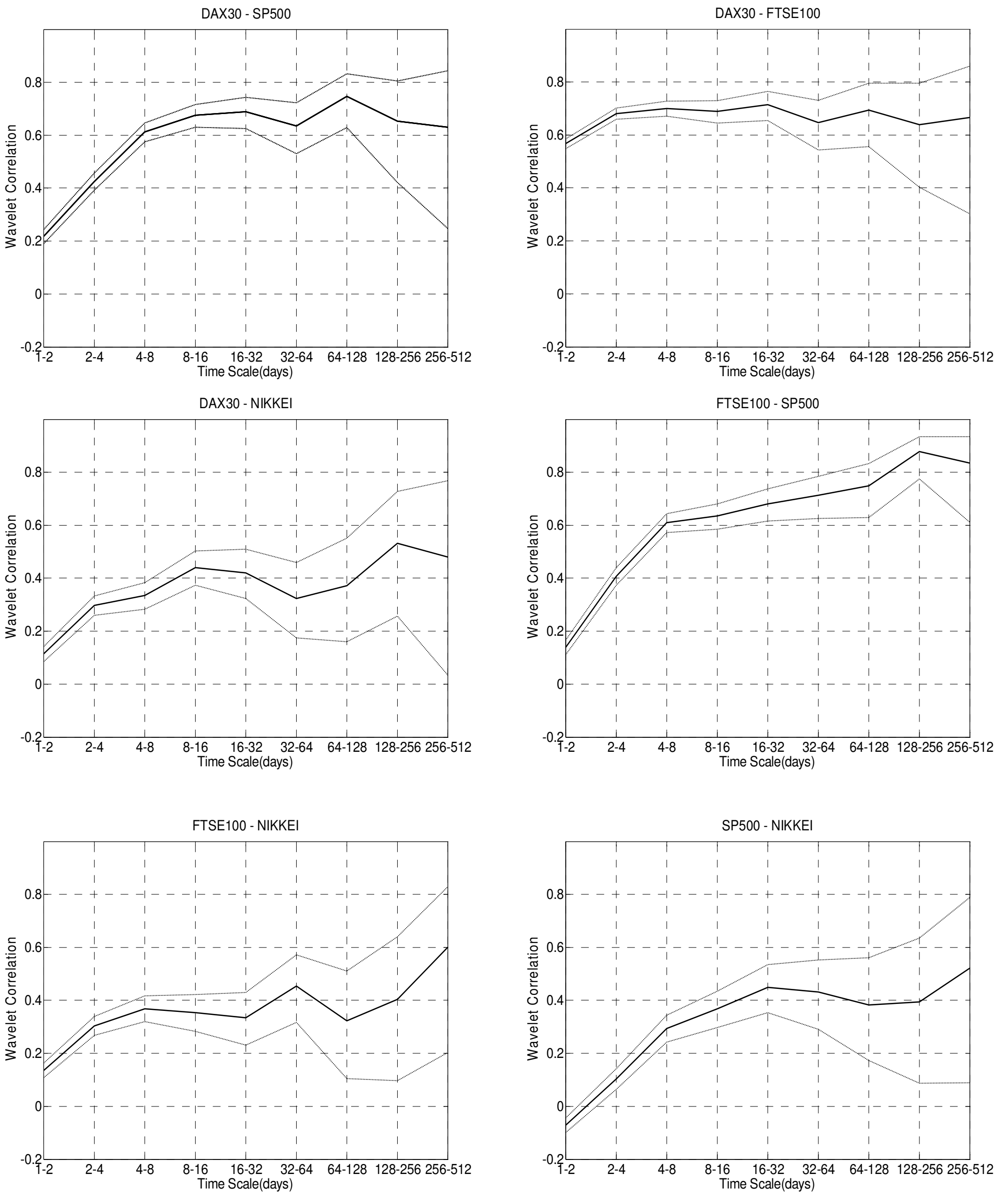

Figure 3. Wavelet correlation of returns between DAX30, FTSE100, S&P500 and Nikkei. Corresponding indices are shown above every sub-figure. Time scale spans from one day to one year in dyadic steps.

For the S&P 500 the increase of correlation from a daily time scale to longer time scales slows down or stops around a time scale of one week. Thereafter the correlation stays approximately at the same level all the way to a time scale of one year. The correlation between Nikkei and the other indices peaks around a time scale of one month. From one month onwards the correlation stays approximately at the same level or even decreases a little between Nikkei and DAX30. The correlation starts to increase again on a time scale of 128-256 days and is significantly stronger on a time scale of one year. Overall the correlations with Nikkei are the smallest among the studied correlations, regardless of the time horizon. It is often argued in the literature that especially the US market affects Japan but not vice versa. Lin et al. (1995) however argue that there is a bi-directional linkage between the US and Tokyo markets on an intra-day timescale. This linkage weakens on longer timescales as can be seen in Figure 3. As expected, the correlations between DAX30 and FTSE100 are strong on every scale.

Table 3 represents four different time scales in detail. There is a ranking in order of increasing correlation for the time scales of day, week, month and year. The ranking is made according to portfolio diversification as the correlations of returns are central factors in efficient portfolio diversification. The correlation between DAX30 and FTSE100 is strong on every time scale so they take the last places in the diversification ranking. One clear aspect of Table 5 is that Nikkei listed stocks should be included in portfolios regardless of the time scale studied. The top three for every time horizon consists of Nikkei and some other index. On the time scales of a day and a week, the most diversified portfolios are formed by combining the SP500 and Nikkei listed stocks. If the investment horizon is longer (the time scales of a month and a year), the best results are obtained by using the European stocks (DAX30, FTSE100) and the Nikkei listed stocks. It is also preferable to use the DAX30 listed stocks in portfolio forming, especially if the investment horizon is longer (time scale of one year). These results support the work of Morano & Beltratti (2008). They find that overall integration between markets has increased, expect for Japan. In their study Nikkei has also significantly lower correlations with other markets.

Table 3. Correlation diversification ranking on four different time scales. Time scales used are a day(scale 1, 1-2 days), a week(scale 3, 4-8 days), a month(scale 5, 16-32 days) and a year(scale 9, 256-512 days). Ranking is formed on the basis of portfolio diversification efficiency. Correlations are in the parentheses. 5% significance level is marked with *.

Rank	Day	Week	Month	Year
1	SP500 - NIKKEI (-0,071*)	SP500 - NIKKEI (0,294*)	FTSE100 - NIKKEI (0,334*)	DAX30 - NIKKEI (0,480*)
2	DAX30 - NIKKEI (0,114*)	DAX30 - NIKKEI (0,334*)	DAX30 - NIKKEI (0,420*)	SP500 - NIKKEI (0,521*)
3	FTSE100 - NIKKEI (0,135*)	FTSE100 - NIKKEI (0,369*)	SP500 - NIKKEI (0,449*)	FTSE100 - NIKKEI (0,602*)
4	FTSE100 - SP500 (0,139*)	FTSE100 - SP500 (0,609*)	FTSE100 - SP500 (0,681*)	DAX30 - SP500 (0,630*)
5	DAX30 - SP500 (0,218*)	DAX30 - SP500 (0,612*)	DAX30 - SP500 (0,689*)	DAX30 - FTSE100 (0,665*)
6	DAX30 - FTSE100 (0,567*)	DAX30 - FTSE100 (0,701*)	DAX30 - FTSE100 (0,714*)	FTSE100 - SP500 (0,834*)

The other topic of this study is the scale-based examination of the cross-correlation of volatilities of stock indices. The purpose is to acquire more information about linkages between the major equity markets and thus, clarifying the understanding of dynamic structures between them. The MODWT based wavelet cross correlation functions are used as an estimator. An example of these functions is presented in figure 4 with the wavelet cross-correlation functions between DAX30 and SP500.

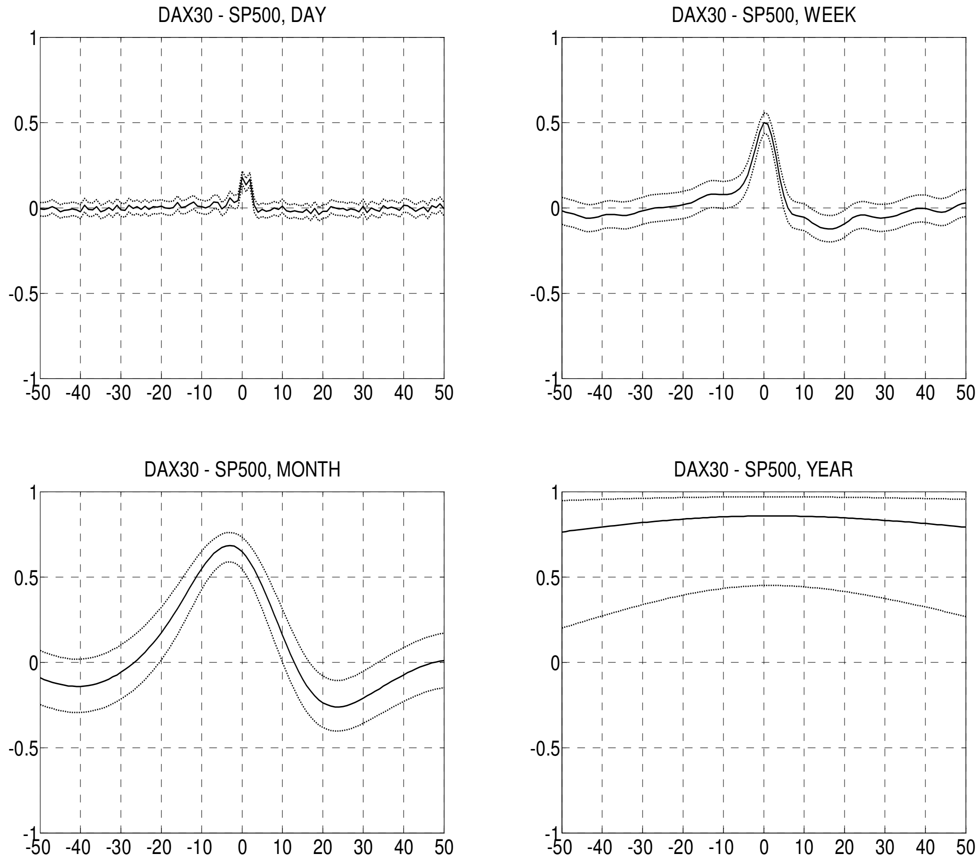

Figure 4. Example of wavelet cross-correlations on four different time scales (1-2, 4-8, 16-32 and 256-512 days). Indices studied are DAX30 and SP500. The skew to the left means leading DAX30 and the skew to the right means leading SP500. On the horizontal line are lags in days and on the vertical line are correlations.

Four different time scales are studied, namely a day, a week, a month and a year. Like with the returns, there is a trend of strengthening contemporaneous correlation when the time horizon gets longer. From the figure can be seen, that the cross correlation on the time scale of a day skews to the right. This means that the volatility of SP500 is leading the volatility of DAX30. When the volatility of SP500 increases (decreases), the volatility of DAX30 increases (decreases) 1-2 days later. The same kind of conclusion, in the other direction, can be made on the time scale of a month. Now the leading index is DAX30. When dealing with one month long averages, changes in the volatility of DAX30 are followed by similar changes in the volatility of SP500. The whole cross-correlation analysis data between every index and on every time scale are available upon request from the author.

Cross-correlation functions of volatilities can be used to analyze volatility spillovers between different markets. The calculated wavelet cross-correlation functions were used for the subjective analysis of volatility spillovers similar to the previous paragraph. These results are presented in figure 5.

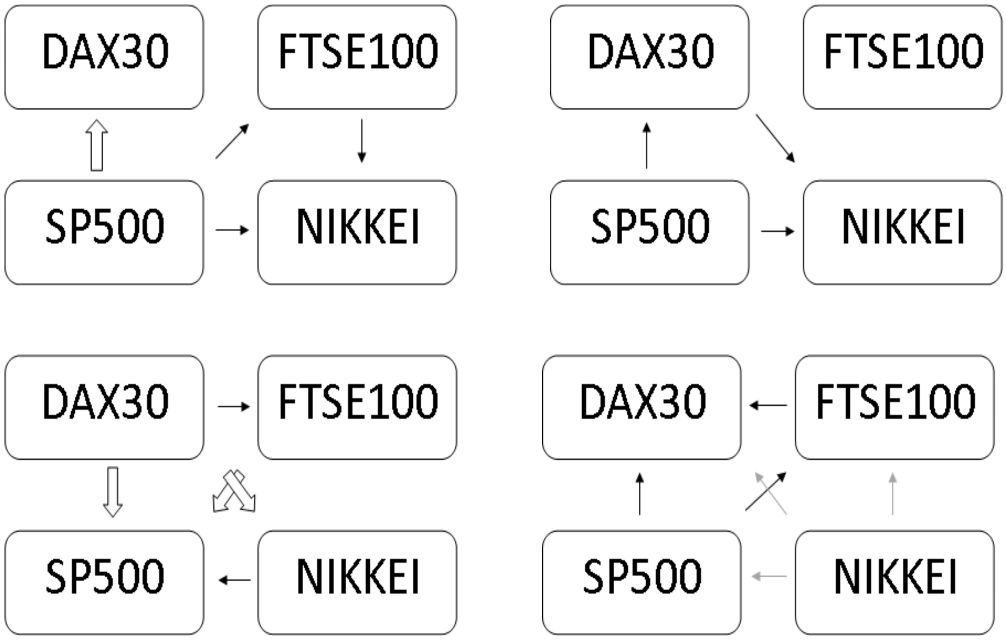

Figure 5. Volatility spillover flow charts for four different time scales. Flows have been visually estimated using the wavelet cross-correlation functions. From the upper-left corner the time scales are a day(1-2 days), a week(4-8 days), a month(16-32 days) and a year(256-512 days). Black arrows were statistically significant at a 5% level, while grey arrows were not. Big arrows describe a very strong volatility spillover.

There are four simple diagrams presenting the same time scales for a day, a week, a month and a year. On the time scales of a day and a week, there is a clear flow of volatility from SP500 to other indices. It is stronger on the shortest time scale, but still clear on the time scale of a week. Things are different when we study the time scale of a month. There is a flow of volatility from the European indices, especially DAX30, to SP500 and Nikkei. This result is an interesting result and one that has not been documented before in the literature. This result however could be an outcome of the chosen indices. DAX30 differs from other indices in the amount of companies included in the index. In the DAX30 there are 30 of the largest German companies and in the SP500 there are 500 companies. Therefore there is a substantial difference between these two indices. On the contrary to the return analysis and results of Morana & Beltratti (2008) (and somewhat also for the study of Hamao et al. (1990)), there is also a spillover from Nikkei to SP500 on the monthly timescale. The timescale of a year is again similar to the shorter time scales. The volatility of SP500 is affecting the volatility of other indices with a lag. One different aspect on the longest time scale is that Nikkei is also affecting other indices, which contradicts the results of some previous research and support for example conclusions of Lin et al. (1995). These correlations were, however, statistically significant only at a 10% level.

2.4 Conclusion

In this chapter, the linkages between major world stock indices are studied. The methodology is based on wavelet correlation (and cross-correlation), which decomposes correlation of a time series on a scale by scale basis using the non-decimated discrete wavelet transform. The wavelet methods give us multiresolution analysis for correlation. Therefore we can study correlation's dependence on a time scale. This is important because different investors have different investment horizons and wavelet analysis can be used to improve decision making in the practical situations of risk management, portfolio allocation and asset pricing.

There is a clear trend that the correlation increases, when the time horizon gets longer. The previous research argues that correlations decrease when we move from the intraday time scales to the daily timescale (Wongsman 2006). The results of this study show how the correlations increase from the daily time scale to longer time scales. Thus along the scale dimension the correlations appear to be the smallest on a daily time scale. The correlations between Nikkei and other indices are the smallest on every scale. Morana & Beltratti (2008) find similar results on a time scale of one month and now this result is extended to other timescales. Therefore, from the standpoint of portfolio diversification, Nikkei listed stocks should always be included in the portfolio. The difference is that on shorter time scales, Nikkei listed stocks should accompany stocks from SP500, while on longer time scales, European stocks should be used. DAX30 has smaller correlations with SP500 and Nikkei than FTSE100 and is a better choice in investment strategies including European stocks.

The cross-correlation analysis of volatilities on a scale by scale basis is used to analyze volatility spillover effects. There again, dependence on a time scale was observed. On shorter time scales there was a volatility spillover from SP500 to other indices. Things change when we move to the time scale of one month. Volatility spillover from the European indices, especially DAX30, to SP500 and Nikkei was observed. On the longest time scale things are again similar to the shorter time scales, where the changes of volatility of SP500 lead changes in the other indices. Different aspect compared to the short time horizons is the observation of a weak volatility spillover from Nikkei to other indices. The results follow the previous literature. Morana & Beltratti (2008) observe the flow from the US to other markets and the separate nature of the Japanese market. Also on certain scales there is support for the results of Lin et al. (1994) for the influence of the Nikkei market to other markets. The strong spillover from the DAX30 index to other indices is something new which has not been documented before.

Correlation between returns and volatilities are extensively studied subjects in the literature. Above analysis includes the multiresolution analysis to the big picture. Decomposing correlation and cross-correlation functions on a scale by scale basis allows the study of their time scale dependence. As the results indicate, the correlation between returns and the cross-correlation between volatilities are dependent on the time scale examined. Investors also should take into account their investment horizon when they make risk management and portfolio allocation decisions based on the correlation structure between markets. The correlation structure diverges when the investment horizon spans over many years in contrast to over a few days.

3 CONTAGION AMONG MAJOR EQUITY MARKETS*

In this chapter, an analysis of contagion among the major world markets during the last 25 years is carried out. The analysis uses a novel way to study contagion with the help of wavelet methods. Clear signs of contagion among the major markets are found. The results show that short time scale correlation increases during a major crisis. At the same time long time scale correlations remain approximately at the same level indicating contagion. Also the overall increase of interdependence is found.

3.1 Introduction

The debate around a phenomenon called contagion has been active in recent years. Forbes & Rigobon (2002) define contagion as an increase of correlation between markets after some crisis. This is a narrow definition which is not universally accepted as a definition of contagion. A more broad definition argues that contagion occurs whenever a shock to one country is transmitted to another country, even if there are no significant changes in cross-market relationships (Forbes & Rigobon 2002). Some researchers argue that contagion cannot be defined based on changes in cross-market linkages. Instead, they argue that the analysis of contagion should be based on the analysis of shock propagation from one country to another and that only certain types of transmission mechanisms constitute contagion However, the definition of Forbes and Rigobon has been the most popular in recent papers discussing contagion. This definition is also adopted in this study with a slightly different perspective. Contagion is defined as a temporary increase of short time-scale correlation. By examining how the structure of correlation along the scale-dimension changes after some crisis this study aims to avoid the heteroscedasticity problem that has plagued contagion research based on correlation coefficients. Using wavelets as a tool, linkages between markets can be studied on different time scales. If this structure along the scale dimension changes in periods of turmoil, it should be an indication of contagion.

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* An article based on this chapter received the best paper award at the 2009 Northern Finance Association meeting. The article was accepted for publication in the International Journal of Managerial Finance.

A significant increase of interest over contagion phenomenon occurred after the 1987 stock market crash (See for example Claessens et. al (2000) for a good survey of the contagion literature before the new millennium). King and Wadhwani (1990) focus on the major stock markets and find an increase in stock market correlations after the 1987 stock market crash, i.e. contagion. Lee and Kim (1993) add developing countries to the study and also find evidence of contagion. The overall consensus during the nineties was that contagion exists. Forbes and Rigobon (2002) argue that previous studies found contagion, because they did not correct the correlation measure for heteroscedasticity. Using a heteroscedasticity corrected correlation measure they find that contagion does not exist. Following the guidelines of Forbes & Rigobon, many other studies end up to similar conclusions. For example Collins and Biekpe (2003) study the integration of African countries in the world financial markets and find very little evidence of contagion. Lee et al. (2007) find that the South-East Asia tsunami did not trigger contagion in the international stock markets (although they find some signs of contagion in the foreign exchange markets). Recently the conclusions of Forbes & Rigobon (2002) have been criticized. Corsetti et al. (2005) argue that the findings of Forbes and Rigobon are a result of an assumed model. They note that the model assumes unrealistic restrictions on the variance of country-specific shocks. Bartram and Wang (2005) note that the bias Forbes and Rigobon document follows directly from the assumptions of their analysis (see also Pesaran and Pick 2007). Many other corrections for the model of Forbes and Rigobon have been proposed. Hon et al. (2007) use a GARCH-model to deal with the heteroscedasticity (see also Jokiipii & Lucey 2007).

Rodriguez (2007) uses a copula approach to investigate contagion and find that the dependence structure of stock markets is different when studying tail dependence compared to overall dependence. The tail dependence exhibit strong changes during the Asian and Mexican crises and is a clear sign of contagion. Taking this recent criticism into account, the overall consensus has changed from a "no contagion"- to an "at least some contagion"-conclusion or in some cases to very strong signs of contagion (see for example Yang and Bessler 2008, Dungey et al. 2007).

The correlation coefficient has been widely used as a measure of interdependence in financial research. It is also widely used in contagion studies. Correlation literature was already surveyed in the previous chapter. Seminal papers in the research area are Lin et al. (1994) on the study of correlation of the volatility of New York and Tokyo markets and Longin & Solnik (2001) on the study correlation and trend. Other studies are Ramchand & Susmel (1997), Andersen et al. (2001), Ball & Torous (2000), Kearney (2000), Cifarelli and Paladino (2005), Baele (2003) and Morana and Belratti (2008).

In this study, contagion is examined using wavelet correlation and wavelet coherence methods. Wavelet correlation methods are achieving an increasing popularity in financial time series analysis. A survey of wavelet correlation methods was made in the two previous chapters. Wavelet coherence is similar to wavelet correlation. It is calculated using the continuous wavelet transform instead of the discrete wavelet transform. A wavelet coherence estimator was introduced by Grinsted et al. (2004), Torrence & Webster (1999) and Torrence & Compo (1998). Wavelet coherence appears to be applied only once in financial and economic research. Rua and Nunes (2009) analyze the comovements of stock market returns using a similar wavelet coherence method as in this chapter. Their focus is however somewhat different. They examine the overall dependence of the developed markets on an aggregate level and also separated to different sectors. One of the main conclusions of their paper is that on the side of analyzing time-varying properties of the comovements, it is also of utmost importance to analyze the frequency-varying properties of the comovements.

This study aims to overtake the debate around the correlation measure being a biased measure of contagion by studying correlation on different time scales. In addition, making conclusions about correlation as a function of time, conclusions as a function of time scale (frequency) are made. If a short time scale correlation changes (increases), while a long time-scale correlation remains approximately the same, we have contagion. That is the main assumption in the following study. This approach avoids the problems of the heteroscedasticity bias of Forbes & Rigobon (2002), because volatility should affect both short and long time scale correlations. The empirical study is divided into two different parts. The first study uses wavelet coherence of the continuous wavelet transform similar to Rua and Nunes (2009). This study includes the main contribution of this study. The second study uses the estimator of wavelet correlation calculated with the maximal overlap discrete wavelet transform. The purpose of the second study is to analyze the findings of the wavelet coherence study in more detail.

3.1 Wavelet coherence and rolling correlation

3.2.1 Rolling wavelet correlation

The Maximal Overlap Discrete Wavelet Transform (MODWT) (Percival & Walden 2000) is similar to the Discrete Wavelet Transform (DWT) in that the high-pass and low-pass filters are applied to the input signal on each level. However, in the MODWT, the output signal is never subsampled (not decimated). Instead, the filters are upsampled on each level. The theory behind the MODWT based wavelet correlation was introduced in the previous chapter. The study of this chapter uses a slight modification of the wavelet correlation. Using a simple rolling window approach, the estimator is used to calculate a time series of correlation values.

As was introduced in the previous chapter, the output of the MODWT are the detail coefficients $d_1[n], d_2[n], d_3[n], \ldots, d_{J_0}[n]$ and the approximation coefficients $a_{J_0}[n]$. These coefficients are acquired from the convolution equations

$$a_{j+1}[n] = h_{j+1}[n] * a_j[n] = \sum_k h_{j+1}[n-k] a_j[k] \tag{9}$$

$$d_{j+1}[n] = g_{j+1}[n] * a_j[n] = \sum_k g_{j+1}[n-k] a_j[k]. \tag{10}$$

These equations are applied to the rolling window and this window is rolled forward one day at a time. The equations

$$\rho_{XY}(\lambda_j) \equiv \frac{\mathrm{cov}_{XY}(\lambda_j)}{\sqrt{v_X(\lambda_j) v_Y(\lambda_j)}} \tag{11}$$

and

$$\left[\tanh\left\{h\left[\rho_{XY}(\lambda_j)\right] - \frac{\Phi^{-1}(1-p)}{\sqrt{N_j - 3}}\right\}, \tanh\left\{h\left[\rho_{XY}(\lambda_j)\right] + \frac{\Phi^{-1}(1-p)}{\sqrt{N_j - 3}}\right\}\right] \tag{12}$$

are then used to calculate the wavelet correlation and confidence intervals for every window. This method gives us an estimation of correlation both in time- and scale-space.

3.2.2 Wavelet coherence

The second method used to study the presence of contagion effects is the wavelet coherence method introduced by Torrence & Compo (1998) and Grinsted et al (2004). Instead of the discrete wavelet transform, the estimator for interdependence is now based on the continuous wavelet transform. A wavelet $\psi(t)$ is a function of time that obeys the admissibility condition

$$C_{\psi} = \int_{0}^{\infty} \frac{\left|\Psi(f)\right|}{f} df < \infty, \tag{13}$$

where $\Psi(f)$ is the Fourier transform of $\psi(t)$. The continuous wavelet transform is defined as

$$W^{X}(u,s) = \int_{-\infty}^{\infty} x(t)\psi_{u,s}(t)\,dt, \tag{14}$$

where

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right)$$

is the translated and dilated version of the original wavelet function. The wavelet coherence of two time series is defined as

$$R_n^2(s) = \frac{\left|S\left(s^{-1}W_n^{XY}(s)\right)\right|^2}{S\left(s^{-1}\left|W_n^X(s)\right|^2\right)\cdot S\left(s^{-1}\left|W_n^Y(s)\right|^2\right)}, \tag{15}$$

where S is a smoothing operator, s is a wavelet scale, $W_n^X(s)$ is the continuous wavelet transform of the time series *X*, $W_n^Y(s)$ is the continuous wavelet transform of the time series *Y* and $W_n^{XY}(s) = W_n^X W_n^{Y*}$ is a cross wavelet transform of the two time series *X* and *Y* (Grinsted et al. 2004 and Torrence & Webster 1999). The best wavelet for feature extraction purposes is the Morlet wavelet, since it provides a good balance between time- and frequency localization. Also for the Morlet wavelet the Fourier period is almost equal to the wavelet scale used (Grinsted et al. 2004). The smoothing operator is defined to be similar to the wavelet used. It is written as

$$S(W) = S_{scale}\left(S_{time}\left(W_n(s)\right)\right), \tag{16}$$

where $S_{time}(W)\big|_S = \left(W_n(s) * c_1^{\frac{-t^2}{2s^2}}\right)$ and $S_{scale}(W)\big|_S = \left(W_n(s) * c_2\Pi(0.6s)\right)\big|_n$ (see Torrence & Webster 1999 for more details). c_1 and c_2 are normalization constants and Π is a rectangle function. The factor of 0.6 is empirically determined and follows Torrence & Compo (1998). The statistical significance levels of the wavelet coherence are determined using Monte Carlo methods. The guidelines of Grinsted et al. (2004) are followed, where the reader is advised to look for more detailed information.

3.3 Empirical analysis

3.3.1 Empirical data

The empirical data consists of four major stock indices. Included are DAX 30 (Germany), FTSE 100 (Great Britain), S&P 500 Composite (the US) and Nikkei 225 (Japan). The sample period starts from January 2, 1984 and ends at January 8, 2009, including 6529 daily closing prices for the studied indices. The estimator of wavelet correlation is calculated from the series using the MODWT. The dependence of the indices is also examined using wavelet coherence analysis. Based on the descriptive analysis of interdependence structure, contagion is tested using two different wavelet time scales, namely 2-4 days and 8-16 days (chosen time scales are explained later). A test statistic for the differences between correlations before and after an incident (defined later) is calculated. Figure 6 presents a time series of examined indices. In the figure are also marked most of the incidents that potentially might have had global influence on financial markets during the last 25 years. Abbreviations used in the figures are explained in table 4

Table 4. A description of abbreviations used in the text and figures presented in chronological order.

BM	The Black Monday - The major collapse of the US stock market on October 19, 1987
GW	The gulf war - August 2, 1990, when Saddam Hussein attacked Kuwait
MX	The Mexican peso crisis. The date chosen is December 19, 1994 (Forbes and Rigobon, 2002)
EA	The East Asian financial crisis. The date chosen is July 15, 1997.
RU	The Russian financial crisis. The date chosen is August 13, 1998.
IT+	The peak of SP500 index during the Dot-Com bubble. The date is March 24, 2000.
WTC	The suicide attacks of al-Qaeda upon the United States on September 11, 2001.
IT-	The lowest point of SP500 after the Dot-Com bubble burst. The date is October 2, 2002.
GF+	The peak of SP500 during the last bull market before the global financial crisis of 2007-2009.
GFc	The crash of the global stock markets during the global financial crisis of 2007-2009.

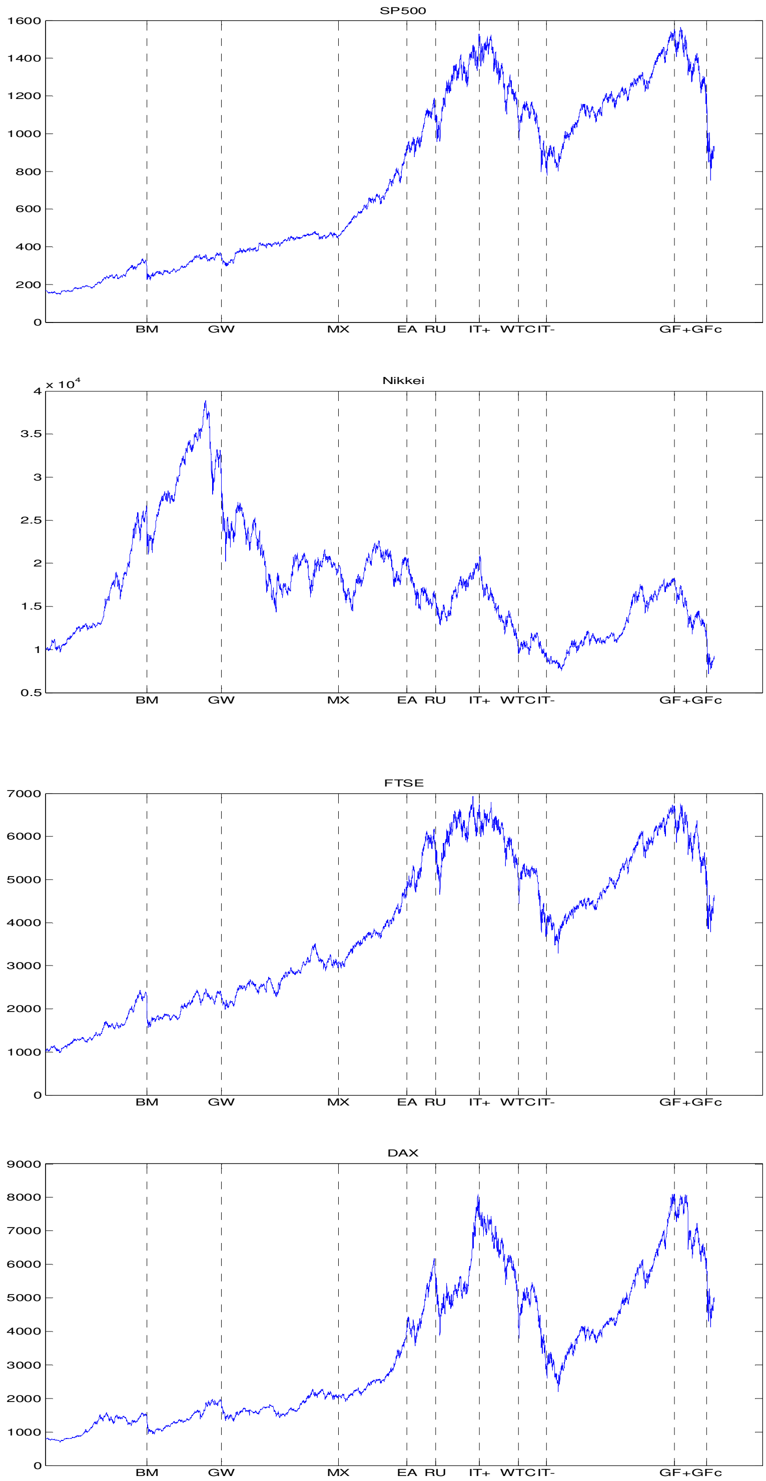


Figure 6. Prices of four major world indices. The sample period is from January 2, 1984 to January 8, 2009. Abbreviations used in the figures are explained in table 4.

3.3.2 Empirical results

Forbes & Rigobon (2002) argue that the method of using an ordinary comparison of correlation coefficients during the periods of turmoil and stable is biased because of the heteroscedasticity present in the data. Their arguments where questioned for example by Bartram & Wang (2005) and Corsetti et al. (2005). They question the assumptions made on the variance of the county-specific noise. The debate continues. Forbes & Rigobon define contagion simply as an increase of the correlation coefficient as a result of some financial crisis. With the introduction of multiresolution analysis, these issues can be separated on different time scales. If there is an increase in correlation on shorter time scales, longer time scales remaining approximately the same, will this result in contagion? This is the main assumption of this paper. Such a change in the correlation structure around some financial crisis indicates just contagion. This should be a quite plausible assumption, because correlations are compared together and in principle volatility should not play a role here. Conclusions are made by analyzing the correlation dynamics along the scale dimension. If the significance of short timescale correlations in the overall correlation structure increase, there is contagion. If this kind of concentration on short timescale correlation is not seen, there is no contagion.

Wavelet coherence maps are used as a descriptive tool to analyze correlation structure. In the last section at the end of this chapter are the wavelet coherence figures between the major markets. The structure of the figures is as follows: The shortest time scale in the figures is one week. From the time scale of one week to the time scale of 25 days, the sample period has been divided into three different figures. This has been done for the sake of clarity. Presented below these three figures are the time scales from 26 to 700 days for the whole sample period. The wavelet coherence figures give us good tools for a descriptive analysis of contagion. If the signal of contagion is the increase of short time-scale correlation, the figures give many indications of contagion. The area, where short time scale correlation increases, varies between different crises. Sometimes there is an increase of correlation already on a seven day time scale, sometimes the increase is around two week - one month time scale. Also the breaking point between a changing short time-scale correlation and an approximately constant long time-scale correlation varies from around 100 and 200 days. Below is a short list of results from the wavelet coherence maps.

- An overall increase of correlation during the last 25 years. The increase is slightly weaker with Nikkei but still clearly visible. On the other hand the European indices and SP500 have experienced a very strong increase of correlation.
- The black Monday causes clear contagion effects. The increase of short timescale correlation is strongest with FTSE and SP500 but other indices show also clear contagion effects.
- Around the Gulf war there is also signs of contagion. The signs are however not as clear as with the Black Monday.
- Probably the strongest signs of contagion can be seen around the ongoing global financial crisis.
- Around the East-Asian financial crisis and the Russian financial crisis there are some signs of contagion. The signs are, however, quite weak.
- With the gradually increasing correlation during the last 25 years and the contagion effects of the ongoing financial crisis, markets are very strongly correlated at the moment. Especially SP500, FTSE and DAX are highly correlated at every timescale at the moment.

Previous studies have concentrated mainly on the study of ordinary correlation analysis and on its different modifications of it. The wavelet coherence figures show clearly that this is not enough. The time scale has to be included in the study. Otherwise signs of contagion could be missed because we are studying the wrong time scale.

A study of short time-scale correlation using a discrete version of the wavelet transform is used to accompany wavelet coherence analysis for purposes of comparison. A multiresolution analysis with the focus on two short time scales is performed. The first scale represents 2-4 day averages and the second scale represents 16-32 day averages. The time-scale of 2-4 days was chosen as the shortest time scale to avoid the bias of different closing times. The longer time scale was chosen to be 16-32, because it includes one month on its scale making comparison with earlier studies easier. 95% confidence intervals are used to analyze the statistical significance. After experimenting with a few different wavelet filters, the Haar filter was utilized in the MODWT. Being the simplest of all wavelet filters, it mostly avoids the boundary problems of filtering and still achieves a quite a good band-pass performance.

Figures 7 and 8 present a rolling wavelet correlation series for two different time scales. Timescales used are 2-4 days and 16-32 days. These figures provide support for the findings of the wavelet coherence figures. There are clear signs that correlations increase after the 1987 stock market crash, at the beginning of the Gulf War and at the beginning of the global financial crisis of 2008. There is also an increase of correlation at the end of 90's. This increase cannot be attributed to one specific crisis so easily. The increase begins around the East Asian crisis and is strongest during the Russian crisis. Also the figures show that during the tranquil periods and bull markets, correlations tend to decrease. These all conclusion apply to both time scales being studied.

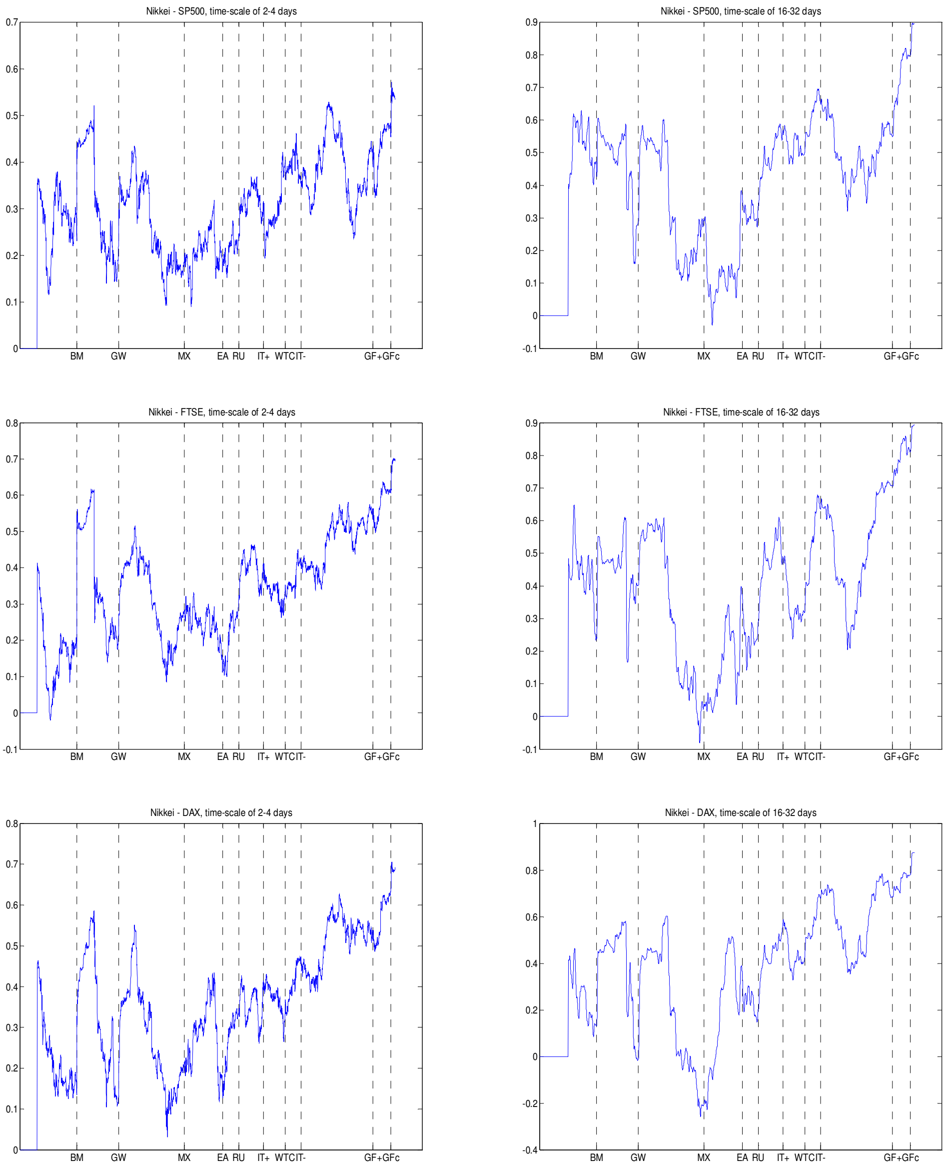


Figure 7. Rolling wavelet correlations using 200 day rolling window. On the horizontal line are the studied dates of major incidents. On the left are the correlations of 2-4 day wavelet averages. On the right are the 16-32 day wavelet averages.

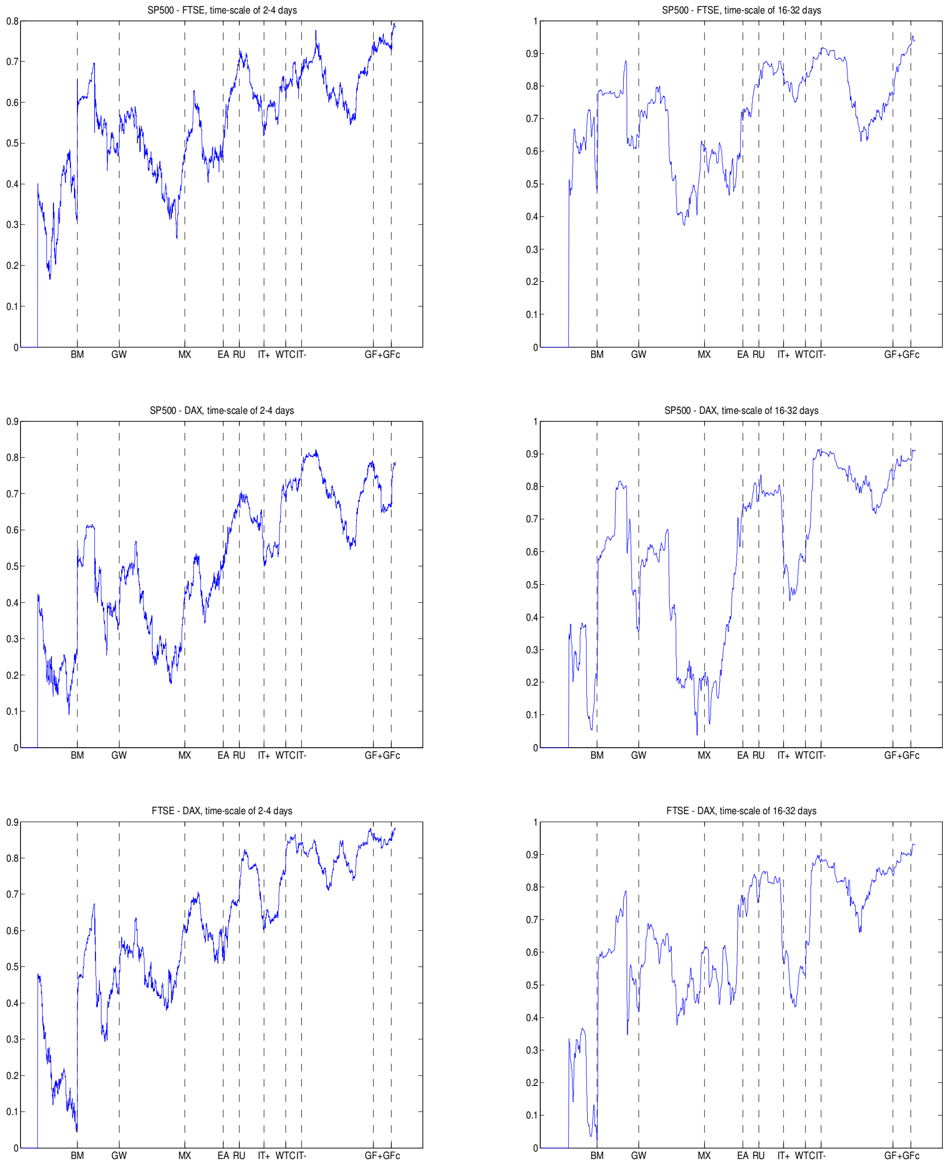


Figure 8. Continues from previous page. Rolling wavelet correlations using 200 day rolling window.

As an elementary test for contagion, a t-test is applied to evaluate if there is a significant increase in correlation coefficients after an incident. The correlation coefficient is calculated using a 250 day time window before and after the incident. This was a compromise. A shorter window would not have had enough independent data points to study longer time scales. On the other hand a longer time scale would not have isolated the immediate surroundings of the incident and the effects of many incidents would have mixed up in the correlation coefficient.

If ρ_1 is the correlation coefficient before the incident and ρ_2 the correlation coefficient after the incident, the test hypotheses are

$$\begin{aligned} H_0 &: \rho_1 > \rho_2 \\ H_1 &: \rho_1 = \rho_2 \end{aligned} \tag{17}$$

The estimated correlation coefficients are shown in table 5. The table presents two different time scales. A time scale of 2-4 days and a time scale of 8-16 days. The smallest scale of 1-2 days was not chosen, because it is affected by the different closing times of the studied markets. It would have been useful to include longer time scales for the contagion test. The degrees of freedom of the test decrease quickly with the scale. The time scale of 8-16 days was the last scale with reasonably solid results. The focus in this study is the comparison of these two timescales around an incident.

The results indicate that there have been contagion effects between the major markets at least three times; during the 1987 financial crisis, at the beginning of the Gulf War and now during the global financial crisis. In these cases there is a significant increase of correlation between every index studied. The increase is often significant even at a one percent level, giving support to the results of wavelet coherence analysis. One exception is SP500 and Nikkei during the 1987 crisis. There is no significant increase in the correlation on a longer time scale. Another exception is SP500 and FTSE during the present crisis. At least with the sample period that ends to January 8, 2009, there is no significant increase in the correlation for both time scales. Somewhat weaker signs of contagion are seen after the Gulf War. Indications of contagion are also present with the East Asian crisis and the burst of the dot-com bubble on a shorter time-scale. These significant increases of the correlation are absent on a longer time scale. Therefore it can be alleged that the time scale studied is essential when making conclusions about contagion. In the light of these results we can conclude that contagion does exist and, especially during the ongoing global market crisis, its' presence is clear through all the examined markets.

Table 5. The results of t-tests comparing equality of correlation coefficients. The correlation coefficients were calculated using 250 day sample periods before and after some significant date. 5% and 1% significance levels are marked with * and **. 5% significance level was taken as a limit of contagion

	2-4 days (df 87)				8-16 days (df 8)			
	Corr. before	Corr. after	Test statistic	Contagion	Corr. before	Corr. after	Test statistic	Contagion
October 17, 1987 (BM)								
SP500-Nikkei	0.266	0.507	2.209*	Yes	0.391	0.596	0.978	No
SP500-FTSE	0.268	0.704	4.631**	Yes	0.289	0.832	3.199**	Yes
SP500-DAX	0.266	0.613	3.403**	Yes	0.128	0.675	2.463*	Yes
Nikkei-FTSE	0.186	0.633	4.312**	Yes	0.044	0.710	3.007**	Yes
Nikkei-DAX	0.129	0.602	4.373**	Yes	0.087	0.696	2.755**	Yes
FTSE-DAX	0.008	0.677	6.293**	Yes	-0.271	0.819	5.106**	Yes
August 2, 1990 (GW)								
SP500-Nikkei	0.161	0.426	2.258*	Yes	0.319	0.581	1.189	No
SP500-FTSE	0.454	0.602	1.591	No	0.613	0.698	0.532	No
SP500-DAX	0.347	0.561	2.096*	Yes	0.433	0.675	1.268	No
Nikkei-FTSE	0.172	0.516	3.064**	Yes	0.380	0.591	0.992	No
Nikkei-DAX	0.103	0.546	3.925**	Yes	0.088	0.766	3.284**	Yes
FTSE-DAX	0.463	0.620	1.720*	Yes	0.525	0.733	1.253	No
December 19, 1994 (MX)								
SP500-Nikkei	0.224	0.229	0.042	No	0.469	0.066	-1-576	No
SP500-FTSE	0.499	0.554	0.584	No	0.545	0.660	0.654	No
SP500-DAX	0.421	0.587	0.366	No	0.261	0.366	0.419	No
Nikkei-FTSE	0.278	0.231	-0.385	No	0.189	0.355	0.642	No
Nikkei-DAX	0.194	0.293	0.811	No	-0.103	0.451	2.098*	Yes
FTSE-DAX	0.589	0.656	0.814	No	0.594	0.460	-0.668	No
July 15, 1997 (EA)								
SP500-Nikkei	0.156	0.301	1.179	No	0.298	0.440	0.587	No
SP500-FTSE	0.566	0.715	1.965*	Yes	0.682	0.833	1.299	No
SP500-DAX	0.531	0.702	2.155*	Yes	0.693	0.847	1.398	No
Nikkei-FTSE	0.095	0.420	2.713**	Yes	0.072	0.449	1.464	No
Nikkei-DAX	0.207	0.385	1.510	No	0.200	0.388	0.735	No
FTSE-DAX	0.583	0.780	2.929**	Yes	0.633	0.842	1.711	No
August 13, 1998 (RU)								
SP500-Nikkei	0.263	0.352	0.758	No	0.352	0.550	0.893	No
SP500-FTSE	0.707	0.605	-1.386	No	0.808	0.763	-0.420	No
SP500-DAX	0.673	0.628	-0.594	No	0.789	0.762	-0.235	No
Nikkei-FTSE	0.345	0.449	0.952	No	0.358	0.487	0.562	No
Nikkei-DAX	0.361	0.396	0.309	No	0.300	0.573	1.223	No
FTSE-DAX	0.725	0.780	0.980	No	0.794	0.790	-0.037	No
March 24, 2000 (IT)								
SP500-Nikkei	0.186	0.407	1.877*	Yes	0.496	0.462	-0.160	No
SP500-FTSE	0.564	0.655	1.124	No	0.701	0.830	1.137	No
SP500-DAX	0.518	0.719	2.565**	Yes	0.529	0.845	2.316*	Yes
Nikkei-FTSE	0.376	0.295	-0.706	No	0.386	0.307	-0.320	No
Nikkei-DAX	0.389	0.346	-0.381	No	0.487	0.357	-0.565	No
FTSE-DAX	0.631	0.761	1.983*	Yes	0.504	0.827	2.229*	Yes
September 11, 2001 (WTC)								
SP500-Nikkei	0.393	0.352	-0.367	No	0.518	0.612	0.495	No
SP500-FTSE	0.654	0.655	0.021	No	0.821	0.898	1.072	No
SP500-DAX	0.748	0.741	-0.133	No	0.895	0.860	-0.549	No
Nikkei-FTSE	0.358	0.394	0.325	No	0.374	0.636	1.278	No
Nikkei-DAX	0.340	0.458	1.092	No	0.446	0.738	1.658	No
FTSE-DAX	0.815	0.834	0.474	No	0.864	0.875	0.162	No
July 15, 2007 (GF)								
SP500-Nikkei	0.356	0.605	2.530**	Yes	0.633	0.883	2.297*	Yes
SP500-FTSE	0.738	0.777	0.711	No	0.852	0.886	0.486	No
SP500-DAX	0.652	0.873	4.387**	Yes	0.798	0.958	2.951**	Yes
Nikkei-FTSE	0.538	0.702	2.079*	Yes	0.751	0.907	1.899*	Yes
Nikkei-DAX	0.531	0.707	2.226*	Yes	0.689	0.893	2.111*	Yes
FTSE-DAX	0.844	0.912	2.357**	Yes	0.881	0.967	2.377*	Yes

3.4 Conclusion

This chapter studies the presence of contagion between major world markets. Contagion has been a widely studied subject for two decades. Papers after the 1987 stock markets crash provided evidence of contagion between markets (King and Wadhwani 1990, Lee and Kim 1993). Somewhat later many papers examined the presence of contagion in developing markets. There the results were mainly similar and provided evidence of contagion (see for example Calvo and Reinhart 1996). Forbes and Rigobon (2002) end up in a different conclusion. They argue that the heteroscedasticity of the return series causes a bias to the correlation and therefore contagion mostly does not exist. These conclusions are criticized at least by Corsetti et al (2005) and Bartram & Wang (2005). They note that the results of Forbes and Rigobon are caused by the assumed model. These days, the consensus of contagion lies somewhere in the "some contagion" -zone.

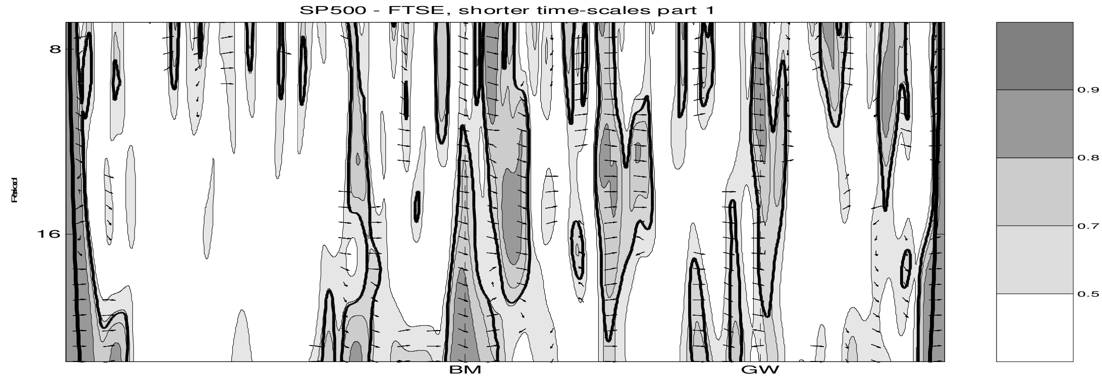
This chapter extends the contagion literature by adding time scale dimension to the picture. Different time scales are analyzed using the continuous wavelet transform based wavelet coherence and the discrete wavelet transform based wavelet correlation. The results show how the correlations change as a function of the scale. As Rua and Nunes (2009) note, much more thorough analysis of interrelations can be achieved using the wavelet methods. This applies also to contagion study. The correlation structure changes that are found with wavelet methods might be missed with ordinary correlation analysis, since the correlation in time possibly changes only on a certain time scale.

The definition of contagion follows Forbes and Rigobon (2002). If there is an increase in correlation after some crisis point, we have contagion. In this paper contagion is defined to be a change in the short time scale correlations, long time-scale correlations remaining the same. Using this definition, clear signs of contagion are found. Correlations on shorter time scales increase significantly while longer time scales remain approximately the same. This is most clearly seen with the 1987 stock market crash, the Gulf War and the ongoing global financial crisis. Some signs of contagion are seen with other crisis, especially in the wavelet coherence analysis. However, these changes are not significant at 5% level. The results also show how the short time-scale correlations decrease during tranquil periods (bull markets) giving support to the conclusions of Longin and Solnik (2001). Also long time-scale correlations indicate an overall increase of interdependence during the time period studied. This increase in interdependence (Forbes & Rigobon, 2002) plus contagion during the ongoing crisis makes the markets very highly correlated on every scale at the moment.

The inclusion of a multiresolution analysis, i.e. different time scales, proves out to be very important. As a matter of fact, they are almost vital as correlations change quickly as a function of scale. Therefore, many changes are seen only during certain time scales. The results show that contagion has been a major factor between major markets few times in the last 25 years. Contagion phenomenon is not disappearing since almost the strongest signs of contagion can be seen during the ongoing financial crisis.

3.5 Wavelet coherence diagrams

The following pages present the wavelet coherence maps for the studied indices. These figures were separated from the main text to maintain readability. On every page there are four figures. Three upmost figures represent time scales for 7 to 25 days. These scales were separated to three figures to make the figures more informative. The last figure on every page presents time scales from 25 to 600 days for the whole sample period. The information title is seen above every figure.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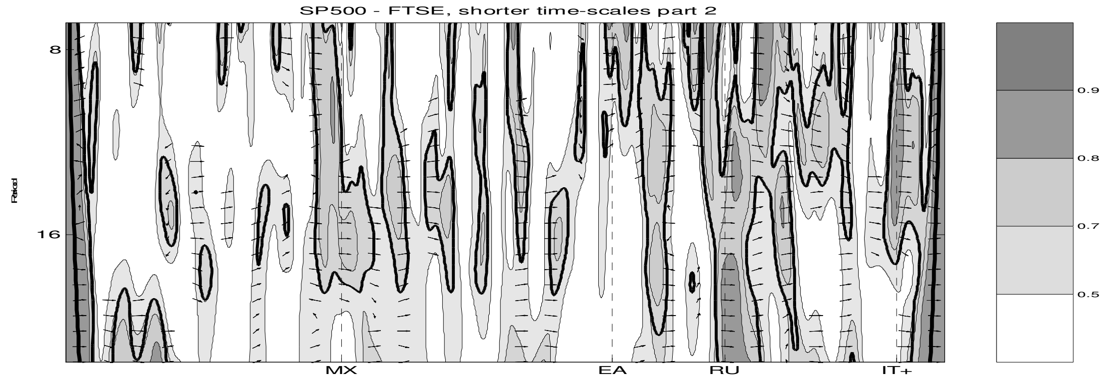
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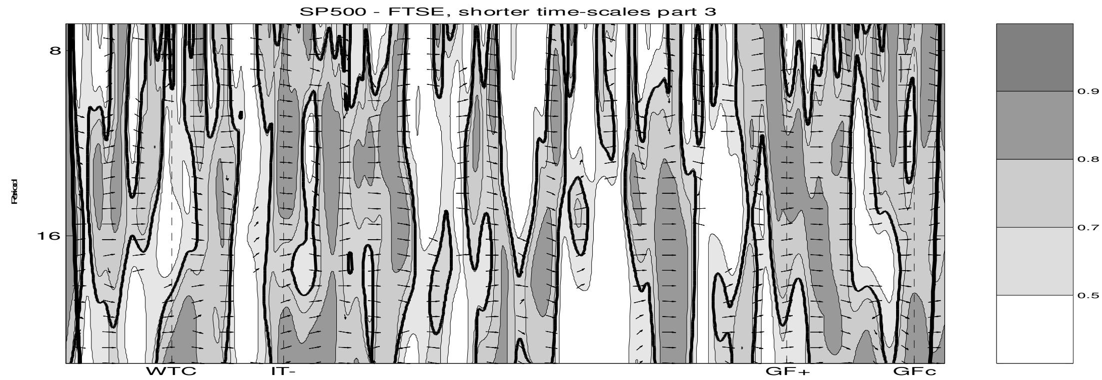
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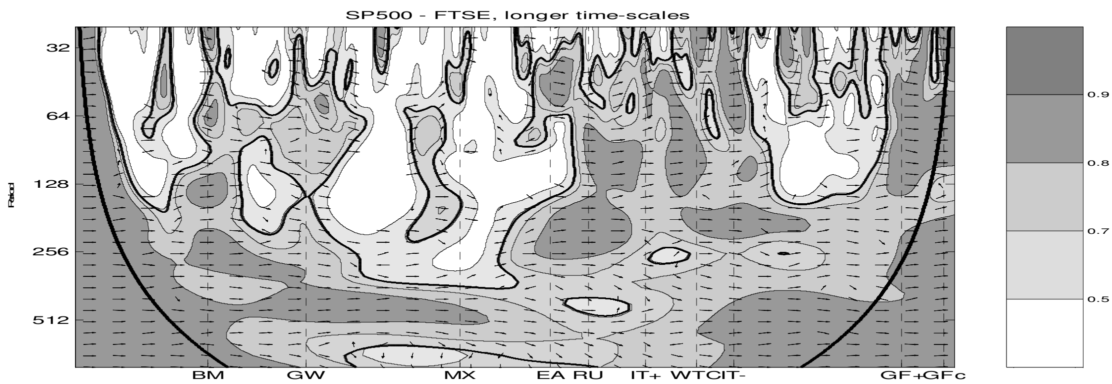
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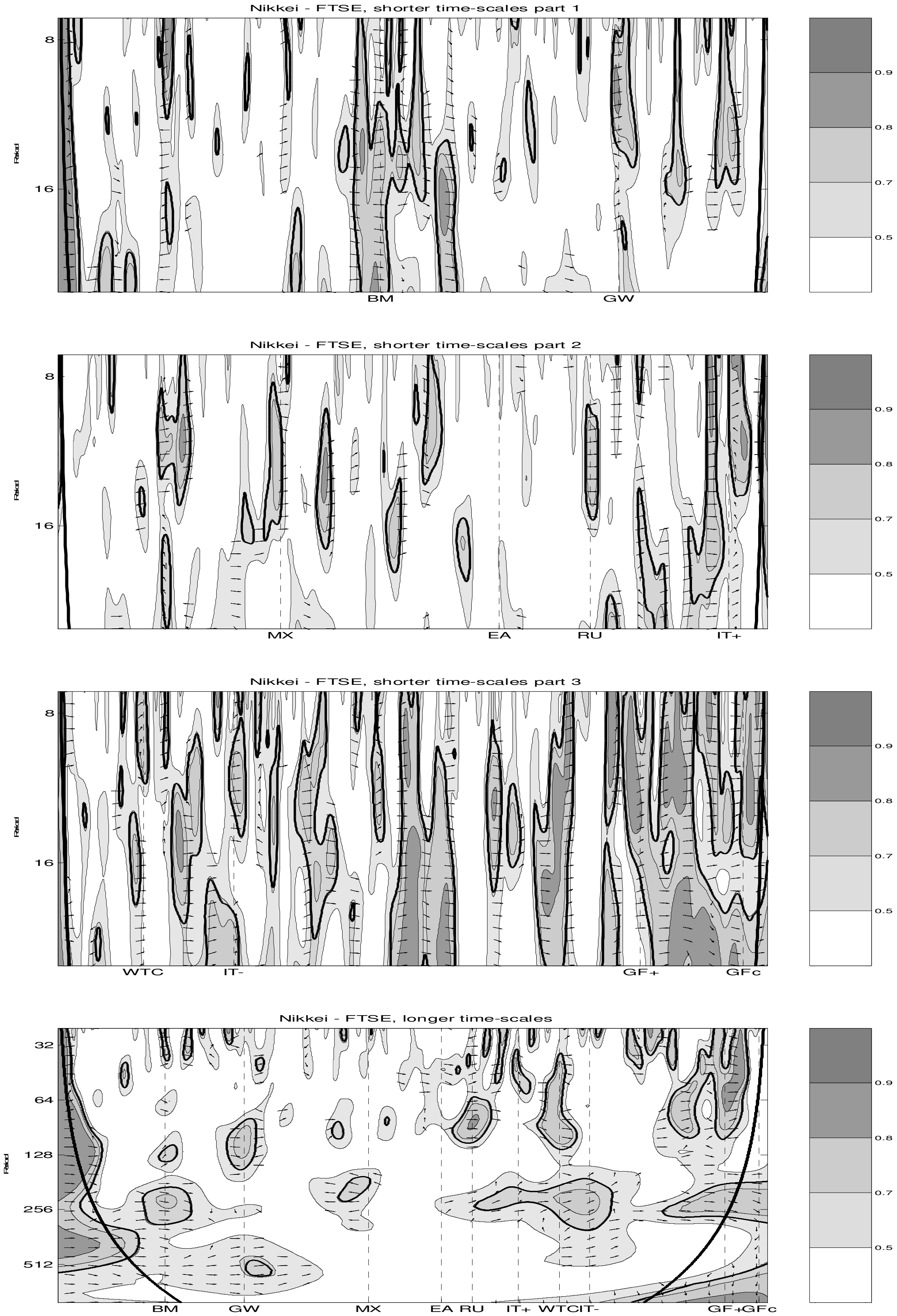
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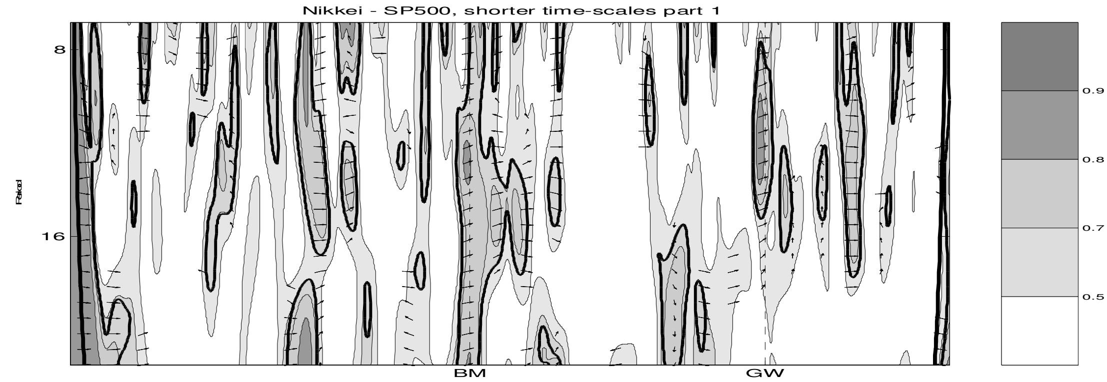
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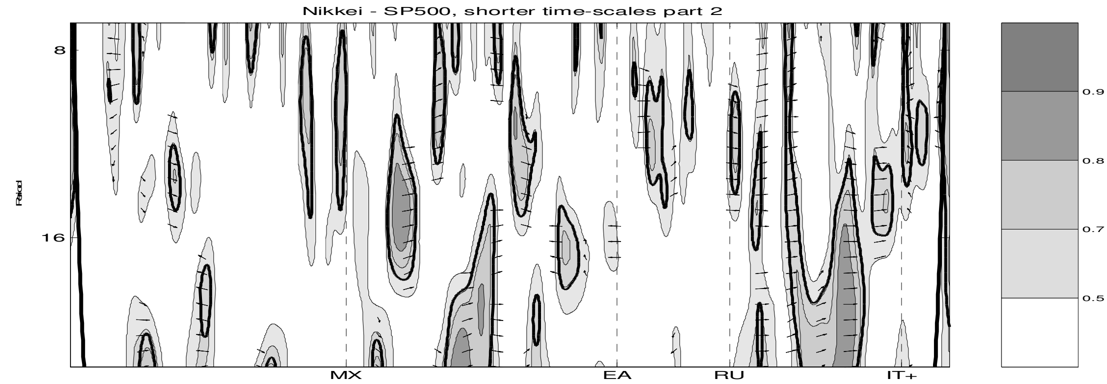
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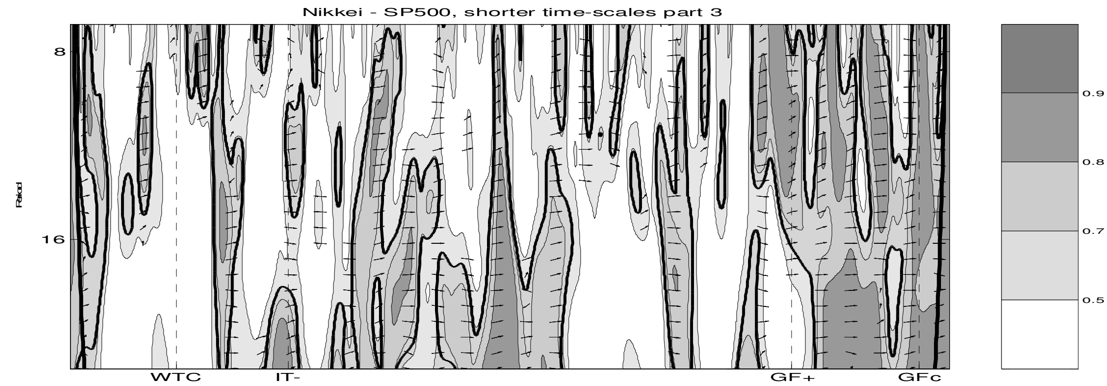
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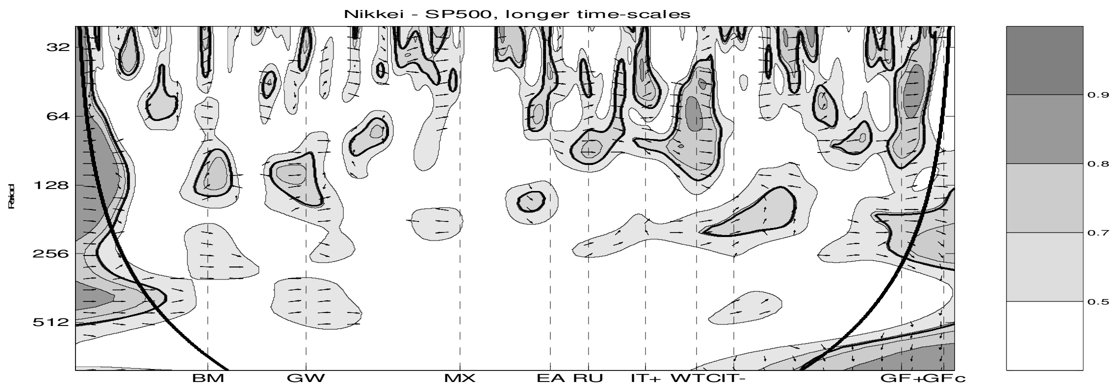
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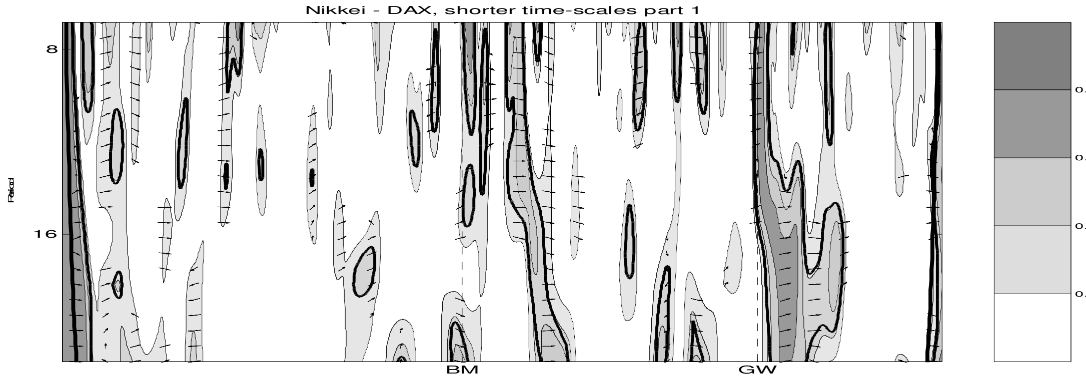
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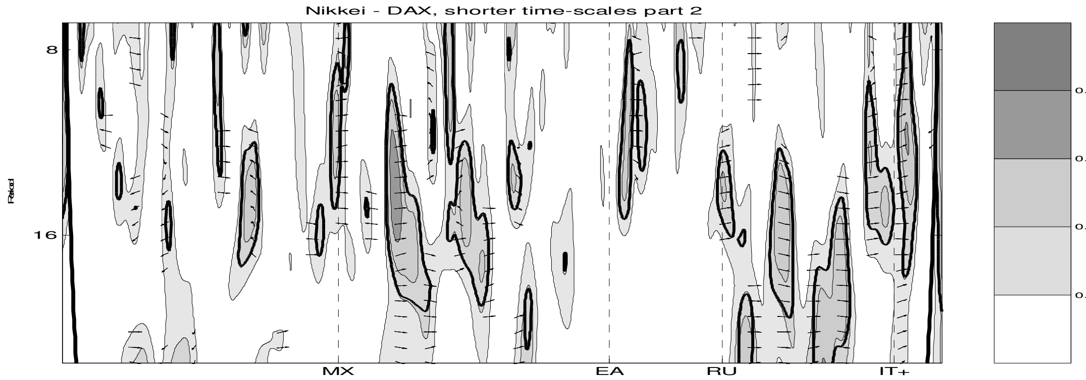
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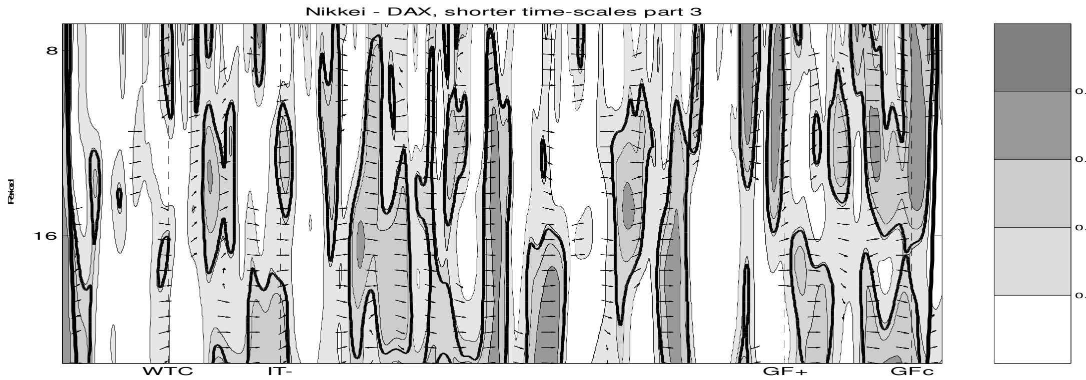
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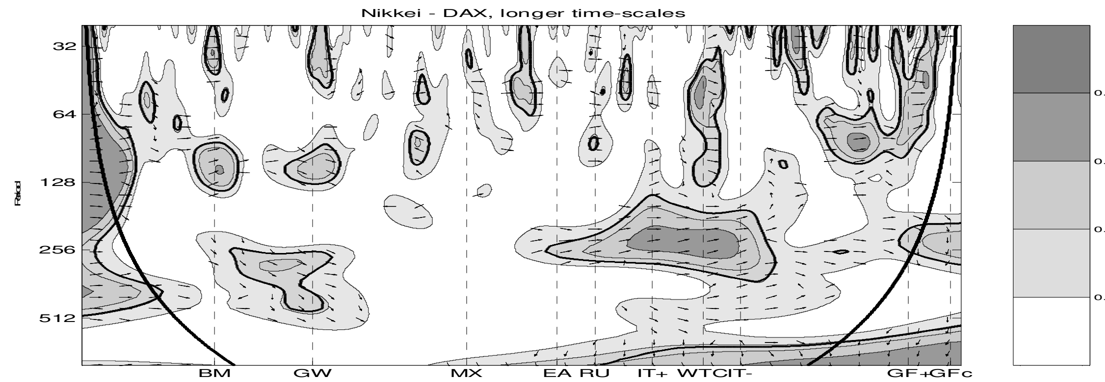
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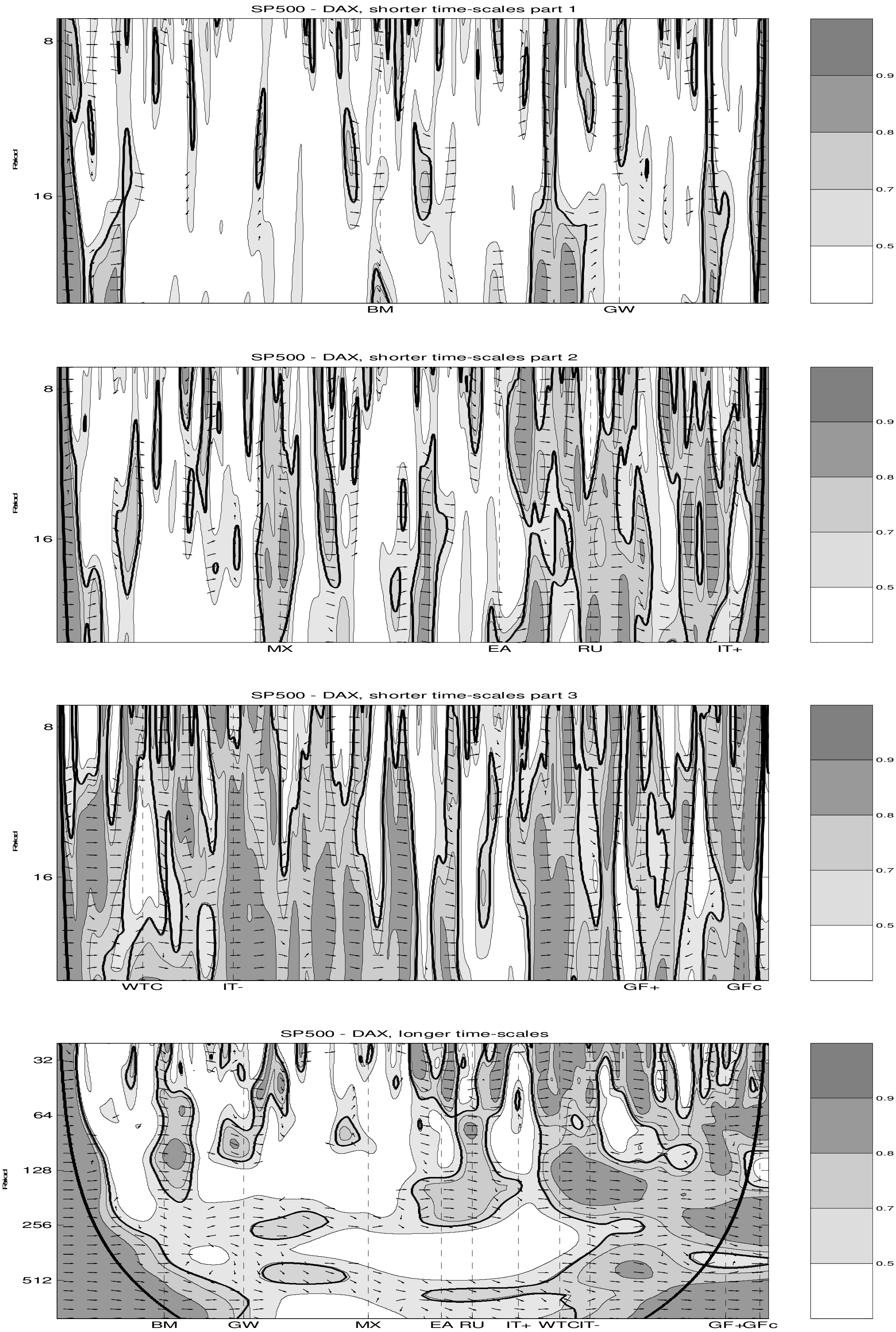
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SP500 - DAX, shorter time-scales part 3
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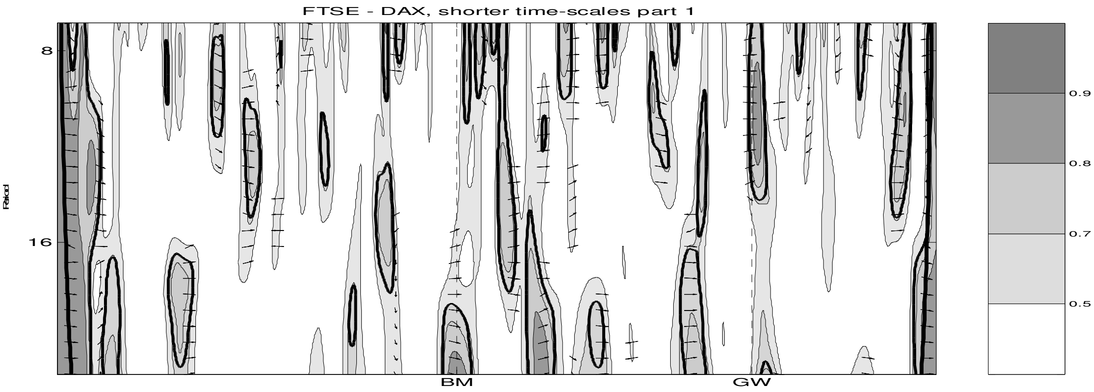
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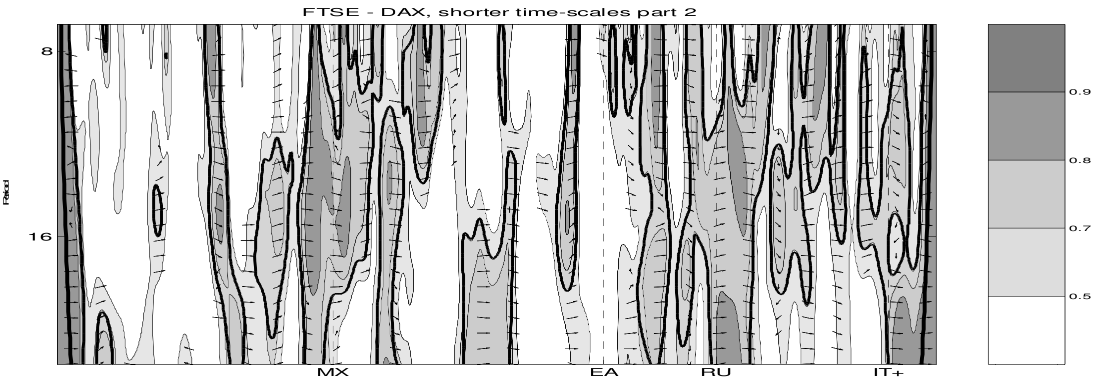
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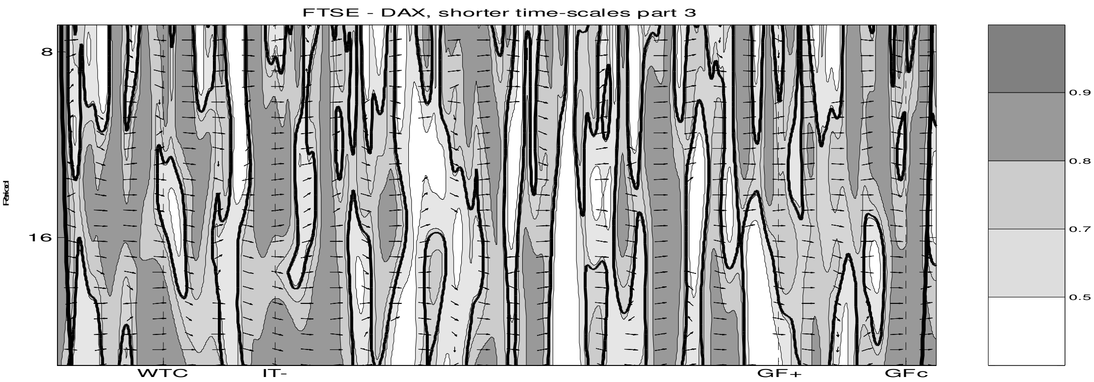
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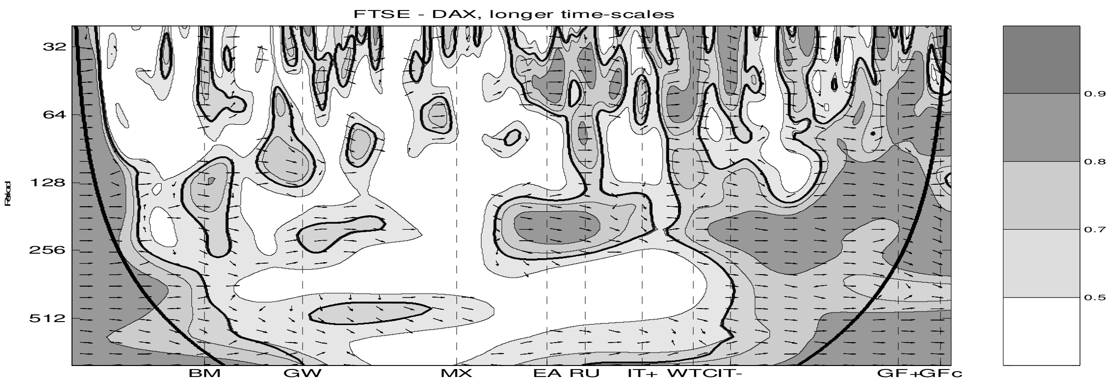
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4 CROSS-CORRELATION BETWEEN MAJOR EUROPEAN EXCHANGE RATES

This chapter examines the lead-lag relations of the major European currencies using wavelet cross-correlation. The estimators of wavelet cross-correlation are constructed using the maximal overlap discrete wavelet transform. The results indicate that the euro and the Swiss franc lead the British pound on larger scales. On a one month and longer time scales the lag of the Pound is obvious. Overall wavelet cross-correlation reveals a rich dynamical structure between the exchange rates.

4.1 Introduction

There are many kinds of investors participating in markets. There are investors with a short time investment strategy and there are institutional investors whose investment time scale might be years. This heterogeneous structure should be implemented in the study of market relations. Wavelet methods can handle this kind of structure for time series analysis. Wavelet techniques possess a natural ability to decompose a time series into several sub-series which may be associated with particular time scales. Therefore, wavelet multiresolution analysis (MRA) makes it possible to analyze these sub-processes separately.

Lead-lag relations in the financial time series analysis are widely studied. Knif et al. (1995) analyze lead-lag relations between the Swedish and Finnish stock markets using the methods of univariate spectral analysis and cross-spectral analysis. They find that differences between the return spectra of the two markets are significant and that Swedish market leads about ten days for the period 1977-1985 and five days for the period 1986-1989. Lead-lag relations between options-market and the related spot market have attracted a significant attention. Boyle et al. (1999) for example show that the S&P 500 index option market leads the cash index. In their paper they develop a model that relates the bias in implied volatility to the lead-lag structure between the two markets. Their results also show that implied volatility is statistically significantly biased due the lead-lag relationship.

Linkages between major exchange rates have also been extensively studied. The overall dynamics of the foreign exchange market are analyzed for example in Mahajan and Wagner (1999), Cai et al. (2008), Gadea et al. (2004), MacDonald & Marsh (2004), Tabak & Cajueiro (2006) and Ramsey & Zhang (1997). As mentioned, these articles study overall dynamics. Some papers that specifically concentrate on the linkages between exchange rates are presented below. Hong (2001) proposes a new test for volatility spillovers and applies this method to exchange rates. The series which are scrutinized are weekly exchange rates of the Deutsche mark and the Japanese yen against the US dollar from the first week of 1976 to the last week of 1995. The results mainly indicate that there is not much causality in mean and variance between these exchange rates. There is however some indication that a change in the Deutsche mark volatility Granger causes a change in Japanese yen volatility. Matsushita et. al. (2007) study how closely the British pound follows the euro. They find differences in the dynamics of these two currencies and conclude that these two currencies should be considered as different. Aroskar et al. (2004) study the effect of the 1992 European financial market crisis on foreign exchange markets. Using cointegration methods they identify a long-term relationship between European currencies expect for the British pound which acts somewhat separately from the others. The results also include the Deutsche mark dominance against other currencies, especially during longer time scales. They also find that this dominance almost vanishes during the crisis. Brooks and Hinch (1999) study the lead/lag relationships between Sterling-denominated exchange rates and find that "bigger" currencies lead "smaller" currencies. They also note that the interrelations change significantly with time. Krylova et al. (2009) focus on the linkages of major exchange rates by studying the cross-dynamics of volatility term structure slopes implied by foreign exchange options. Their findings provide interesting new insights to the interrelations of exchange rates. For example foreign exchange options indicate that the euro is the dominant currency. The implied volatility term structure of the euro affects all the other volatility term structures while the term structure of the euro appears to be virtually unaffected by other currencies. Nikkinen et al. (2006) also use options to study linkages in expected future volatilities among major European currencies. The results indicate that the market expectations of future exchange rate volatilities are closely linked. The leading role of the euro against the pound and the franc is observed again. Wu includes time-scales to the study of linkages between USD/DEM and USD/JPY exchange rates. In this study intrinsic mode functions and the Hilbert transform are used to characterize the behaviors of pricing transmissions. The results indicate that the correlations are stronger in the daily time scale than in longer time scales. The correlations also weakened during the observation period of 1986-1993.

Wavelet analysis has been applied to exchange rate analysis several times. Gençay, Selçuk and Whitcher (2002b) apply a wavelet multiscaling approach to financial time series. They study the properties of foreign exchange volatility using a five-minute data. Composing the variance of a time series on different scales they find that volatilities follow different scaling laws on different horizons, the break point being one day. Their short investigation of wavelet cross-correlation between volatilities indicated that the association between two volatilities is stronger at lower frequencies. Similar studies with results that indicate importance of a scale-based analysis of exchange rates are for example Nekhili et al. (2002) exploring exchange rate returns on different time horizons, Lee (2004) in the study of spillover effects between the different geographical location of markets and Gençay & Selçuk (2004) studying volatility-return dynamics. The research above, which is extended in this chapter, clearly shows the importance of time scale based analysis in exchange rate analysis.

The purpose of this chapter is to examine the lead-lag relations of major European exchange rates using the wavelet cross-correlation methods. Although exchange rates are widely studied, time-scale based analyses are rare. The research of exchange rates has mainly focused on the study of temporal interrelations in time. With the introduction of wavelet methods, these interrelations can be studied in more detail. Understanding the dynamic behavior of exchange rates is considered important because it has important practical implications on the implementation of the investment and risk management strategies. Wavelet methods improve the understanding of this dynamic behavior. With the discrete wavelet transform it is possible to analyze different scale structures or 'processes' forming the original time series. Lead-lag relations that could not be distinguished in the usual cross-correlation analysis can be analyzed. Given that the foreign exchange market is by far the largest financial market in the world, understanding the dynamic behavior of exchange rates is considered essential. Results of this paper directly attack the understanding of this dynamic behavior. The focus in this research is the dynamic behavior of foreign exchange markets inside Europe.

4.2 Wavelet cross-covariance and cross-correlation

The estimators of wavelet cross-correlation and cross-covariance are based on the maximal overlap discrete wavelet transform (MODWT) which was introduced in the second chapter. The MODWT is a variation of the orthonormal discrete wavelet transform (DWT). It is computed similarly to the ordinary DWT but without subsampling. Estimators calculated using the MODWT are considered more preferable because they are asymptotically more efficient than the estimator based on the DWT (Percival, 1995). Furthermore, the ordinary DWT is not suitable for cross analyses, because its lack of translational invariance disrupts the lag-resolution of the wavelet cross-covariance and cross-correlation (Percival & Walden, 2002). The derivation naturally follows closely to the derivation of wavelet covariance and wavelet correlation estimators.

Let $\{h_1\}=\{h_{1,0},\ldots,h_{1,L-1}\}$ and $\{g_1\}=\{g_{1,0},\ldots,g_{1,L-1}\}$ denote the MODWT wavelet filter and scaling filter coefficients from a Daubechies compactly supported wavelet family (Daubechies, 1992). Let

$$H_{1,k}=\sum_{m=0}^{N-1}h_{1,m}e^{-i2\pi mk/N},\ k=0,\ldots,N-1,\ N\geq L,\ h_{1,m}=0 \text{ for } m\geq L \tag{18}$$

be the discrete Fourier transform (DFT) of $\{h_1\}$ and define $G_{1,k}$ similarly for $\{g_1\}$. The wavelet filter $\{h_j\}$ for scale $\lambda_j=2^{j-1}$ is defined as the inverse DFT of

$$H_{j,k}=H_{1,2^{j-1}k \bmod N}\prod_{l=0}^{j-2}G_{1,2^l k \bmod N},\quad k=0,\ldots,N-1 \tag{19}$$

and the scaling filter for scale $2\lambda_J$ as the inverse DFT of

$$G_{J,k}=\prod_{l=0}^{J-1}G_{1,2^l k \bmod N},\quad k=0,\ldots,N-1\,. \tag{20}$$

The vector of MODWT coefficients $\mathbf{W}_j$, $j=1,\ldots,J$, defining a *j*th order partial MODWT of time series $\{x_i\}$, is defined to be the inverse DFT of $H_{j,k}X_k$ where $\{X_k\}$ is the DFT of $\{x_i\}$. These coefficients are associated with the changes of scale λ_j. The vector of MODWT scaling coefficients $\mathbf{V}_J$ is defined similarly by the inverse DFT of $G_{J,k}X_k$ and is associated with averages of scale $2\lambda_J$ and higher. The wavelet cross-covariance decomposes the cross-covariance between two stochastic processes on a scale-by-scale basis and the wavelet cross-correlation similarly decomposes the cross-correlation between processes. In the following, the MODWT coefficients are used to construct estimators of wavelet cross-covariance and cross-correlation.

Let $\{x_t\}=\{\ldots,x_{-1},x_0,x_1,\ldots\}$ and $\{y_t\}=\{\ldots,y_{-1},y_0,y_1,\ldots\}$ be stochastic processes whose d_xth and d_yth order backward differences are stationary Gaussian processes. The wavelet cross-covariance for scale $\lambda_j=2^{j-1}$ and lag τ is defined to be

$$\gamma_{\tau,xy}\left(\lambda_j\right)=\operatorname{cov}\left\{\mathbf{W}_{j,t}^{(x)},\mathbf{W}_{j,t+\tau}^{(y)}\right\} \tag{21}$$

where $\left\{\mathbf{W}_{j,t}^{(x)}\right\}$ and $\left\{\mathbf{W}_{j,t+\tau}^{(y)}\right\}$ are the scale λ_j MODWT coefficients for $\{x_t\}$ and $\{y_{t+\tau}\}$, respectively. The MODWT coefficients have a mean of zero, when the order of the wavelet filter is $L \geq 2 \cdot \max\{d_x, d_y\}$, and therefore, $\gamma_{\tau,xy}(\lambda_j) = \mathrm{E}\{\mathbf{W}_{j,t}^{(x)}, \mathbf{W}_{j,t+\tau}^{(y)}\}$. When calculating an estimate for the wavelet cross-covariance, the boundary effects of wavelet filtering have to be considered. Assuming $x_0, \ldots, x_{N-1}$ and $y_0, \ldots, y_{N-1}$ as realizations of portions of the processes $\{x_t\}$ and $\{y_t\}$, define $\bar{\mathbf{W}}_{j,t} = \mathbf{W}_{j,t}$ for those indices *t* where $\mathbf{W}_{j,t}$ is unaffected by the boundary of realizations. We define a biased estimator $\bar{\gamma}_{\tau,xy}(\lambda_j)$ of the wavelet cross-covariance as in Whitcher et al. (1999). The MODWT based estimator is defined as

$$\bar{\gamma}_{\tau,xy}(\lambda_j) = \begin{cases} \dfrac{1}{N - L_j + 1} \displaystyle\sum_{l=L_j-1}^{N-\tau-1} \bar{\mathbf{W}}_{j,l}^{(x)} \bar{\mathbf{W}}_{j,l+\tau}^{(y)}, & \tau = 0, \ldots, N - L_j; \\ \dfrac{1}{N - L_j + 1} \displaystyle\sum_{l=L_j-1-\tau}^{N-1} \bar{\mathbf{W}}_{j,l}^{(x)} \bar{\mathbf{W}}_{j,l+\tau}^{(y)}, & \tau = -1, \ldots, -(N - L_j); \\ 0, & \text{otherwise.} \end{cases} \tag{22}$$

We can further define the wavelet cross-correlation for scale λ_j and lag τ as

$$\rho_{\tau,xy}(\lambda_j) = \frac{\operatorname{cov}\{\mathbf{W}_{j,t}^{(x)}, \mathbf{W}_{j,t+\tau}^{(y)}\}}{\left(\operatorname{var}\{\mathbf{W}_{j,t}^{(x)}\} \operatorname{var}\{\mathbf{W}_{j,t}^{(y)}\}\right)^{1/2}} = \frac{\gamma_{\tau,xy}(\lambda_j)}{\sqrt{\nu_x(\lambda_j)\nu_y(\lambda_j)}}, \tag{23}$$

where $\nu_x(\lambda_j)$ and $\nu_y(\lambda_j)$ are the wavelet variances of stochastic processes introduced by Percival (1995). Because of the definition of the correlation coefficient, $-1 \leq \rho_{\tau,xy}(\lambda_j) \leq 1$ for all τ, j. The wavelet cross-correlation is similar to its Fourier counterpart – the magnitude squared coherence – but it is related to bands of frequencies (scales). The wavelet cross-correlation provides lead-lag relationships between two processes on a scale by scale basis. Since it is simply made up of the wavelet cross-covariance for $\{x_t, y_t\}$ and wavelet variances for $\{x_t\}$ and $\{y_t\}$, an unbiased estimator of the wavelet correlation based on the MODWT is given by

$$\bar{\rho}_{\tau,xy}(\lambda_j) = \frac{\bar{\gamma}_{\tau,xy}(\lambda_j)}{\sqrt{\bar{\nu}_x(\lambda_j)\bar{\nu}_y(\lambda_j)}}. \tag{24}$$

To calculate the confidence intervals of wavelet cross-correlation, we use the results of Whitcher et al. (1999, 2000). To ensure that the confidence intervals are between the interval $[-1,1]$, Fisher's z-transformation

$$h(\rho)=\frac{1}{2}\log\left(\frac{1+\rho}{1-\rho}\right)=\tanh^{-1}\rho \tag{25}$$

is used. For the estimated correlation coefficient $\bar{\rho}$, based on n independent samples, $\sqrt{n-3}\left(h(\hat{\rho})-h(\rho)\right)$ has approximately a $N(0,1)$ distribution. An approximate $100(1-2p)\%$ confidence interval for $\rho_{xy}(\lambda_j)$ based on the MODWT is

$$\left[\tanh\left\{h\left[\bar{\rho}_{xy}(\lambda_j)\right]-\frac{\Phi^{-1}(1-p)}{\sqrt{N-L_j'-3}}\right\},\tanh\left\{h\left[\bar{\rho}_{xy}(\lambda_j)\right]+\frac{\Phi^{-1}(1-p)}{\sqrt{N-L_j'-3}}\right\}\right] \tag{26}$$

where $L_j'=\left\lceil (L-2)(1-2^{-j})\right\rceil$ is the number of DWT coefficients associated with scale λ_j. We use the number of wavelet coefficients as if $\bar{\rho}(\lambda_j)$ had been computed using the DWT because, under the assumptions of Fisher's z-transformation, the denominator should consist of the number of independent samples used in the construction of the correlation coefficient (Whitcher et al. 2000). The DWT is known to approximately decorrelate a wide range of time series and thus provides a reasonable measure of the scale-dependent sample size. This property does not hold for the number of MODWT coefficients because of its lack of downsampling.

4.3 Empirical analysis

4.3.1 Empirical data

The data consists of daily returns of exchange rates between major European currencies and the US dollar. The European currencies used are the British pound, the euro and the Swiss franc. The sample period is between 12.15.1998 – 10.18.2005 and includes 2500 observations. The time period starts from the introduction of the euro and includes enough observations so that wavelet cross correlations of at least four month averages (scale 7 in the MODWT) can be analyzed with reasonable confidence intervals. The data was acquired from www.oanda.com.

Table 6 presents the descriptive statistics for the three series. For every series, the skewness and mean are slightly negative suggesting average negative returns. The skewness of USD-GBP is slightly less negative and standard deviation somewhat smaller than others. However, the skewness does not statistically significantly vary from zero in any instance. The excess kurtosis compared to the normal distribution is quite significant for all cases.

Table 6. Descriptive statistics for the return data of three euro exchange rates. The sample period starts December 15, 1998 and ends October 18, 2005.

Descriptive statistics	USD-EUR	USD-CHF	USD-GBP
Mean (%)	-0.00023	-0.00053	-0.00065
Standard deviation (%)	0.24	0.25	0.19
excess kurtosis	1.88	2.06	2.09
t-value (kurtosis = 3)	19,17	21,09	21,36
Skewness	-0.055	-0.088	-0.019
t-value (skewness = 0)	-1.19	-1.79	-0.38
Range	0.020	0.022	0.017
Minimum	-0.010	-0.012	-0.009
Maximum	0.010	0.011	0.008
Count	2500	2500	2500

Figures 8-10 present the MODWT wavelet coefficients of the first eight scales for the three series. Figure 8 presents coefficients for the USD-EUR series, figure 9 for the USD-CHF series and figure 10 for the USD-GBP return series. A somewhat coarse look of the coefficient series is caused by the wavelet filter. After a visual comparison, the Coiflet(6) wavelet filter was chosen for the analysis, being the best compromise between filters (not too long, not too short) and because it is the most symmetric of all wavelet filters. Shorter wavelet filters are not as good band-pass filters and longer wavelet filters suffer from the boundary effects. Coiflet(6) has a sharp spike in the middle which causes this coarse look for the coefficient series. The comparison was done between the Haar, Daubechies(4), Coiflet(6) and Least Asymmetric(8) filters.

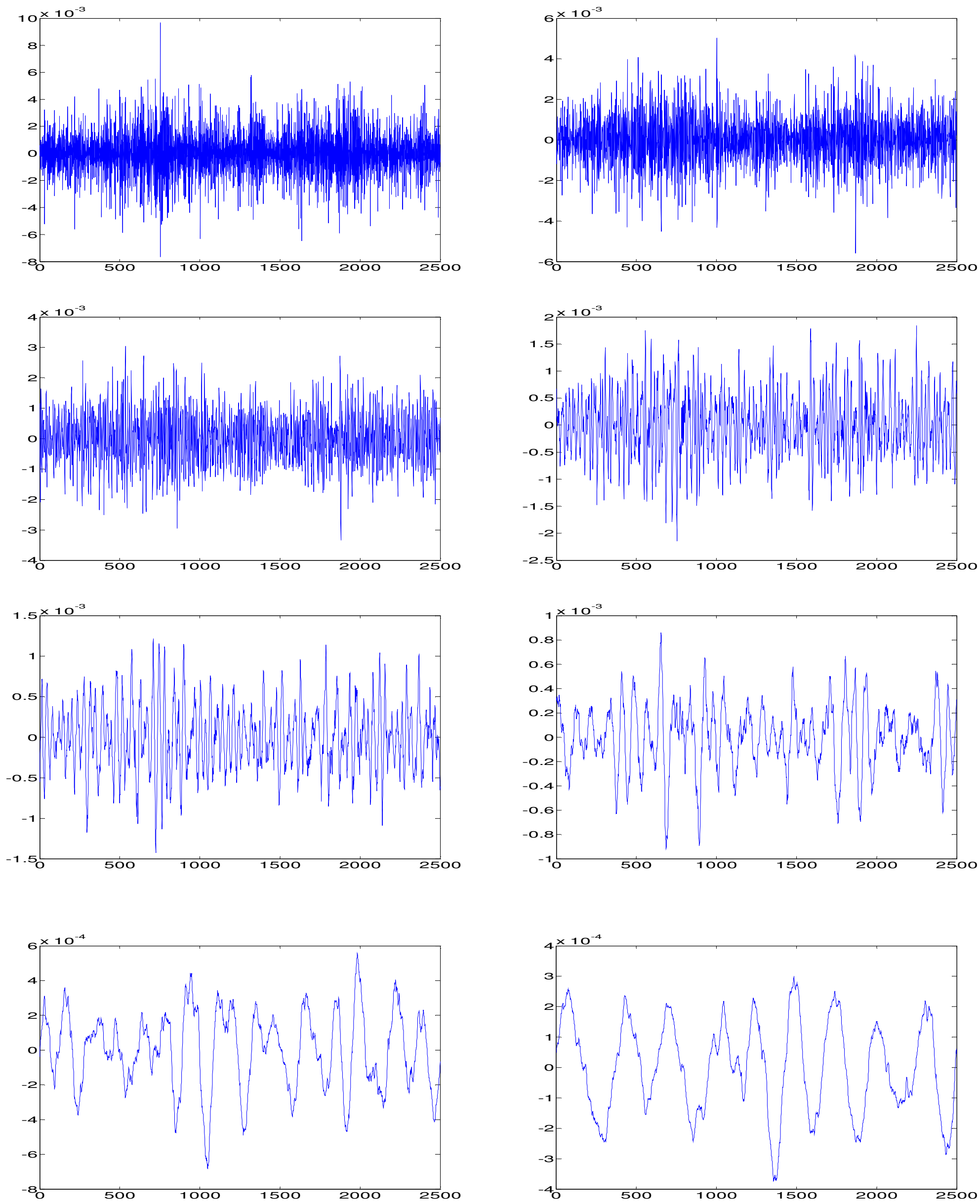

Figure 9. Wavelet coefficients for first eight scales of the USD-EUR return series. The time period is 12.15.1998 – 10.18.2005 using a daily sample frequency

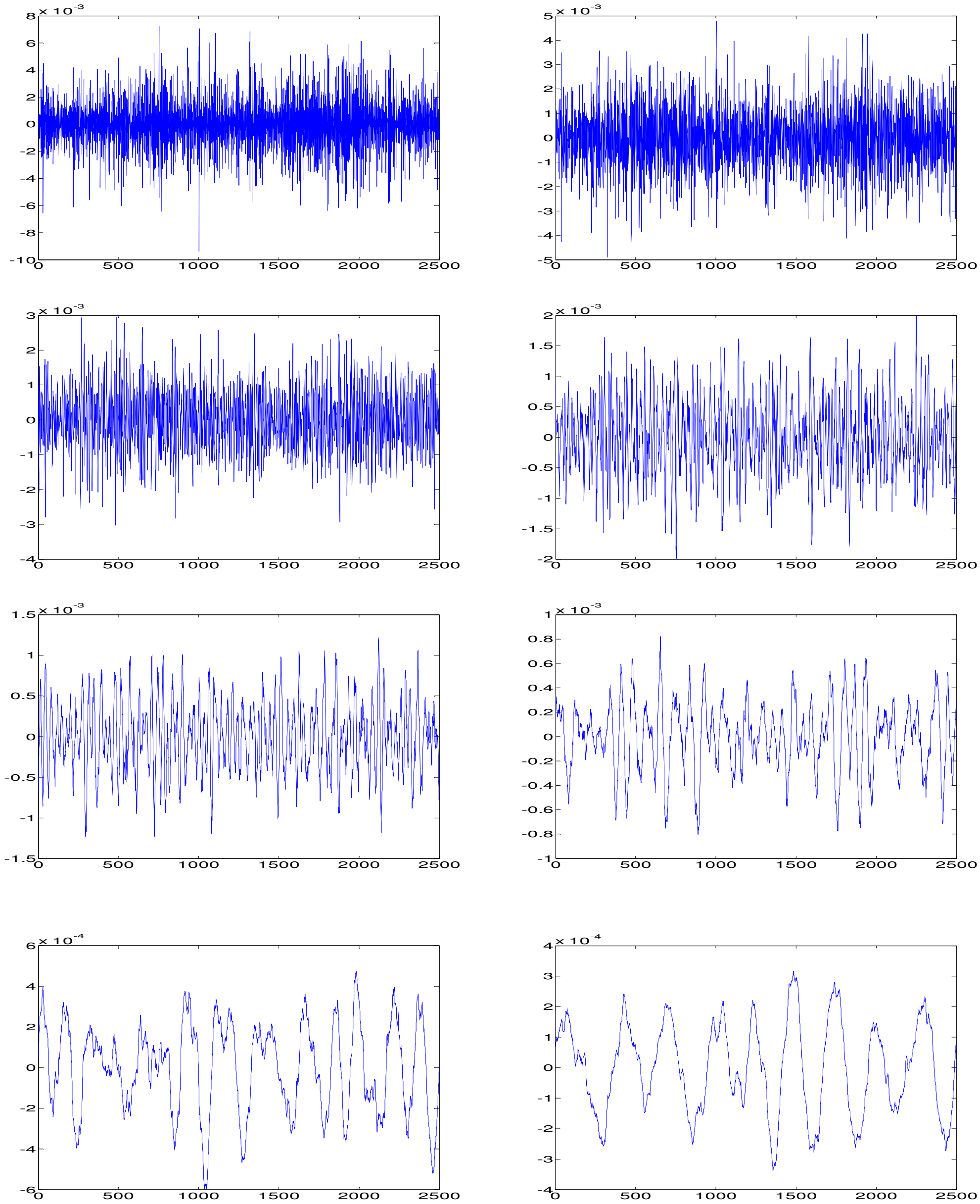

Figure 10. Wavelet coefficients for the first eight scales of the USD-CHF return series. The time period is 12.15.1998 – 10.18.2005 using daily sample frequency.

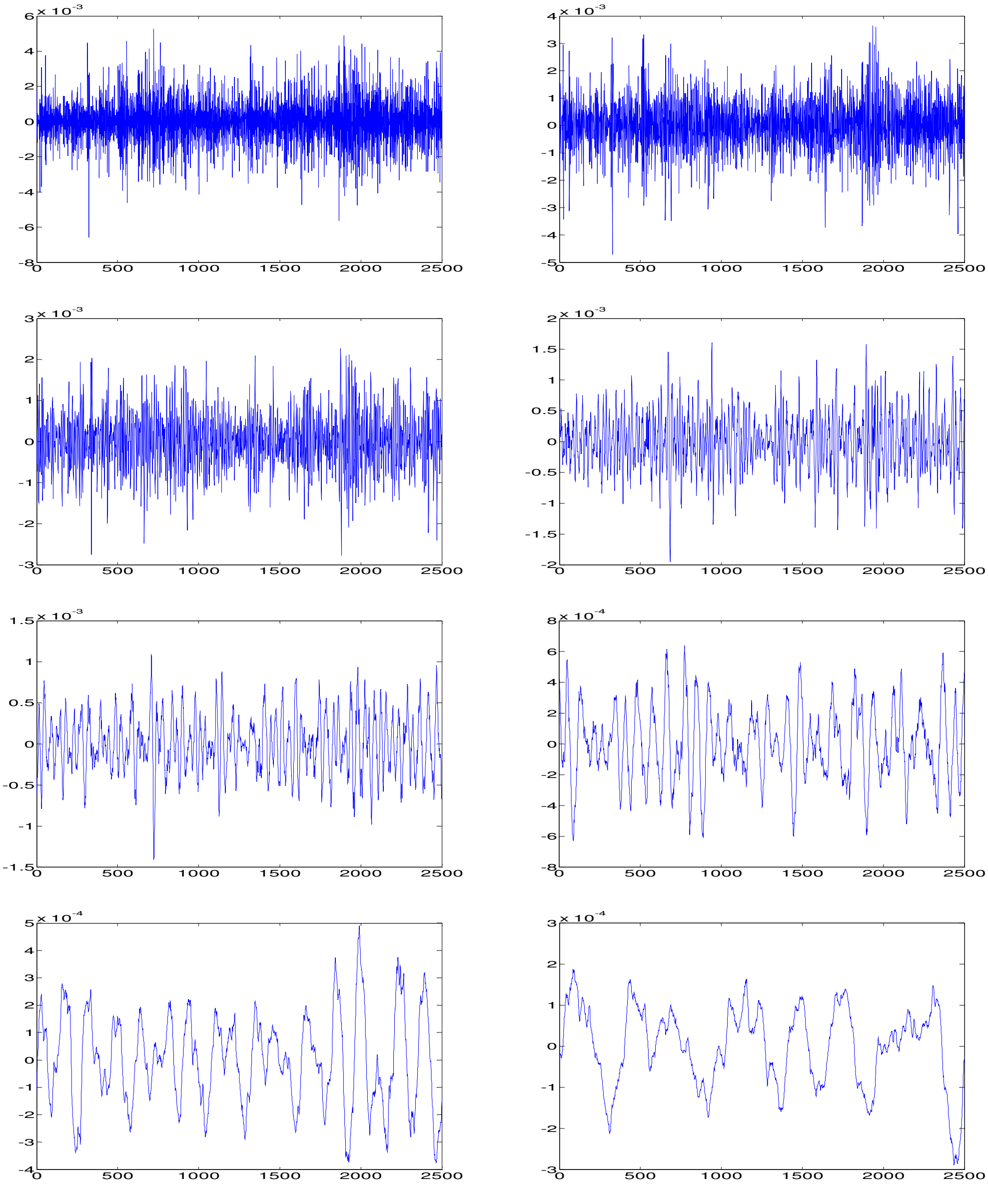

Figure 11. Wavelet coefficients for the first eight scales of the USD-GBP return series. The time period is 12.15.1998 – 10.18.2005 using a daily sample frequency

4.3.2 Empirical results

Figures 12-17 present the cross-correlation diagram of the MODWT wavelet coefficients on eight levels. The scales are associated with periods from 1-2 to 128-256 days in dyadic steps. Some experiments were made on scale 9, which is associated with periods of 256-512 days. This scale is interesting because a one year scale belongs to this. However, the sample data did not have enough independent data points for this scale and there were no significant correlations. The dotted lines around the cross-correlation function are 95% confidence intervals.

Figures 12-13 present cross-correlations between the euro and the Swiss franc. The first scale, associated with the periods of 1-2 days, shows a very strong positive contemporaneous correlation associated with negative correlations on both sides of the positive correlation. The negative correlations are probably artifacts caused by strong contemporaneous correlation and the form of the wavelet filter Coiflet(6). The significant negative correlation around the lag of -26 days and the positive correlation around the lag of 43 days are interesting details.

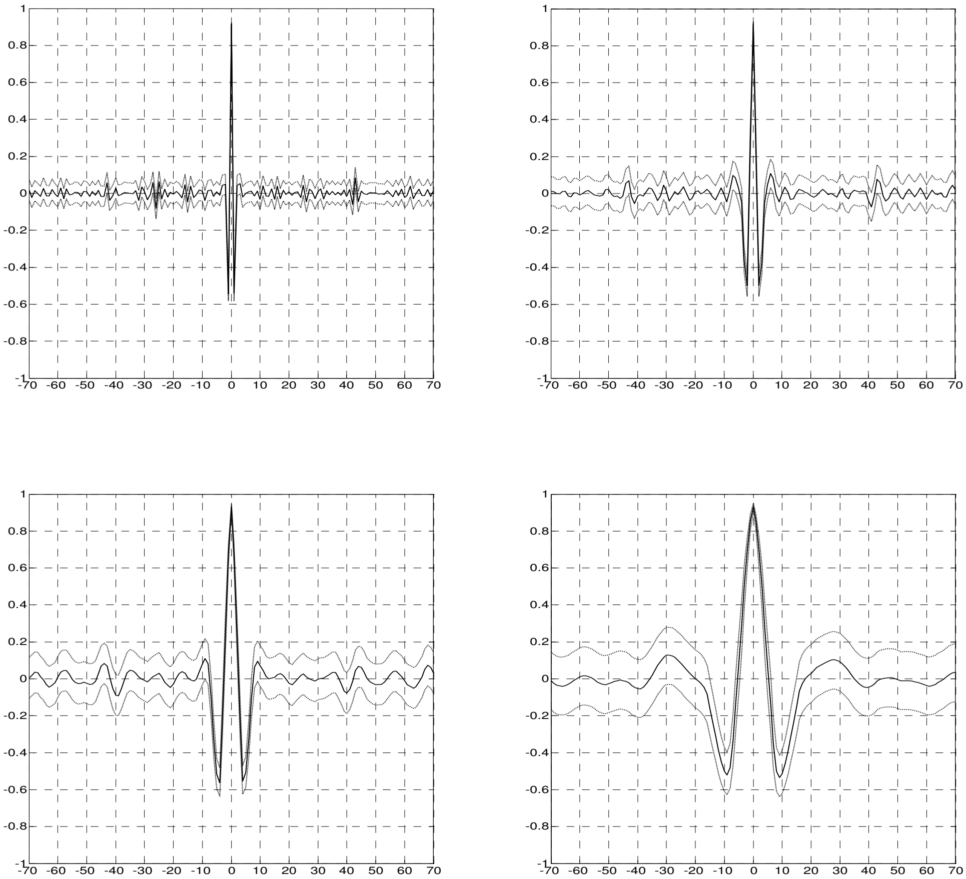

Figure 12. Cross-correlation between the wavelet coefficients of levels 1-4 for the Euro and the Swiss franc. Diagrams present lags from -70 days to +70 days.

The next six scales have only a strong positive contemporaneous correlation with artificial ‘side-lobes’. Otherwise there are no significant correlations at any other lags. The graphs are also quite symmetric, so between the euro and Swiss franc there seems to be no significant flow of information on the first seven scales. The last scale, associated with the periods of 128-256 days, shows minor asymmetry. The strongest positive correlation is on lag 2, which means a two day lead for the Swiss Franc against the euro. The asymmetry is also present in the last significant positive correlations, which are on lags 37 and -28.

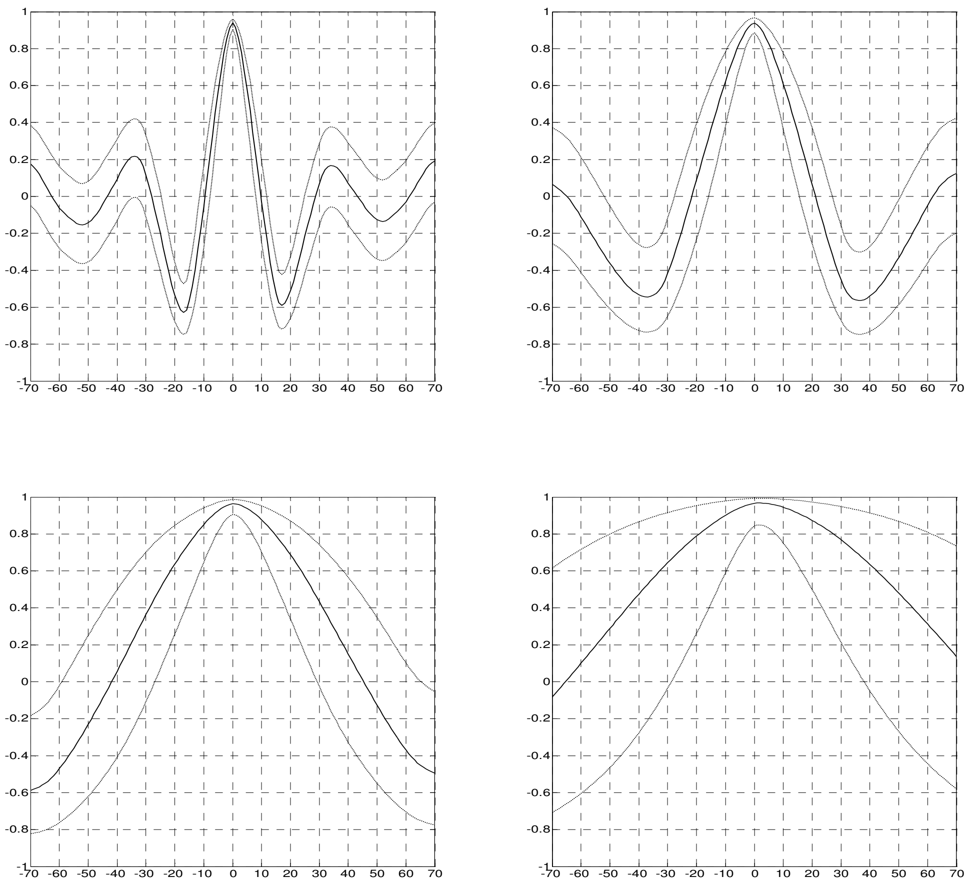

Figure 13. Cross-correlation between the wavelet coefficients of levels 5-8 for the Euro and the Swiss franc. Diagrams present lags from -70 days to +70 days.

Figure 14-15 present the cross-correlations between the euro and the British pound. Again, the first scale has a very strong contemporaneous correlation, although not as strong as between the euro and the franc. There are some significant correlations on the lags of -30 – (-25) the strongest being the negative correlation on the lag of -29 days. Another significant negative correlation is on the lag of 42 days. Preceding scales are somewhat similar. On scale six, which is associated with the periods of 32-64 days, there are significant negative correlations between the lags of 26-48 with a maximum around 30 days, i.e. one month.

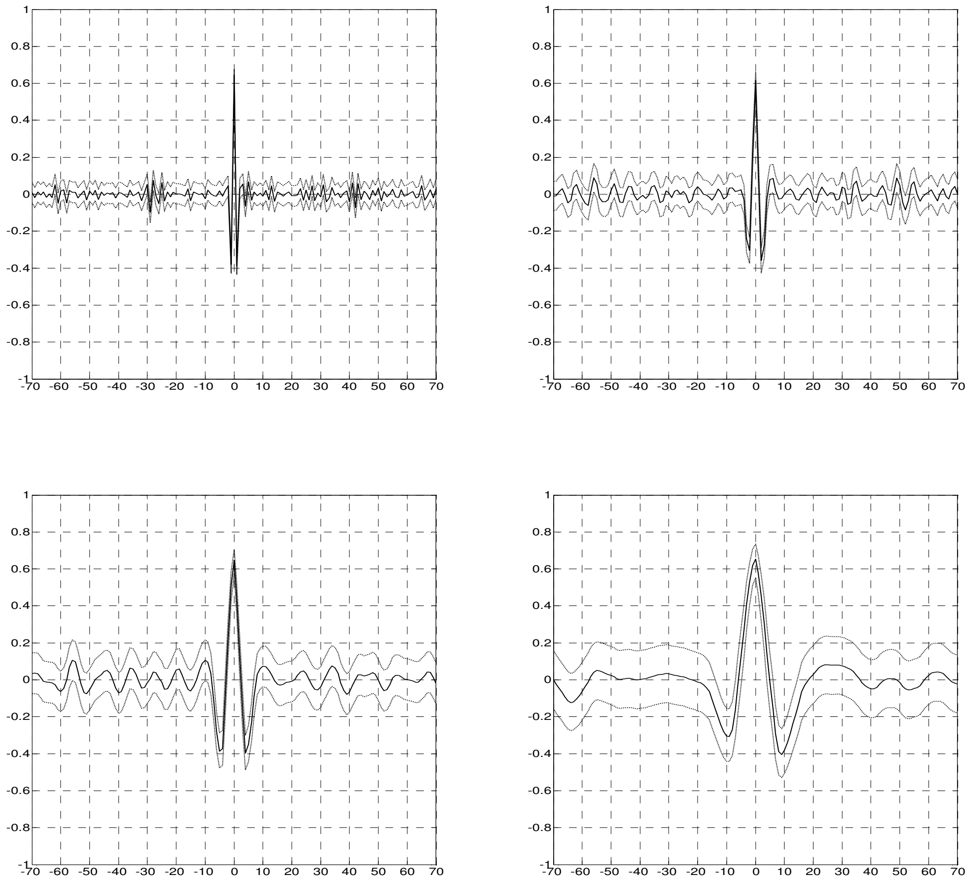

Figure 14. Cross-correlation between the wavelet coefficients of levels 1-4 for the Euro and the British pound. Diagrams present lags from -70 days to +70 days.

The leading role of the euro against the pound starts to appear from scale four onwards. The asymmetry of cross-correlation function towards the euro becomes more pronounced as the scale increases. On scale seven, the last significant correlations on both sides are at lags -31 and 17 so the asymmetry is quite significant. For scale eight these lags are at -32 and 18; scale six at -17 and 11.

This asymmetry means that when we are dealing with long time averages (one month and longer), the present values of the Euro are positively correlated with the future values of the pound.

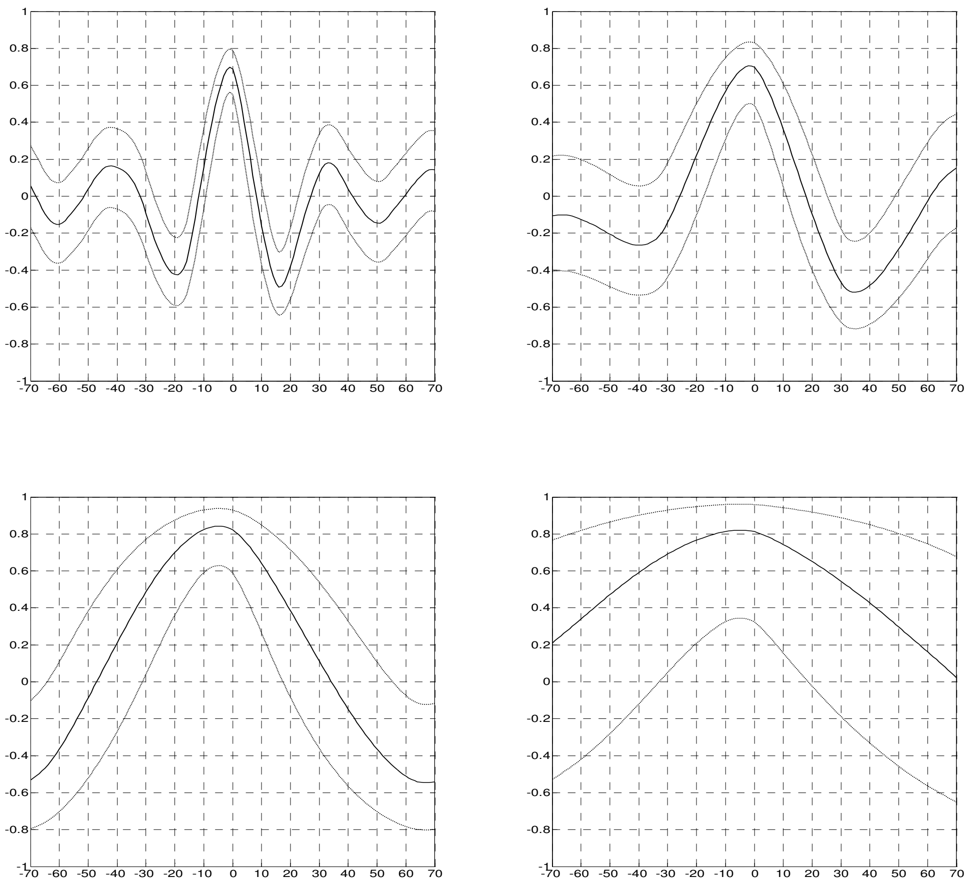

Figure 15. Cross-correlation between the wavelet coefficients of levels 5-8 for the Euro and the British Pound. Diagrams present lags from -70 days to +70 days.

Figures 16-17 present the cross-correlations between the Swiss franc and the British pound. The structure appears to be quite similar to the euro – pound case. There are significant 'spikes' on the first scale on locations -29, 27 and 42. The significant negative correlation around 30 days on scale six is even stronger than in the euro – pound case. The asymmetry of the cross-correlations is also similar to the euro case. For scale six the values are -17 and 11 and for scale seven -31 and 16. On scale eight the asymmetry is much stronger, the critical values being at -32 and 13.

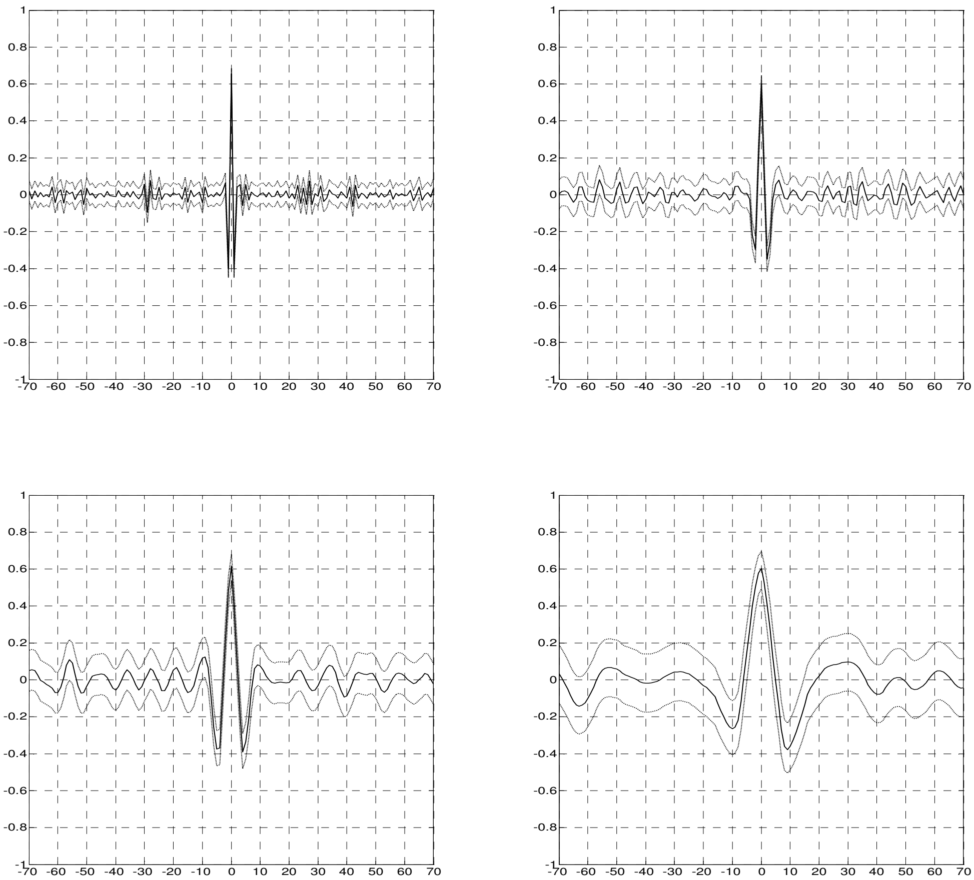

Figure 16. Cross-correlation between the wavelet coefficients of levels 1-4 for the Swiss Franc and the British Pound. Diagrams present lags from -70 days to +70 days.

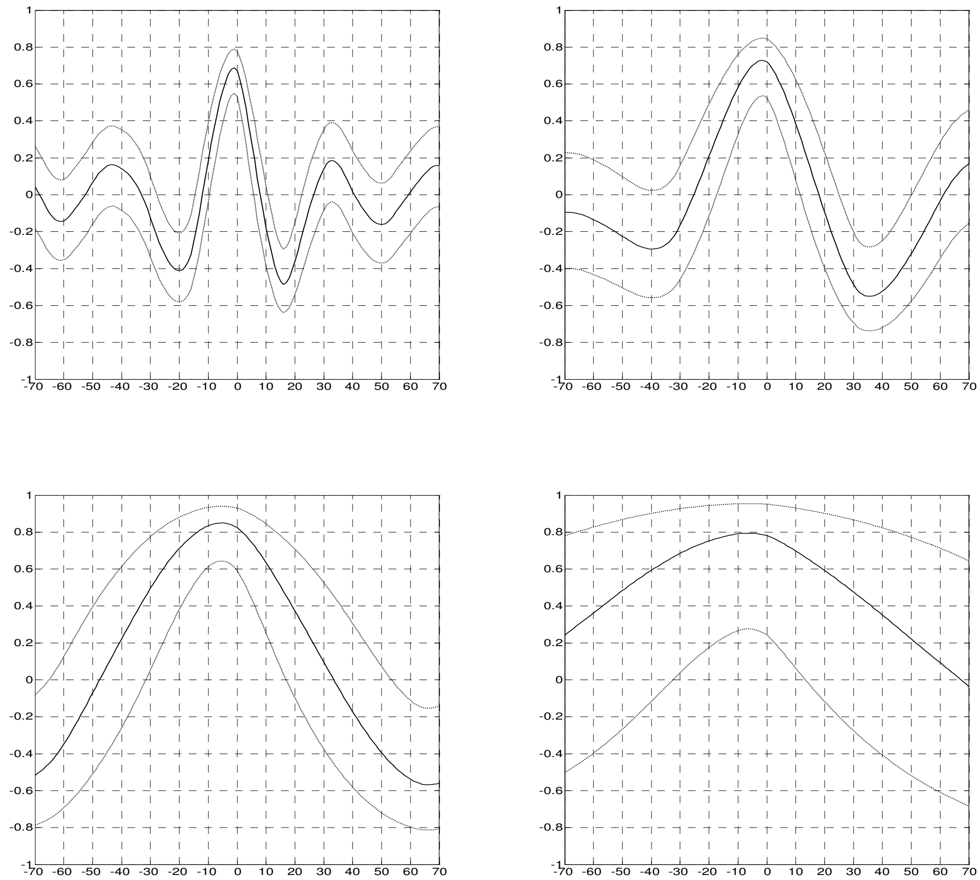

Figure 17. Cross-correlation between the wavelet coefficients of levels 5-8 for the Swiss Franc and the British Pound. Diagrams present lags from -70 days to +70 days.

To check if the dynamics observed were transient, the data was split in two halves. The MODWT transform was then applied for both halves and similar cross correlation functions were calculated. These functions were then compared to the original cross-correlation function for the whole series. Almost every previously mentioned aspect was persistent. Especially the leading roles of the euro and Swiss franc were evident. However, the negative correlation on scale six for the euro-pound and franc-pound cases disappeared in the second half and was present only during the first half. Figure 18 presents an example of cross-correlation functions for both halves of the Euro-Pound case and on scales 6 and 7. When compared to the cross-correlation functions of the whole series in Figure 15, similarities are evident.

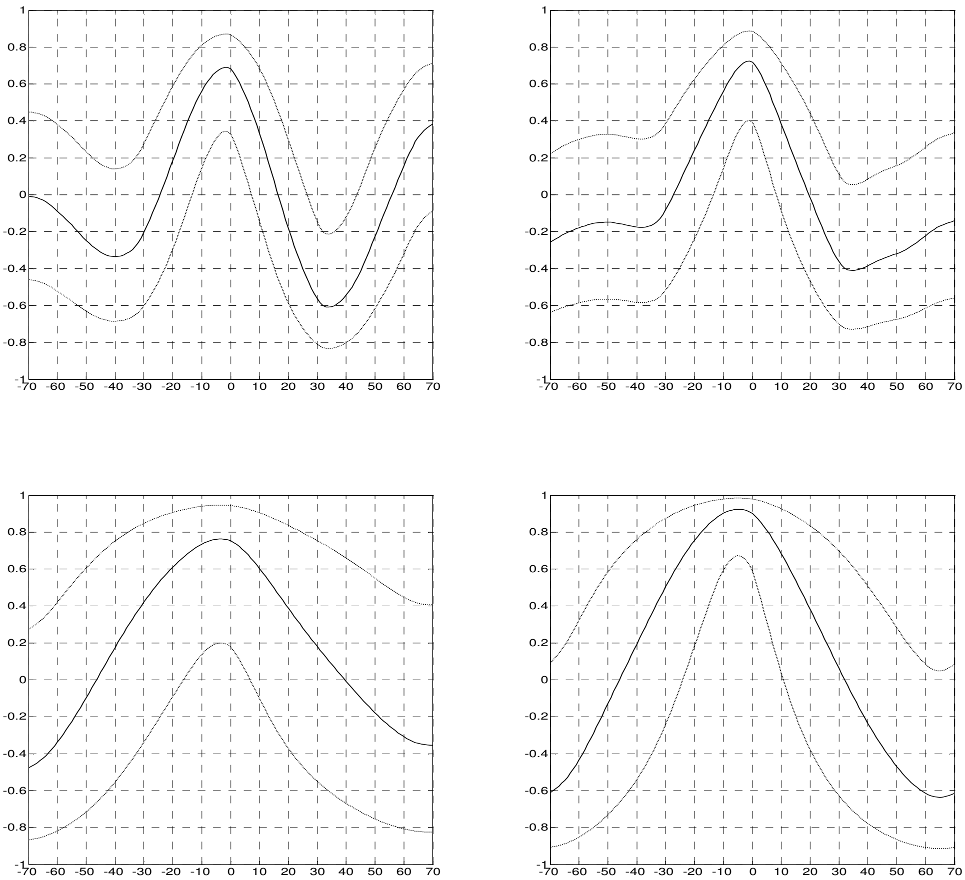

Figure 18. Cross-correlation functions between the wavelet coefficients of levels 6 and 7 for the euro and the British pound. The pictures at the top contain level 6; the lower pictures contain level 7. The first half is on the left, the second half is on the right. Diagrams present lags from -70 days to +70 days.

4.4 Conclusions

This chapter examines the lead-lag relations of the three major European currencies using the wavelet cross-correlation methods. The interrelations of European exchange rates have been studied quite extensively. The novelty of this study is the use of wavelets, which make it possible to investigate the scale dimension of the linkages of exchange rates. The maximal overlap discrete wavelet transform was used to decompose original series into different scale wavelet coefficient series and cross-correlation functions were then calculated between the coefficient series to analyze the dynamics of cross-dependence of the exchange rates on different time scales.

The euro and Swiss franc had very symmetric cross-correlation functions on all scales with a very strong positive contemporaneous correlation. This suggests that the euro and the Swiss franc move closely together without any significant lead-lag dynamics. This result is similar to the conclusions of Nikkinen et al. (2006). Krylova et al. (2009) find an evidence of nonlinear relationship between the Swiss Franc and the euro. There are some very weak findings that support this observation as the lead/lag -relations between the euro and the franc change direction when we move from shorter time scales to longer time scales.

Between the euro and the pound there appears to be much more variation between scales. The contemporaneous correlation is much weaker than in the euro-franc case. There are few other significant correlations on the first two scales. Notable was also the significant negative correlation around 30 days on the scale six. This correlation was probably caused by some extraordinary phenomenon, because it was not present in the second half of the series. These results support the findings of Matsushita et al. (2007) who argue that the pound and euro behave differently and should not be considered as the same currency. The asymmetry towards the euro on larger scales suggests the leading role of the euro against the pound which is similar to the results of Krylova et al. (2009) and Nikkinen et al. (2006).

Cross-correlation functions for the franc-pound case are quite similar to that of the euro-pound case. This follows on from the fact that the franc is closely connected to the euro with a strong contemporaneous correlation. The features found in the euro-pound case, like the asymmetry, are slightly even stronger.

The only other study that considers the interrelations of exchange rates on different time scales is Wu (2007) and this study only examines the USD/DEM and USD/JPY exchange rates. However there is one clear difference between the results of Wu and the results found using the wavelet cross-correlation methods. Wu argues that the correlations between exchange rates are stronger on a daily time scale than on longer time scales. However the wavelet cross-correlation diagrams suggest just the opposite and almost without exceptions the correlations become stronger when the time scale increases.

Overall the maximal overlap discrete wavelet transform based estimator of cross-correlation gives good insight to the time scale dependant dynamics of exchange rates. Including time-scales a more complete picture of the interrelations can be drawn. Participants in the markets naturally have different time horizons in their investment plans. Therefore, the wavelet methods are just the right tool for this purpose because this way they can extract from the wavelet analysis the time-scale that interests them most and make decisions according to this time-scale.

5 CROSS DYNAMICS OF EXCHANGE RATE EXPECTATIONS*

This chapter provides a novel wavelet analysis on the cross-dynamics of exchange rate expectations. Over-the-counter currency options on the euro, the Japanese yen, and the British pound vis-à-vis the U.S. dollar are used to extract expected probability density functions of future exchange rates and apply recent wavelet cross-correlation techniques are applied to analyze linkages in market expectations. Significant lead-lag relationships between the expected probability densities of major exchange rates are found regardless of time scales. At higher frequencies, the USD/JPY exchange rate is found to affect the expected distributions of the EUR/USD and GBP/USD exchange rates. However, at lower frequencies, there are also significant feedback effects from the EUR/USD density functions to the USD/JPY densities. These findings suggest that the dynamic structure of the relations between exchange rate expectations varies over different time scales.

5.1 Introduction

The crisis of the European exchange rate mechanism in 1992 and the Asian currency crisis in the autumn of 1997 demonstrate that uncertainty in one exchange rate may spread to other exchange rates and cause a chain reaction of contagion throughout the foreign exchange markets (see e.g., Baur, 2003; Kallberg et al., 2005). These events, together with recent empirical evidence (see e.g., Kearney and Patton, 2000; Krylova et al., 2005; Nikkinen et al., 2006; Pérez-Rodríguez, 2006, Inagaki, 2007), suggest that market expectations and uncertainty about exchange rate movements are affected not only by country specific economic fundamentals and monetary policy but also by common uncertainty factors.

In this chapter, focus is on the linkages in exchange rate expectations. Although it is not directly observable, market participants' exchange rate expectations may be inferred from the prices of currency options. Provided that market participants are rational, market prices of currency options should incorporate all the available

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* An article based on this chapter is co-authored with Jussi Nikkinen, Seppo Pynnönen and Sami Vähämaa. The article was presented at the 2009 Southern Finance Association meeting, the 2008 Eastern Finance Association meeting, the 2007 Midwest Finance Association and the 2007 Southwestern Finance Association meeting.

information about market's assessment of future exchange rate developments. Data on over-the-counter currency option on the euro, the British pound, and the Japanese yen vis-à-vis the U.S. dollar are used to extract expected probability density functions of future exchange rates. Furthermore, wavelet techniques are utilized again to examine the cross-correlation structures of the option-implied probability densities among the major exchange rates over different time scales. By focusing on linkages in expected exchange rate distributions over different time scales, this chapter provides novel insights into the dynamics of foreign exchange markets.

Over the past few years, several studies have applied wavelet techniques to analyze financial time-series. Gencay et al. (2001) and Nekhili et al. (2002) use wavelets to examine the scaling properties of exchange rate returns and volatility, while Karuppiah and Los (2005) analyze the dynamic structure of Asian spot exchange rates over the Asian currency crisis in 1997. Fernandez (2006, 2008) adopts wavelet-based variances to analyze the effects of the Asian currency crisis, the September 11 terrorist attacks, and the second Gulf war on stock market volatility. Kim and In (2005) use wavelet correlations to examine the relationships between stock returns and inflation, and Kim and In (2007) between stock prices and bond yields. In and Kim (2006) focus on the lead-lag relationships between stock and futures markets, while Elder and Serletis (2008) apply wavelets to analyze the dynamics of energy futures prices.

This chapter extends the existing literature by providing a wavelet cross-correlation analysis of the behavior of exchange rate expectations. This is the first attempt to directly address the cross-dynamics of exchange rate expectations, as measured by option-implied probability density functions of future exchange rates. Given that the foreign exchange market is by far the largest financial market in the world, understanding the dynamic behavior of market participants' exchange rate expectations may be considered a high priority task. The analysis presented in this chapter may also have important practical implications, as linkages in expectations across exchange rates have a direct impact on the formulation and implementation of investment and risk management strategies. Moreover, from the viewpoint of monetary policy authorities, it is important to consider to what extent the expectations of future exchange rates are affected by common uncertainty factors and spillover effects which are beyond the control of local monetary policy. Finally, by focusing on the linkages in option-implied probability distributions of future exchange rates, this chapter may also offer useful insights into the behavior of option markets.

Empirical findings demonstrate that market expectations of the three major exchange rates are closely linked. Regardless of time-scales, significant lead-lag relationships between the expected probability densities of exchange rates are found. However, findings also suggest that the dynamic structure of exchange rate expectations may vary considerably over different time-scales. In terms of short-run cross-dynamics of volatility expectations, the Japanese yen seems to have a leading role among the exchange rate triplet. On a longer scale, however, significant feedback effects from the GBP/USD volatility expectations to the JPY/USD volatility are found. The wavelet cross-correlations of the higher-order moments of option-implied exchange rate distributions indicate that the expectations of the JPY/USD exchange rate are virtually unrelated to the developments of the European currencies, while the higher-order moments of the EUR/USD and GBP/USD densities appear strongly linked with each other.

The remainder of this chapter is organized as follows. Section 2 describes the data on OTC currency options. Section 3 presents the methodology used to extract implied probability density functions from option prices. Section 4 discusses the wavelet cross-correlation technique applied in this paper. Section 5 reports the empirical findings on the cross-dynamics of exchange rate expectations. Finally, Section 6 provides concluding remarks.

5.2 OTC currency option data

Over-the-counter (OTC) currency options are used to extract the expected probability density functions of future exchange rates. The data consist of daily one-month implied volatility quotes for at-the-money forward options, 25-delta strangles, and 25-delta risk reversals on the EUR/USD, GBP/USD, and JPY/USD exchange rates. According to the Bank for International Settlements (2007), these three exchange rates together account for about 52 % of trading in the foreign exchange markets with a combined average daily turnover of about 1.6 trillion U.S. dollars, and are thereby decidedly the three most actively traded exchange rates. The currency options data set used in the analysis extends from October 1, 2001 through December 31, 2007, for a total of 1610 trading days.

At-the-money forward options are the most actively traded instruments in the OTC currency options markets. They are European-style currency options for which the strike price equals, or is very close to, the forward exchange rate with the same maturity as the option. OTC currency options are typically quoted in implied volatilities with respect to deltas rather than strike prices. For the at-the-money forward options, the delta is, by definition, equal to 0.5, and hence these instruments are also commonly referred to as 50-delta options.

Strangles and risk reversals are standardized OTC contracts, which are both combinations of out-of-the-money call and put options. A 25-delta strangle consists of a simultaneous purchase of a 25-delta call option and a 25-delta put option. The implied volatility quote for a strangle is the spread of the implied volatility of the 25-delta call and put options, or the strangle volatility, over the implied volatility of an at-the-money forward option. Non-zero volatility quote for a strangle reflects market participants' expectations about the likelihood of large future exchange rate movements. A 25-delta risk reversal combines a long position in a 25-delta call option with a short position in a 25-delta put option. The volatility quote for the risk reversal is the implied volatility differential between the 25-delta call and put options. Risk reversal quotes are nonzero if the market expectations about future exchange rate movements are asymmetrically distributed. Hence, the volatility quotations for strangles and risk reversals provide information regarding the distributional shape of exchange rate expectations.

There are several advantages in using OTC currency option data, rather than data on exchange-traded options, to estimate implied probability densities of future exchange rates. First, OTC currency options have superior liquidity in comparison to exchange-traded options. A recent survey by the Bank for International Settlements (2007) shows that the notional amount of outstanding exchange-traded currency options is less than 1 % of the amount of OTC options. Moreover, the OTC currency options market has been growing considerably over recent years, with about 95 % increase in the average daily turnover during the sample period. Second, OTC options have a constant time to maturity, whereas the maturity of exchange-traded options varies from day to day. As a consequence, estimation problems caused by the time-to-maturity effects of option prices may be avoided by using OTC data. Third, as OTC options are quoted in terms of deltas, they have a fixed distance between the strike price of the option and the current forward rate. Exchange-traded options, in contrast, have fixed strike prices, and thus the exact moneyness of these contracts varies from day to day. Finally, as documented by Christoffersen and Mazzotta (2005), data on OTC currency options is of superior quality for volatility forecasting purposes.

5.3 Probability density functions implied by option prices

Let c_t denote the time t value of a call option written on exchange rate S_t, with a single expiration date T, and a contractual terminal payoff function $\max\left[S_T - K, 0\right]$, where K is the strike price of the option. The theoretical value of the call option at time t is equal to the discounted expected value of the terminal payoff function:

$$c_t = e^{-r(T-t)} E_t^{\tilde{P}} \left[\max\left(S_T - K, 0\right) \right], \tag{27}$$

where r is the risk-free interest rate and $E_t^{\tilde{P}}$ denotes the conditional expectations operator under the risk-neutral probability measure $\tilde{P}$. Since the expected rate of return for all assets is, by definition, equal to the risk-free interest rate under $\tilde{P}$, the expectation of the option's terminal payoff can be discounted at the risk-free rate. Given the risk-neutral probability density function of the underlying exchange rate price at the maturity of the option, $f\left(S_T\right)$, the time t value of the call option can be equivalently expressed as:

$$c_t = e^{-r(T-t)} \int_{-\infty}^{\infty} \max\left(S_T - K, 0\right) f\left(S_T\right) dS_T. \tag{28}$$

Because the price of the option is a function of the risk-neutral probability density of the underlying exchange rate price at the maturity of the option, a set of observed option prices with the same maturity but with different strike prices implicitly contain information about market participants' expectations regarding the distribution of the underlying exchange rate at the maturity of the option.

Several methods for extracting the expected probability density function from option prices have been proposed in literature. Extensive reviews of these alternative methods are provided e.g. in Bahra (1997), Jackwerth (1999), and Bliss and Panigirtzoglou (2002). In general, the techniques for estimating implied density functions may be broadly classified to parametric and nonparametric methods. Whereas the parametric methods postulate a certain parametric form for the terminal underlying asset price distribution, the nonparametric methods utilize some flexible functions to fit the observed option prices as well as possible, and then apply the results derived by Breeden and Litzenberger (1978) to extract the implied probability density.

Empirical comparisons of alternative methods for estimating implied probability density functions are provided in Campa, Chang and Reider (1998), Bliss and Panigirtzoglou (2002), and Andersson and Lomakka (2005). Although Campa et al. (1998) show that different methodological approaches lead to virtually similar implied distributions, the findings reported in Bliss and Panigirtzoglou (2002) and Andersson and Lomakka (2005) indicate that the nonparametric volatility smoothing methods initially suggested by Shimko (1993) produce more accurate estimates of implied probability density functions.

The implied probability densities from the OTC currency option data are estimated with the nonparametric volatility-smoothing method proposed by Malz (1997). If the option pricing function can be expressed as a continuous function of the strike price, the Breeden-Litzenberger (1978) result can be utilized to extract the implied probability density. As shown by Breeden and Litzenberger (1978), the discounted risk-neutral probability density function of the underlying asset price is given by the second partial derivative of Equation (10) with respect to the strike price of the option:

$$f(S_T) = e^{-r\tau} \frac{\partial^2 c(K,T,t)}{\partial K^2}. \tag{29}$$

Unfortunately, only a discrete set of option prices can be observed in the market, and thus Equation (3), per se, is only of limited use. The apparent solution is to approximate $c(K,T,t)$ by interpolating a smooth function through the discrete set of observable prices. As discussed above, OTC option data used contains implied volatility quotations for at-the-money (50-delta) options and two option combinations consisting of out-of-the-money (25-delta) call and put options. Given the three quotations, we can infer the implied volatilities for 25-delta, 50-delta, and 75-delta options, which then in turn can be used to interpolate implied volatilities as a function of option deltas. Malz (1997) shows that the implied volatility/delta space can be approximated by fitting a spline function with parabolic endpoints to the three data points:

$$\sigma_\delta = \sigma_{0.50} - (\delta - 0.5)(2\sigma_{0.25} - 2\sigma_{0.75}) + (\delta - 0.5)^2 (8\sigma_{0.75} + 8\sigma_{0.25} - 16\sigma_{0.50}), \tag{30}$$

where σ_δ denotes the implied volatility for an option with delta equal to δ.

Equation (4) provides a continuous function of implied volatilities in terms of option deltas. By utilizing the Garman-Kohlhagen (1983) version of the Black-Scholes (1973) option pricing model, the continuous implied volatility function is converted numerically from the implied volatility/delta space into the option price/strike price space to obtain a continuous option pricing function. Then finally, the Breeden-Litzenberger result given by Equation (3) is applied to calculate the implied probability density function of the underlying exchange rate.

Table 7 reports descriptive statistics for the moments of the estimated option-implied probability density functions of the EUR/USD, GBP/USD, and JPY/USD exchange rates over the period of October 1, 2001 through to December 31, 2007.

Table 7. The table reports descriptive statistics for the moments of the estimated option-implied probability density functions of the EUR/USD, GBP/USD, and JPY/USD exchange rates over the period of October 1, 2001 through to December 31, 2007.

	Mean	Median	St.Dev.	Min	Max
Implied volatility:					
EUR/USD	0.094	0.095	0.018	0.050	0.136
GBP/USD	0.084	0.084	0.013	0.050	0.121
JPY/USD	0.099	0.097	0.016	0.064	0.179
Implied skewness:					
EUR/USD	0.052	0.050	0.083	-0.175	0.282
GBP/USD	0.011	0.014	0.086	-0.300	0.228
JPY/USD	0.191	0.172	0.145	-0.243	0.656
Implied kurtosis:					
EUR/USD	3.106	3.100	0.032	3.048	3.206
GBP/USD	3.109	3.102	0.032	3.057	3.230
JPY/USD	3.164	3.166	0.146	2.499	3.905

The table shows that the implied volatility for the JPY/USD exchange rate is, on average, around 10 %, while the EUR/USD and GBP/USD rates exhibit somewhat lower volatility with mean estimates of 9.4 % and 8.4 %, respectively. Implied volatility estimates, however, have varied considerably over the sample period, ranging from 5.0 % for the EUR/USD and GBP/USD exchange rates to 17.9 % for the USD/JPY rate. Furthermore, the table demonstrates that the implied probability densities for the EUR/USD, GBP/USD, and JPY/USD rates tend to be positively skewed. This positive skewness indicates that, during the sample period, market participants have on average attached higher probabilities for sharp U.S. dollar depreciations against the euro, the British pound and the Japanese yen than for dollar appreciations. However, the range of implied skewness estimates is relatively large, and thereby suggests that asymmetries in exchange rate expectations may vary considerably over time. Finally, the kurtosis estimates show that the option-implied probability distributions are slightly fat-tailed for all three exchange rates.

5.5 Wavelet cross-correlations between option-implied probability densities

The wavelet cross-correlations between the moments of the option-implied probability densities of the EUR/USD, GBP/USD and JPY/USD exchange rates are presented in Tables 8-10. The cross-correlations for the time-scales of 4–8 and 64–128 trading days are reported. The shorter time-scale (i.e., higher frequency) may be interpreted to reflect changes in the short-run market expectations, while the longer time-scale (lower frequency) should reflect the expectations related to general trends.

Table 8 reports the wavelet cross-correlations of option-implied exchange rate volatilities. As can be noted from the table, the cross-correlations are positive and statistically highly significant at lag zero, except between the EUR/USD and JPY/USD volatilities on a longer scale. These significant cross-correlations at lag zero suggest that the market expectations about future volatilities are contemporaneously closely linked among the three major exchange rates. The strongest contemporaneous relationship is observed between the EUR/USD and GBP/USD volatilities on a longer time-scale, which indicates a particularly close linkage of general trends in the volatility expectations of the European currencies.

The statistically significant cross-correlations between the EUR/USD and GBP/USD implied volatilities stretch from lag –3 to lag +3 on a short scale, and from lag –30 to lag +20 on a long scale. The longer scale cross-correlation function of the EUR/USD-GBP/USD volatilities is distinctly asymmetric, with the largest correlations occurring at negative lags (–4 to –2). Moreover, also on a shorter scale, the correlation coefficients are also slightly higher for negative than for positive lags. These asymmetries in the cross-correlation functions indicate that movements in the EUR/USD volatility expectations are leading movements in the GBP/USD expectations. The longer scale estimates reported in Table 8 suggest that changes in the EUR/USD implied volatility are followed by similar changes in the GBP/USD volatility with a lag of approximately 2–4 trading days.

Table 8. The table reports wavelet cross-correlations between option-implied volatilities of the EUR/USD, GBP/USD, and JPY/USD exchange rates over short (4-8 days) and long (64-128 days) time-scales, respectively. ***, **, and * denote significance at the 0.01, 0.05, and 0.10 levels, respectively.

	EUR/USD↔GBP/USD		EUR/USD↔JPY/USD		GBP/USD↔JPY/USD	
Lag	Short scale	Long scale	Short scale	Long scale	Short scale	Long scale
-50	-0.003	0.196	0.031	0.065	0.017	0.110
-45	-0.025	0.299	0.074	0.128	0.028	0.194
-40	-0.055	0.399	-0.038	0.186	-0.030	0.276
-35	0.080	0.494	0.069	0.241	0.058	0.354
-30	0.000	0.582 **	-0.070	0.291	-0.084	0.424
-25	-0.014	0.661 **	-0.073	0.335	-0.053	0.486
-20	0.067	0.728 ***	-0.030	0.373	-0.040	0.537 *
-18	0.061	0.751 ***	-0.012	0.386	-0.045	0.554 *
-16	0.033	0.771 ***	0.003	0.398	-0.023	0.569 *
-14	0.013	0.789 ***	0.023	0.408	0.025	0.582 **
-12	0.013	0.804 ***	0.037	0.417	0.064	0.592 **
-10	0.006	0.816 ***	0.028	0.424	0.067	0.600 **
-8	-0.039	0.825 ***	-0.033	0.429	0.017	0.606 **
-6	-0.049	0.831 ***	-0.103	0.432	-0.044	0.609 **
-5	0.009	0.833 ***	-0.091	0.433	-0.035	0.609 **
-4	0.121	0.834 ***	-0.028	0.434	0.017	0.609 **
-3	0.305 ***	0.834 ***	0.104	0.434	0.130 *	0.608 **
-2	0.498 ***	0.834 ***	0.258 ***	0.433	0.262 ***	0.606 **
-1	0.645 ***	0.832 ***	0.394 ***	0.432	0.379 ***	0.604 **
0	0.698 ***	0.829 ***	0.470 ***	0.430	0.448 ***	0.601 **
1	0.603 ***	0.824 ***	0.435 ***	0.427	0.419 ***	0.596 **
2	0.427 ***	0.818 ***	0.335 ***	0.424	0.334 ***	0.592 **
3	0.230 ***	0.812 ***	0.207 ***	0.420	0.222 ***	0.586 **
4	0.064	0.804 ***	0.093	0.415	0.115	0.580 **
5	-0.016	0.796 ***	0.037	0.410	0.056	0.573 *
6	-0.039	0.787 ***	0.018	0.405	0.029	0.565 *
8	0.002	0.766 ***	0.033	0.392	0.027	0.548 *
10	0.006	0.743 ***	0.020	0.378	0.022	0.528 *
12	-0.045	0.716 ***	-0.009	0.361	0.007	0.506 *
14	-0.072	0.686 **	-0.002	0.343	0.006	0.481
16	-0.057	0.655 **	0.013	0.324	0.025	0.455
18	-0.039	0.620 **	-0.024	0.303	0.044	0.427
20	-0.036	0.584 **	-0.091	0.281	0.029	0.397
25	0.014	0.485	0.007	0.223	-0.035	0.316
30	-0.031	0.377	-0.039	0.163	-0.055	0.230
35	-0.003	0.263	-0.030	0.099	0.023	0.142
40	-0.041	0.146	-0.067	0.036	-0.150 **	0.053
45	0.024	0.030	-0.030	-0.026	-0.041	-0.032
50	0.005	-0.083	-0.001	-0.084	-0.002	-0.111

As can be seen from Table 8, the wavelet cross-correlations between the implied volatilities of the EUR/USD and JPY/USD exchange rates are significantly positive on a short time-scale from lag –2 to lag +3. Again, the cross-correlation function is slightly asymmetric, now towards positive lags. This asymmetry towards positive lags indicates that movements in market expectations about the future JPY/USD volatility are leading the volatility expectations of the EUR/USD rate. On a longer scale, however, all the cross-correlations between the EUR/USD and JPY/USD volatilities appear statistically insignificant.

Interestingly, the cross-correlations between the GBP/USD and JPY/USD implied volatilities seem to behave somewhat differently on two time-scales. On a shorter scale, there is a slight asymmetry in the cross-correlation function towards positive lags, suggesting that changes in the expected JPY/USD volatilities are leading the GBP/USD volatility expectations. In contrast, on a longer scale of 64–128 days, the asymmetry in the cross-correlation function is much stronger and now towards negative lags. This correlation structure shows that on a longer time-scale changes in market expectations of the GBP/USD volatility are followed by changes in the JPY/USD volatility with an approximate lag of 2–8 trading days. Thus, Table 8 provides slight evidence for the lead of the yen in terms of short-run market expectations, and somewhat stronger evidence for the lead of the pound in terms of expectations related to general trends.

Another interesting feature in Table 8 is that the higher frequency cross-correlation between the GBP/USD and JPY/USD volatilities appears negative and statistically significant at lag +40. This negative coefficient would suggest that movements in the JPY/USD volatility expectations lead to reversed movements in the GBP/USD expectations with a lag of 40 trading days.

The wavelet-based lead-lag relations in volatility expectations are summarized in Figure 19. The figure shows that the EUR/USD volatility expectations affect the expectations about the future GBP/USD volatility both on short and long time-scales. Moreover, the Japanese yen seems to have a leading role in terms of short-run market expectations, as the implied volatilities of the EUR/USD and GBP/USD exchange rates are strongly affected by the expected JPY/USD volatility on a short time-scale. On a longer scale, however, we find significant feedback effects from the GBP/USD volatility expectations to the JPY/USD volatility.

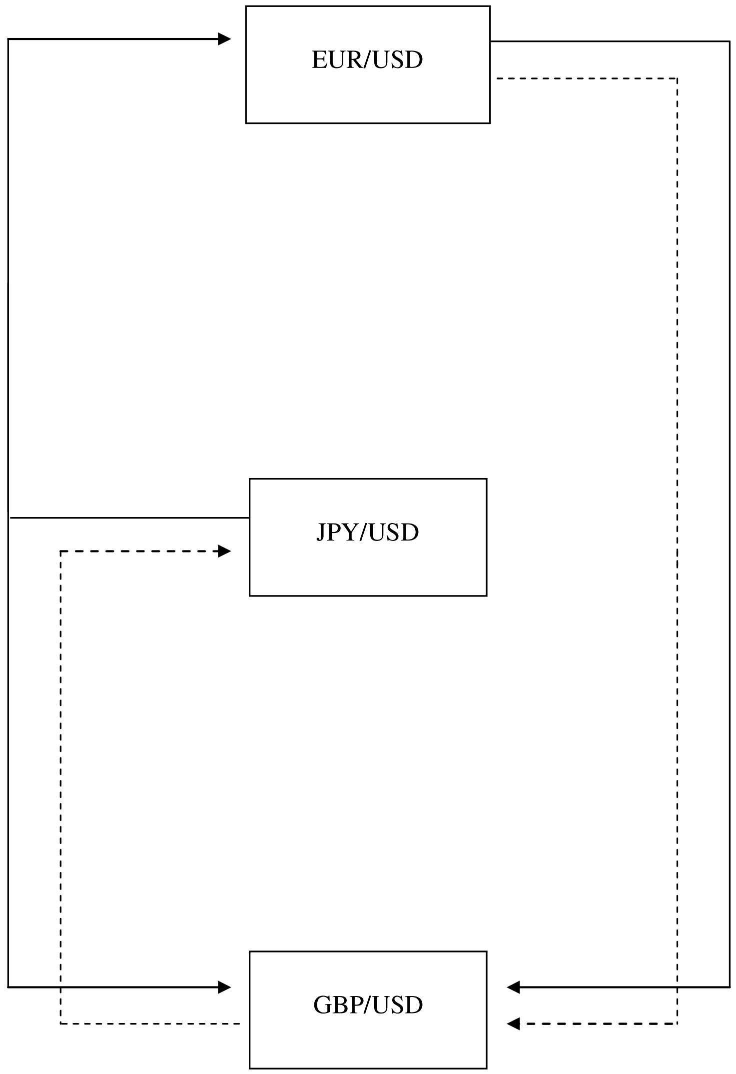

Figure 19. Wavelet cross-correlations of implied volatility coefficients. The solid and dashed lines represent statistically significant causality from exchange rate i to exchange rate j over short (4-8 days) and long (64-128) time-scales, respectively.

Table 9. The table reports wavelet cross-correlations between option-implied skewness coefficients of the EUR/USD, GBP/USD, and JPY/USD probability densities over short (4-8 days) and long (64-128 days) time-scales, respectively. ***, **, and * denote significance at the 0.01, 0.05, and 0.10 levels, respectively.

	EUR/USD↔GBP/USD		EUR/USD↔JPY/USD		GBP/USD↔JPY/USD	
Lag	Short scale	Long scale	Short scale	Long scale	Short scale	Long scale
-50	0.032	0.141	0.023	0.033	0.028	-0.094
-45	-0.065	0.231	-0.068	0.005	-0.051	-0.138
-40	-0.049	0.326	0.022	-0.022	-0.010	-0.175
-35	0.025	0.422	-0.086	-0.045	-0.089	-0.205
-30	-0.047	0.515 *	-0.036	-0.065	-0.039	-0.227
-25	0.006	0.601 **	0.028	-0.080	0.028	-0.240
-20	-0.080	0.679 **	0.059	-0.093	0.002	-0.245
-18	-0.067	0.707 ***	0.082	-0.097	0.006	-0.244
-16	-0.039	0.733 ***	0.015	-0.101	-0.011	-0.242
-14	-0.028	0.756 ***	-0.070	-0.104	-0.035	-0.239
-12	-0.022	0.776 ***	-0.101	-0.107	-0.045	-0.234
-10	-0.014	0.792 ***	-0.093	-0.109	-0.053	-0.228
-8	-0.006	0.805 ***	-0.062	-0.111	-0.065	-0.221
-6	0.052	0.813 ***	0.002	-0.113	-0.047	-0.213
-5	0.137 *	0.815 ***	0.045	-0.115	-0.022	-0.209
-4	0.270 ***	0.816 ***	0.086	-0.116	0.006	-0.204
-3	0.449 ***	0.816 ***	0.125	-0.117	0.038	-0.200
-2	0.614 ***	0.814 ***	0.153 *	-0.119	0.069	-0.195
-1	0.722 ***	0.811 ***	0.162 **	-0.120	0.094	-0.190
0	0.728 ***	0.805 ***	0.152 **	-0.122	0.113	-0.185
1	0.609 ***	0.798 ***	0.120 *	-0.124	0.115	-0.180
2	0.425 ***	0.790 ***	0.083	-0.126	0.104	-0.175
3	0.221 ***	0.780 ***	0.055	-0.128	0.086	-0.170
4	0.047	0.769 ***	0.041	-0.130	0.067	-0.165
5	-0.064	0.757 ***	0.042	-0.132	0.055	-0.160
6	-0.130 *	0.744 ***	0.042	-0.134	0.050	-0.155
8	-0.159 **	0.715 ***	0.013	-0.139	0.046	-0.145
10	-0.116 *	0.682 **	-0.033	-0.144	0.039	-0.135
12	-0.071	0.648 **	-0.028	-0.149	0.044	-0.125
14	-0.079	0.611 **	0.013	-0.154	0.050	-0.116
16	-0.098	0.573 *	0.004	-0.159	0.018	-0.106
18	-0.070	0.534 *	-0.057	-0.163	-0.043	-0.097
20	-0.024	0.494	-0.086	-0.168	-0.077	-0.087
25	-0.022	0.391	0.026	-0.178	-0.025	-0.064
30	-0.088	0.290	0.003	-0.186	0.011	-0.042
35	-0.086	0.192	-0.103	-0.194	-0.083	-0.023
40	0.033	0.100	0.062	-0.200	0.067	-0.007
45	-0.013	0.016	-0.009	-0.205	-0.031	0.006
50	0.051	-0.054	0.002	-0.210	0.019	0.013

Table 9 presents the wavelet cross-correlations of option-implied asymmetries in exchange rate expectations. The observed lead-lag relationships in asymmetries of expectations are summarized in Figure 20. The higher frequency cross-correlations between the EUR/USD and GBP/USD expectations display an interesting pattern. The correlations are significantly positive from lag –5 to lag +3. This asymmetry in the cross-correlation function would indicate that the asymmetries in the expected EUR/USD distributions lead to asymmetries in the GBP/USD distributions with a short lag. However, as can be noted from Table 9, the cross-correlations on the other hand are significantly negative from lag +6 to lag +10, thereby suggesting that asymmetries in market expectations may move into opposite directions. These negative cross-correlations at positive lags imply that increasing asymmetries in the GBP/USD expectations lead to decreasing asymmetries in the EUR/USD expectations with a lag of approximately 6–10 trading days.

Skewness expectations on longer scale cross-correlations between the EUR/USD and GBP/USD are positive and statistically significant from lag –30 to lag +18. The cross-correlation function is distinctly asymmetric, with the largest correlations occurring at lags from –6 to –2. Therefore, in terms of general trends, estimates suggest that increasing asymmetries in the expected EUR/USD distributions lead to increasing asymmetries in the GBP/USD distributions with an approximate lag of about one week.

Asymmetries in market expectations about the JPY/USD exchange rate seem to be almost unrelated to the asymmetries of the European currencies. On a shorter time-scale, the cross-correlations between the EUR/USD and JPY/USD expectations are statistically significant between lags –2 and +1. Again, the cross-correlation function is asymmetric towards negative lags, and thereby suggests that asymmetries in the expected EUR/USD distributions affect the asymmetries in the JPY/USD distributions with a short lag. As can be noted from Table 9, the longer scale cross-correlations between the EUR/USD and JPY/USD expectations and all the cross-correlations between the GBP/USD and JPY/USD expectations appear statistically insignificant

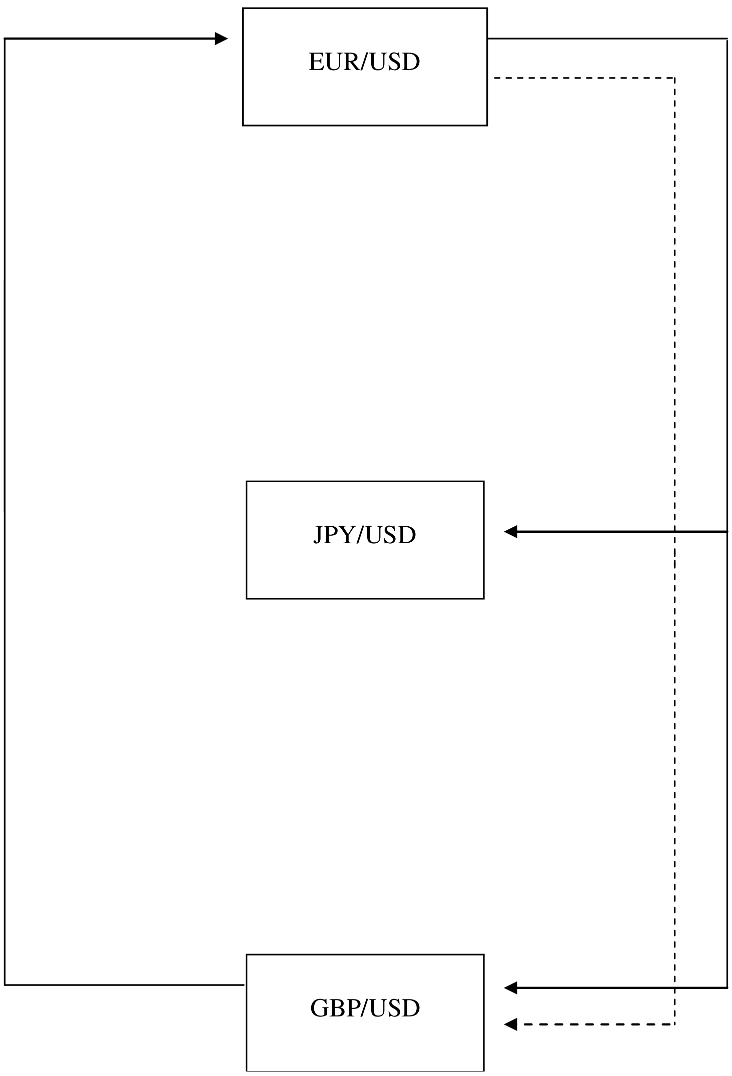

Figure 20. Wavelet cross-correlations of implied skewness coefficients. The solid and dashed lines represent statistically significant causality from exchange rate i to exchange rate j over short (4-8 days) and long (64-128) time-scales, respectively.

Table 10. The table reports wavelet cross-correlations between option-implied kurtosis coefficients of the EUR/USD, GBP/USD, and JPY/USD probability densities over short (4-8 days) and long (64-128 days) time-scales, respectively. ***, **, and * denote significance at the 0.01, 0.05, and 0.10 levels, respectively.

	EUR/USD↔GBP/USD		EUR/USD↔JPY/USD		GBP/USD↔JPY/USD	
Lag	Short scale	Long scale	Short scale	Long scale	Short scale	Long scale
-50	-0.069	0.343	0.012	-0.035	0.002	0.275
-45	-0.057	0.439	0.000	-0.021	-0.032	0.294
-40	0.004	0.529 *	-0.010	0.009	0.069	0.309
-35	0.108	0.611 **	0.046	0.049	-0.044	0.319
-30	0.077	0.680 **	-0.017	0.090	0.034	0.325
-25	0.058	0.733 ***	0.001	0.130	0.011	0.327
-20	-0.040	0.769 ***	-0.029	0.159	-0.033	0.326
-18	-0.080	0.778 ***	0.012	0.167	-0.011	0.325
-16	-0.086	0.783 ***	0.059	0.173	0.027	0.323
-14	-0.039	0.786 ***	0.059	0.176	0.054	0.320
-12	0.038	0.785 ***	-0.006	0.176	0.027	0.317
-10	0.070	0.781 ***	-0.044	0.172	-0.034	0.313
-8	0.040	0.773 ***	-0.019	0.166	-0.044	0.308
-6	0.031	0.762 ***	-0.008	0.159	0.009	0.302
-5	0.067	0.755 ***	-0.009	0.155	0.037	0.299
-4	0.135 *	0.747 ***	-0.008	0.150	0.052	0.296
-3	0.260 ***	0.738 ***	0.007	0.144	0.061	0.292
-2	0.387 ***	0.729 ***	0.038	0.138	0.056	0.288
-1	0.497 ***	0.718 ***	0.063	0.132	0.052	0.284
0	0.551 ***	0.706 ***	0.077	0.126	0.063	0.279
1	0.493 ***	0.694 **	0.065	0.119	0.059	0.274
2	0.386 ***	0.681 **	0.040	0.112	0.061	0.268
3	0.241 ***	0.666 **	0.015	0.105	0.061	0.262
4	0.102	0.651 **	-0.002	0.098	0.041	0.255
5	0.018	0.636 **	-0.002	0.090	0.041	0.249
6	-0.043	0.619 **	-0.006	0.082	0.030	0.242
8	-0.053	0.584 **	-0.039	0.065	-0.020	0.227
10	-0.020	0.547 *	-0.067	0.047	-0.106	0.213
12	0.001	0.507 *	-0.014	0.030	-0.108	0.198
14	-0.006	0.466	0.051	0.013	-0.009	0.182
16	-0.038	0.423	0.041	-0.005	0.046	0.167
18	-0.068	0.379	-0.011	-0.024	0.004	0.152
20	-0.089	0.333	-0.025	-0.041	-0.015	0.137
25	-0.046	0.216	0.027	-0.083	-0.029	0.102
30	-0.007	0.097	-0.066	-0.123	-0.045	0.070
35	-0.032	-0.021	0.023	-0.148	0.047	0.042
40	0.061	-0.133	0.025	-0.156	-0.048	0.012
45	-0.141	-0.235	-0.040	-0.147	0.023	-0.020
50	-0.054	-0.323	0.020	-0.118	0.057	-0.050

The wavelet cross-correlations of option-implied kurtosis estimates are reported in Table 10, and the significant lead-lag relations summarized in Figure 21. Consistent with previous findings on implied volatility and skewness, the cross-correlations of kurtosis coefficients also provide considerable evidence to suggest that the market expectations of the two major European currencies are closely linked to each other. On a shorter time-scale, the statistically significant cross-correlations span from lag –4 to lag +3. This cross-correlation function is slightly asymmetric towards negative lags.

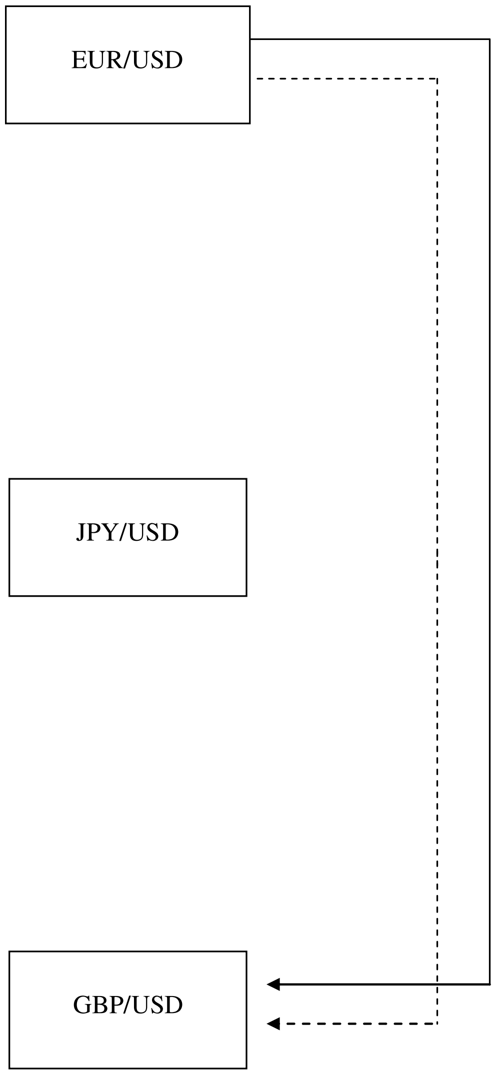

Figure 21. Wavelet cross-correlations of implied kurtosis coefficients. The solid and dashed lines represent statistically significant causality from exchange rate i to exchange rate j over short (4-8 days) and long (64-128) time-scales, respectively.

The longer scale cross-correlations between the kurtosis coefficients of the EUR/USD and GBP/USD densities are positive and significant between lags –40 and +12. The cross-correlation function is strongly asymmetric, with the highest correlations observed at lags –16 to –10. These findings indicate that, in terms of general trends, the expectations of future extreme movements in the EUR/USD exchange rate are leading the expectations about extreme movements in the GBP/USD rate by about two to three weeks. The market expectations regarding the JPY/USD exchange rate appear to be unrelated to the developments of the European currencies, as the cross-correlations between the implied kurtosis estimates are statistically insignificant at all lags on both time-scales.

5.6 Conclusions

This chapter focuses on the cross-dynamics of exchange rate expectations. Over-the-counter currency options on the euro, the Japanese yen, and the British pound vis-à-vis the U.S. dollar are used to extract expected probability density functions of future exchange rates, followed by applying recent wavelet cross-correlation techniques are applied to analyze linkages in these option-implied market expectations over different time-scales. By focusing on the dynamic structure of the relations between expected exchange rate distributions, this paper provides new insights into the dynamics of foreign exchange markets.

Empirical findings demonstrate that market expectations are closely linked among the three major exchange rates. Regardless of time-scales, significant lead-lag relationships between the expected probability densities of exchange rates are found. The linkages in market expectations appear particularly strong between the EUR/USD and GBP/USD exchange rates. On shorter time-scale, the implied volatility of the JPY/USD exchange rate is found to affect the volatilities of the EUR/USD and GBP/USD rates. Thus, the Japanese yen seems to have a leading role among the exchange rate triplet in terms of short-run dynamics of volatility expectations. On a longer scale, however, there are also significant feedback effects from the GBP/USD volatility expectations to the JPY/USD volatility.

The wavelet cross-correlations of the higher-order moments of option-implied exchange rate distributions indicate that the market expectations about the JPY/USD exchange rate are virtually unrelated to the developments of the European currencies. The higher-order moments of the expected EUR/USD and GBP/USD densities are strongly linked to each other, especially on a longer time-scale. The results indicate that movements in the skewness and kurtosis of the expected EUR/USD distributions may lead to movements in the GBP/USD distributions. In general, empirical findings suggest that the dynamic structure of exchange rate expectations may vary considerably over different time-scales.

6 WAVELET NETWORKS IN FINANCIAL FORECASTING

This chapter examines the predictability of the major exchange rates on different time scales. The applied forecasting method is a wavelet network model which is compared to a simple linear forecasting model and a random walk model. It is found that the nonlinear forecast method does not improve forecasting performance. Fit to a training data is always better with the wavelet network, but fit to a testing data is always opposite, the linear model being better. Forecasting with the shorter forecast horizon is better, which is in contrast to the recent results that forecasting performance improves with longer forecasting horizons.

6.1 Introduction

Forecasting a financial time series is a very specific problem in time series forecasting and has very long traditions. Almost a hundred years ago Bachelier (1914) studied the nature of security prices and proposed a random walk as a characteristic for their movement (Lendasse et al. 2000). The consensus thereafter has mainly been that security prices have no memory, i.e. the past cannot be used to predict the future in any meaningful way. A famous paper by Fama (1965) argues that the security prices do follow the random walk model. This result has a close connection to the efficient market hypothesis (EMH), which means in short, that a security price reflects all the information of the market and the security. The seminal work of Meese and Rogoff (1983) provided similar results specifically for exchange rates. They argue that a simple driftless random walk model outperforms models that are based on economic theory.

With the availability of non-linear methods, forecasting has again acquired more interest. In particular, neural networks have gained popularity among researchers. The first results of neural networks in the middle of 20th century were not very encouraging and interest to them decreased significantly. At the end of 1980s, when Hornik et al. (1989, 1990) proved that neural networks are universal approximators, researchers became interested in them again. Thereafter, forecasting economic time series with neural networks has been widely studied (see for example Swanson & White (1997) for further reference). Neural networks particularly in exchange rate forecasting are studied in Majhi et al. (2009); Mitra & Mitra (2006); Shazly & Shazly (1999); Meade (2002); Wong et al. (2003); Yu et al. (2005) and Zhang & Hu (1998). The progress with exchange rates was rather slow. The main conclusion during the nineties was that predictability exists only with very long forecast horizons. Chinn & Meese (1995) compare four different structural exchange rate models. They find that, compared to the random walk model, there is some improvement with these models only on longer horizons.. Faust et al. (2003) question improvements in the exchange rate forecasting. They note that in almost every case, the documented improvements in the forecasting are achieved only with the original data and disappear with a data revision. However recent years have shown some progress in forecasting exchange rates. Carriero et al (2009) use a large Bayesian VAR model to forecast exchange rates. Their model outperforms other forecasting methods systematically in every situation. They also achieve an improved forecasting performance on shorter time horizons. Abutaleb et al. (2003) use a time-varying exchange rate model and also achieve promising results.

Wavelet networks are a special class of neural networks where activation functions are wavelet functions. The same universal approximation result holds also for wavelet networks. It has also been argued in many papers that wavelet networks are usually better in non-linear regression than ordinary feed-forward neural networks (Zhang & Benveniste 1992). The interest among wavelet networks has increased enormously during recent years. The best results among financial time series are achieved by Chauhan et al. (2009). They combine wavelet networks with a differential evolution algorithm (Storn and Price 1997) and achieve very promising results. Their prediction model outperforms previous models in all cases.

The purpose of this chapter is to study the non-linear structure in exchange rates and the performance of wavelet networks in financial forecasting. The comparison was made between a pure linear model, a linear model + wavelet network –model and a random walk model. Including the linear model to the wavelet network, the aim is to let wavelet network focus on the non-linear structure of the data. Results show that the non-linear wavelet network model does not improve forecast and fits only to the noise of the data. In this way the results confirm the original views of Meese and Rogoff (1983) and support the views of Faust et al. (2003). There is also no improvement in predictability as we move from a short forecast horizon to a long forecast horizon. This is in contrast to the recent consensus that forecasting improves at longer time intervals.

6.2 Continuous wavelet transform and wavelet networks

6.2.1 Continuous wavelet transform

Wavelet networks are constructed using radial wavelets, which have a one dimensional dilation parameter regardless of their dimension. Behind the wavelet networks is the theory of continuous wavelet transform Central in the theory of the continuous wavelet transform is the admissibility condition. A pair of radial functions $\varphi, \psi \in L_2\left(\mathbb{R}^d\right)$ is admissible as analysis and synthesis wavelets, if they satisfy the condition

$$\int_0^\infty a^{-1}\hat{\varphi}\left(a\omega\right)\hat{\psi}\left(a\omega\right)da = 1, \quad \forall\omega \in \mathbb{R}^d, \tag{31}$$

where $\hat{\varphi}$ and $\hat{\psi}$ are the Fourier transforms of φ and ψ respectively.

Because the functions φ and ψ are radial, the integral in (31) does not depend on $\omega \neq 0$. Daubechies proves the following theorem (Daubechies 1992).

Let φ and ψ be a pair of radial functions satisfying (31). Then for any function $f \in L_2\left(\mathbb{R}^d\right)$, the following formulae define an isometry between $L_2\left(\mathbb{R}^d\right)$ and $L_2\left(\mathbb{R}_+ \times \mathbb{R}^d\right)$:

$$u\left(a, \mathbf{t}\right) = a^{d-1/2}\int_{\mathbb{R}^d} f\left(\mathbf{x}\right)\varphi\left(a\left(\mathbf{x}-\mathbf{t}\right)\right)d\mathbf{x} \tag{32}$$

$$f\left(\mathbf{x}\right) = \int_{\mathbb{R}_+ \times \mathbb{R}^d} u\left(a, \mathbf{t}\right)\psi\left(a\left(\mathbf{x}-\mathbf{t}\right)\right)a^{d-1/2}dad\mathbf{t}, \tag{33}$$

where $a \in \mathbb{R}_+$ and $\mathbf{t} \in \mathbb{R}^d$ are dilation and translation parameters. Dilation parameter stretches and translation parameter moves a wavelet function along coordinates. Equations (32) and (33) define the continuous wavelet transform of function *f* and its inverse transform.

For this transform to be implementable on digital computers, it has to be discretized. For a discrete version of (33)

$$f\left(\mathbf{x}\right) = \sum_i u_i\psi\left(a_i\mathbf{x} - \mathbf{t}_i\right) \tag{34}$$

to hold, some conditions are required. It can be proven that the family of translated and dilated wavelets

$$\left\{a_i^{d/2}\psi\left(a_i\mathbf{x}-\mathbf{t}_i\right):i\in\mathbb{Z}\right\} \tag{35}$$

can be used to form the discrete reconstruction (34) if the family constitutes a frame (Daubechies 1992).

6.2.2 *Wavelet Network*

The family of wavelets (35) is usually a regular lattice $\left\{\left(a_0^n, mt_0\right):n\in\mathbb{Z}, m\in\mathbb{Z}^d\right\}$. In high dimensional problems, this wavelet basis or frame grows very large. This curse of dimensionality can be dealt in some particular situations, for example when the function *f* is mostly smooth but has localized irregularities. Then we can expect that the wavelet estimator will be more efficient if the wavelet "basis" is constructed according to the training data. This idea of adaptive discretization is behind wavelet networks.

When forming the discrete reconstruction (34), the values of $\left(a_i, \mathbf{t}_i\right)$ can be adaptively determined according to the function *f* or the sampled data. So all the parameters $\left(u_i, a_i, \mathbf{t}_i\right)$ of (34) are adapted and we get something that, in form, closely resembles feed-forward neural networks in form. Therefore this adaptive discrete inverse wavelet transform is called wavelet networks and techniques of neural networks can be applied.

Ordinary feed-forward neural networks used in the context of nonparametric regression are usually first randomly initialized and then trained by a backpropagation procedure. In this respect, wavelet networks have an advantage through their connection to the continuous wavelet transform. For example one could first form the wavelet basis and then use the methods of wavelet shrinkage (Percival & Walden 2000) to reduce the number of wavelets. In this analysis the initialization method proposed by Zhang (1994) is used. This method combines techniques in regression analysis and backpropagation procedures.

6.3 Methodology

The approach in the empirical part follows the method proposed by Zhang (1994). The outline of this approach is as follows:

1. Construct a library *W* of discretely dilated and translated versions of given wavelet ψ that is constructed according to the available training data set.
2. For selecting the best wavelets from the library, the second method of Zhang (1994) is used. It is called stepwise selection by orthogonalization –method. In this method the wavelet that linearly spans with the previous wavelets closest to the space wanted is repeatedly selected.
3. The last step is ordinary backpropagation using a quasi-Newton procedure with steps 1 and 2 as initialization.

Constructing the wavelet library is in principle the same procedure as discretizing the continuous wavelet transform. The standard discretization is a regular lattice

$$\left\{\psi\left(a_0^n\mathbf{x}-\mathbf{m}\cdot\mathbf{t}_0\right):n\in\mathbb{Z},m\in\mathbb{Z}^d\right\} \tag{36}$$

Usually in discretization a dyadic grid is used.

The countable family (36) is too large to be used in wavelet networks. But almost always regression concerns only a compact domain $D\subset\mathbb{R}^d$. Hence in practice $m\in\mathbb{Z}^d$ can be replaced by $m\in S_t$ in (36), with a finite set $S_t\subset\mathbb{Z}^d$. Restrictions on *n* can also be set by focusing on the resolution levels that have significance to the problem at hand. So the family (36) becomes

$$\left\{\psi\left(a_0^n\mathbf{x}-\mathbf{m}\cdot\mathbf{t}_0\right):n\in S_a,m\in S_t\left(n\right)\right\} \tag{37}$$

Some wavelets in (37) do not contain any sample point in their support. So after forming the library (37) the training data is scanned and for each sample point the wavelets in (37) whose supports contain the sample point are determined. This method is preferable in this situation because it is not necessary for the large library to be actually created and allows us to handle problems of relatively large input dimensions. With these methods the library of wavelet regressor candidates are formed

$$W=\left\{\psi_i:\psi_i\left(\mathbf{x}\right)=\alpha_i\psi\left(a_i\left(\mathbf{x}-\mathbf{t}_i\right)\right),\alpha_i=\left(\sum_{k=1}^{N}\left[\psi\left(a_i\left(\mathbf{x}_k-\mathbf{t}_i\right)\right)\right]^2\right)^{-1/2},i=1,\ldots,L\right\} \tag{38}$$

where α_i are normalization factors of wavelets.

The selection of the candidates proceeds in the following way. For the first stage, select the wavelet in *W* that best fits the training data, and then repeatedly select the wavelet that best fits the data while working together with the previously selected wavelets. This method of stepwise selection by orthogonalization is a very straightforward method. It consists of many small steps that are not presented here. Thorough explanation of the method can be found from Zhang (1994). Zhang presents calculations of the computational burden of his three methods for selecting the best wavelets from the library. He finds that in most cases the method applied here is the best compromise between the effectiveness of the regressor selection and the computational burden. He however warns that because these methods are heuristic, one cannot determine a method that is always more effective than others (Zhang 1994).

In the last step this construction is then used as the initialization of a backpropagation procedure that will further refine the wavelet network by adapting its dilation, translation and linear parameters on the training data. A quasi-Newton algorithm is applied in the backpropagation algorithm. Also linear connections are included to capture the linear properties of the empirical data.

6.4 Empirical analysis

6.4.1 Empirical data

The data consists of average returns and volatilities of the exchange rates between the Japanese Yen, the British Pound and the Deutsche Mark vis-à-vis the US dollar. Daily observations cover the period from January 6, 1971 to February 15, 2007. From these daily observations 10 and 30 day average returns and 10 and 30 day volatilities are calculated. The volatilities are calculated as standard deviations of the daily returns.

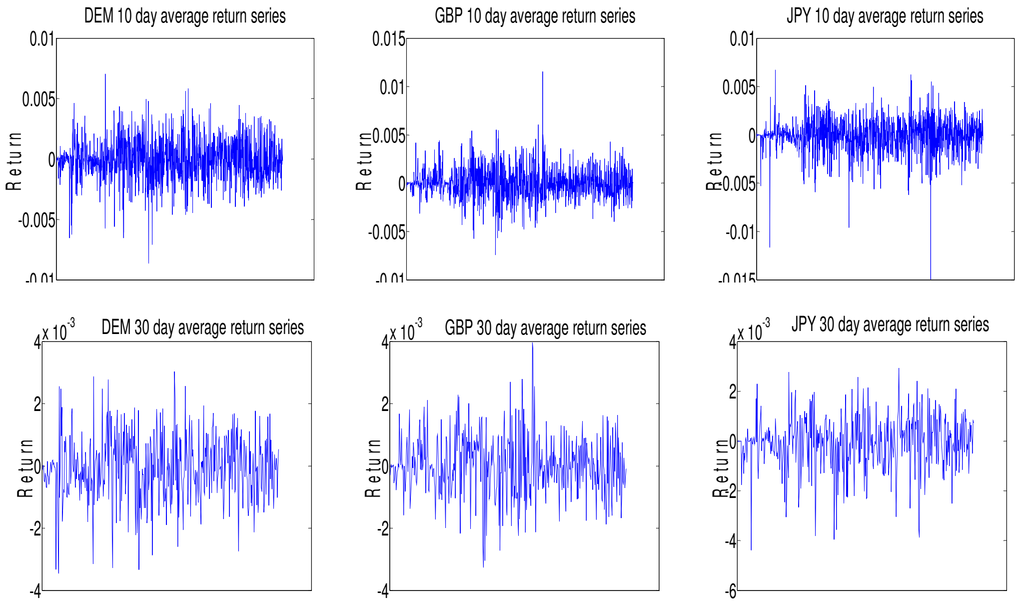

Figure 22. Average returns series for the exchange rates studied. The sample period begins on January 6, 1971 and ends on February 15, 2007

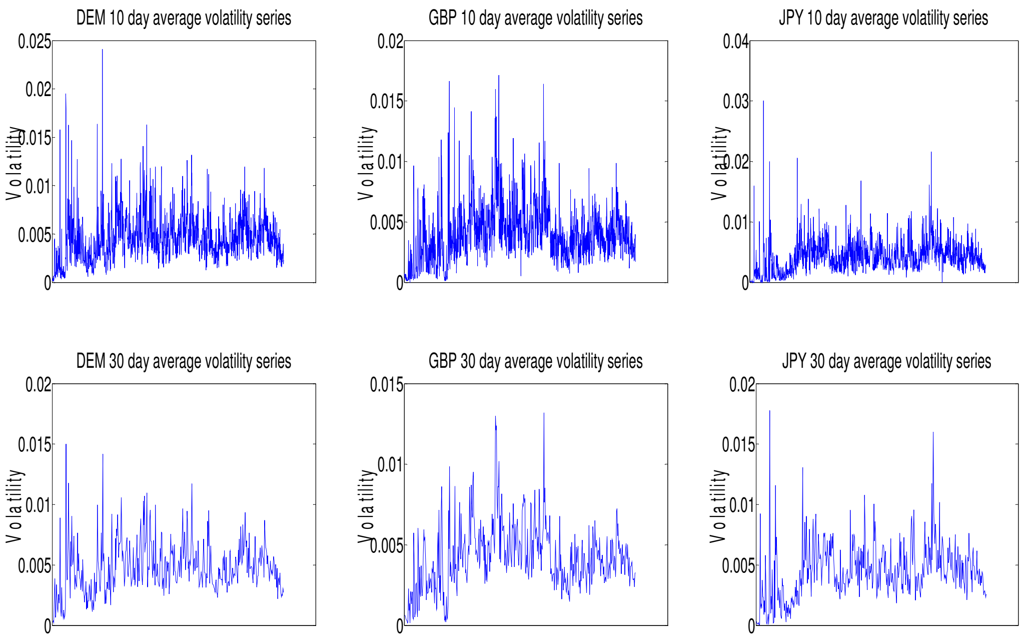

Figure 23. Average volatility series for three exchange rates. The sample period begins on January 6, 1971 and ends on February 15, 2007

Figures 21 and 22 present the studied series and table 11 and 12 descriptive statistics. The mean returns, except the pound, are slightly negative, and the mean volatilities are around 0.4. The skewness is negative for DEM and JPY for the return series but otherwise positive. All series have excess kurtosis. Because of only a few extreme values, the minimum and maximum values are quite large. Despite these statistics, no transformations were made. For example Meese and Rogoff (1983) suggest log-transformation for forecasting purposes. These kinds of transformations were not considered vital in this work, because the main focus is on the comparison, not in the absolute forecasting performance.

Table 11. Descriptive statistics for return series. The statistics are presented on both time horizons. The mean, standard error and standard deviation are presented as percentages to maintain readability.

10 DAY RETURNS

	USD-JPY	*USD-GBP*	*USD-DEM*
Mean (%)	-0.0082	0.0015	-0.0067
t-value (mean=0)	-1.71	0.35	-1.43
Standard Deviation (%)	0.17	0.16	0.17
Excess Kurtosis	6.59	3.45	1.45
Skewness	-1.00	0.36	-0.16
Range	0.022	0.019	0.016
Minimum	-0.015	-0.0074	-0.0086
Maximum	0.0067	0.012	0.0071
Count	1318	1318	1318

30 DAY RETURNS

	USD-JPY	*USD-GBP*	*USD-DEM*
Mean (%)	-0.0082	0.0015	-0.0067
t-value (mean=0)	-1.59	0.33	-1.33
Standard Deviation (%)	0.11	0.097	0.10
Excess Kurtosis	1.55	1.23	0.63
Skewness	-0.58	0.13	-0.11
Range	0.0073	0.0072	0.0065
Minimum	-0.0044	-0.0033	-0.0035
Maximum	0.0029	0.0040	0.0030
Count	439	439	439

Table 12. Descriptive statistics for volatility series. The statistics are presented on both time horizons. The mean, standard error and standard deviation are presented as percentages to maintain readability.

10 DAY VOLATILITIES	*USD-JPY*	*USD-GBP*	*USD-DEM*
Mean (%)	0.43	0.41	0.46
Standard Error (%)	0.0074	0.0065	0.0068
Standard Deviation (%)	0.26	0.24	0.25
Excess Kurtosis	10.18	3.36	5.78
Skewness	1.85	1.26	1.55
Range	0.030	0.017	0.024
Minimum	0	0	0.00015
Maximum	0.030	0.017	0.024
Count	1318	1318	1318
30 DAY VOLATILITIES	*USD-JPY*	*USD-GBP*	*USD-DEM*
Mean (%)	0.45	0.42	0.48
Standard Error (%)	0.011	0.0097	0.010
Standard Deviation (%)	0.23	0.20	0.21
Kurtosis	3.88	2.08	2.21
Skewness	1.04	0.85	0.90
Range	0.018	0.013	0.015
Minimum	0.000048	0.00015	0.00025
Maximum	0.018	0.013	0.015
Count	439	439	439

6.4.2 Empirical results

Figure 23 shows examples of the performance of both forecast methods for an out-of-sample returns data. 10 day averages have a very strong variation around zero. The linear forecast method is not capable in capturing this kind of extreme variation and the fitted series stays close to zero for the whole period. The wavelet forecast method follows these extreme variations somewhat better. However the fitted series is still far from the original series.

Things are somewhat better for the 30 day averages. Variations are slower and the forecast models have a greater chance to follow the data. The linear forecast method still stays quite close to zero. The wavelet forecast method, however, follows the out-of sample data subsequently better. So the results of Chinn & Meese (1995) appear to be correct at least visually. On longer forecast horizons, the variations are not strong and the applied forecast method is able to adapt to the changes. And thus, succeeds an improved forecast performance on longer time horizons

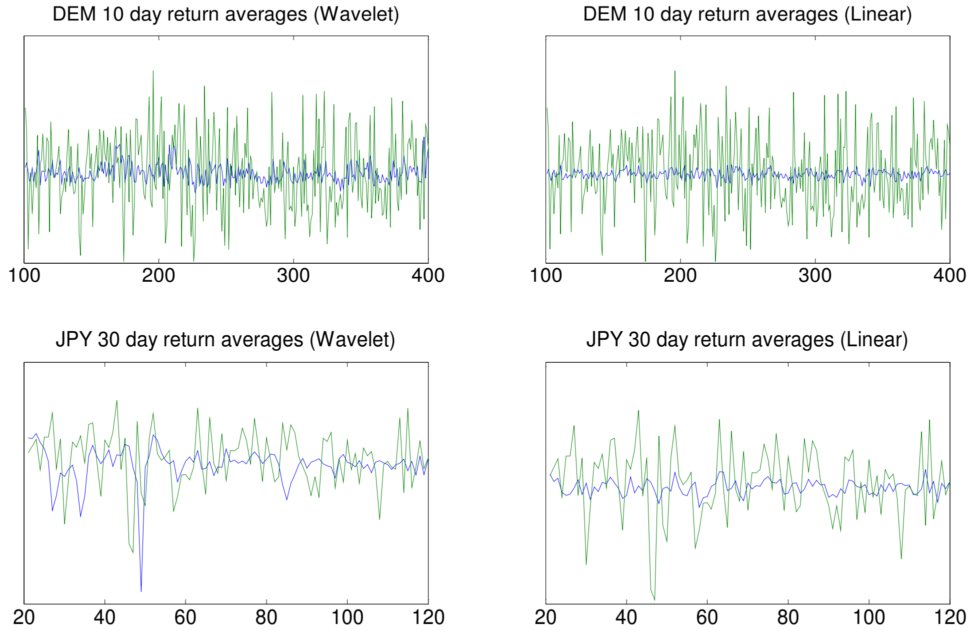

Figure 24. Examples of forecasts for return series. The green line represents the true series and the blue line the forecast for wavelet network and linear model.

Similar conclusion can be made from the volatility forecasts presented in Figure 24. Extreme variations of the shorter forecast horizon makes the forecasting difficult for both forecast methods. Now the linear forecast methods appear to be more suited. For some reason, the wavelet forecast method has very poor forecasts at the beginning of the data after strong volatilities. On the longer forecast horizon, things look quite good for both models. Especially the wavelet forecasts follow the data flawlessly. So again the improvements on the longer forecast horizon are seen.

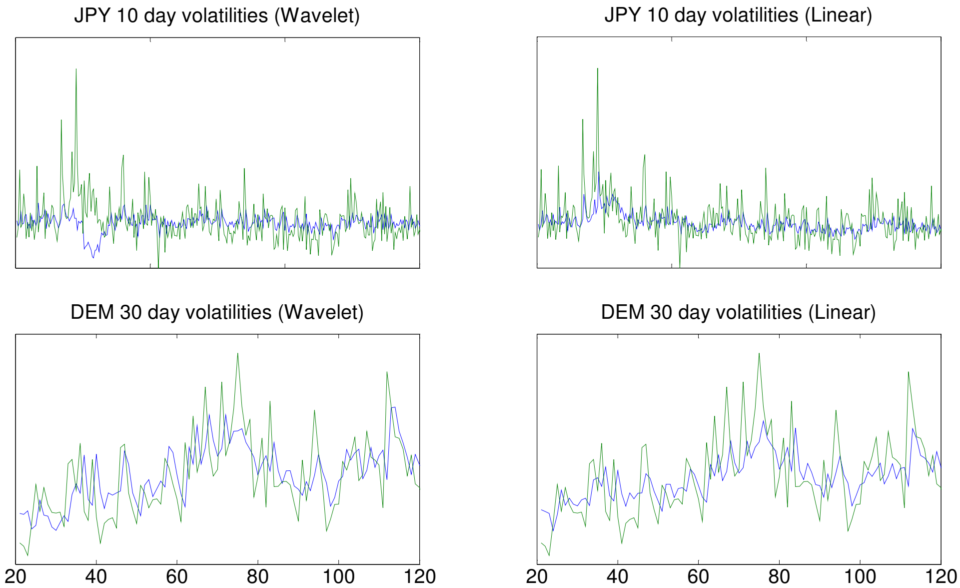

Figure 25. Examples of forecasts for volatility series. The green line represents the true series and the blue line the forecast.

Figure 25 presents mean square errors for the forecasts with different models. In every instance we see that the nonlinear wavelet network does not improve forecasts. The mean square error of the linear model is the smallest closely followed by the wavelet network model. When compared to the forecasts of the random walk model, some improvement in the forecasts can be seen, expect for the 30 day volatility forecasts.

However the improvements are quite modest and the results somewhat support the conclusions of Meese and Rogoff (1983). More important is the results that a complex non-linear forecast method does not improve forecast performance at all. This result supports the conclusions of Faust et al. (2002). Although there has been an increasing popularity in different nonlinear forecasting methods, especially neural network methods, the documented improvements are probably not so significant. The nonlinear methods tend to fit to the noise of the data and do not improve forecasts. Linear methods are more robust. Surprisingly, the MSE results do not support the consensus of the nineties that longer time horizons are easier to forecast (see for example Chinn & Meese 1995). On the contrary there is stronger improvement against the random walk model on the short time horizon. This results are in line with the results of Carriero et al. (2009).

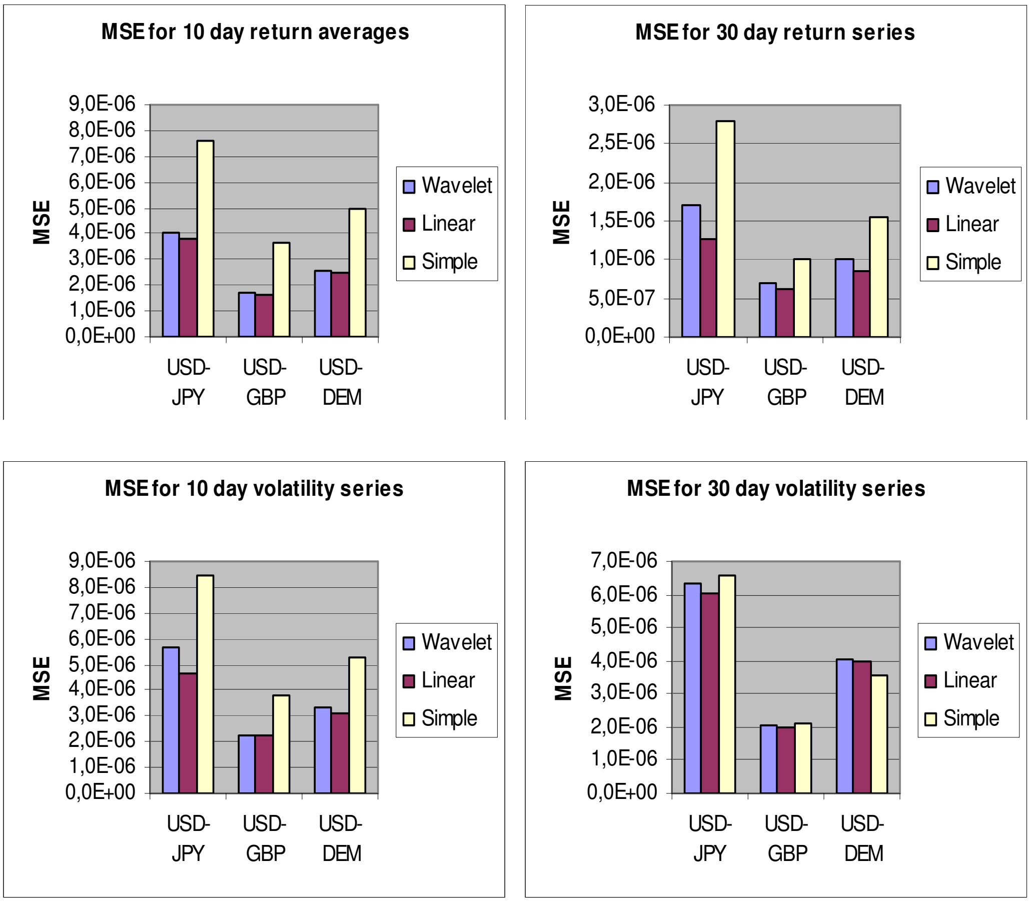

Figure 26. Mean square errors for the forecasts. The errors are presented on two different time horizons. The left side represents the 10 day horizon and on the right side the 30 day horizon. Different colors present different forecasting method.

6.5 Conclusions

This chapter examines the predictability of return and volatility series on different time scales. The results show that using a non-linear forecasting method does not improve forecasting performance. The wavelet network just fits to the noise of the training data and forecasts from the network are worse than forecasts from the linear model. These results support the findings of Faust et al. (2002). They question the improvements of the previous contributions and note that reported improvements are seen only with the data used in the original paper.

Forecasts of both the wavelet network model and the linear model are somewhat better than the random walk model, suggesting that there is predictability in these series. However the improvements are quite modest so in this sense we are still quite close to the conclusions of Meese and Rogoff (1983). The predictability does not improve on the longer forecasting horizons. On the contrary there is a larger difference between the studied models and the random walk model when we are dealing with shorter forecast horizons. This is in contrast to the recent results that forecasts improve when time horizon increases (Chinn & Meese 1995) and somewhat supports the findings of Carriaro et al. (2009). Using even longer forecast horizons might change the picture, which is left for future research.

7 CONCLUDING REMARKS

The previous chapters have presented new applications of wavelet in finance and have extended previous work within the research area.

In the second chapter, the linkages between major world stock indices are studied. The methodology is based on wavelet correlation (and cross-correlation), that give us multiresolution analysis of interdependence between indices. With wavelet correlation we can study correlation's dependence on the time scale. The third chapter studies the presence of contagion between major world markets. This chapter extends the contagion literature by adding the time scale dimension to the picture. Different time scales are analyzed using the continuous wavelet transform based wavelet coherence and the discrete wavelet transform based wavelet correlation. The results show how the correlations change as a function of the time scale. The fourth chapter examines extensively the lead-lag relations of the three major European currencies using the wavelet cross-correlation methods. This makes it possible to investigate scale dimension of the linkages of the exchange rates. The maximal overlap discrete wavelet transform was used to decompose the original series into different scale wavelet coefficient series and cross-correlation functions were then calculated between the coefficient series to analyze the dynamics of cross-dependence of the exchange rates on different time scales. The fifth chapter focuses on the cross-dynamics of exchange rate expectations. Over-the-counter currency options on the euro, the Japanese yen, and the British pound vis-à-vis the U.S. dollar are used to extract expected probability density functions of future exchange rates. The moments of density functions are analyzed using wavelet techniques to study linkages in these option-implied market expectations over different time-scales. Focusing on the dynamic structure of the relations between expected exchange rate distributions, this provides new insights into the dynamics of foreign exchange markets. The sixth chapter examines the predictability of return and volatility series on different time scales. The purpose of this chapter is to study the non-linear structure in the exchange rates and the performance of wavelet networks in financial forecasting.

Results of these chapters are promising. The empirical findings of the second chapter revealed rich structure between stock market indices. There was a clear trend that the correlation between indices increases, when the time horizon gets longer. This research extends the results of the previous research. For example results of Wongsman (2006) are extended to longer scales than solely daily time scales. The correlations between Nikkei and other indices were the smallest on every scale. Therefore this research also gives wider support on the results of Morana & Beltratti (2008) about the separate nature of Nikkei among indices.

The results of the second chapter state that from the standpoint of portfolio diversification, Nikkei listed stocks should always be included in the portfolio. The difference is that on shorter time scales, Nikkei listed stocks should accompany stocks from SP500, while on longer time scales, European stocks should be used.

The short cross-correlation analysis of volatilities in the second chapter revealed interesting structures between volatilities of major indices. On the shorter time scales there was a volatility spillover from SP500 to other indices. On the time scale of one month, volatility spillover from the European indices, especially DAX30, to SP500 and Nikkei is observed. The longest time scale is again similar to the shorter time scales, where the changes of volatility of SP500 lead changes amongst the other indices. The results follow previous literature. Morana & Beltratti (2008) remark on the flow from the US to other markets and the separate nature of the Japanese market. On certain scales there is also support for the results of Lin et al (2004) on the influence of the Nikkei market on other markets. The strong spillover from the DAX30 index to other indices on a month timescale is something new which has not been documented before.

With the novel wavelet coherence method, clear signs of contagion are found in the third chapter. Several times has contagion has been a major factor between markets in the last 25 years. Correlations on shorter time scales increase significantly while longer time scales remain approximately the same. This is most clearly seen with the 1987 stock market crash, the Gulf War and the ongoing global financial crisis. The results also show how the short time-scale correlations decrease at tranquil periods (bull markets) giving support to the conclusions of Longin and Solnik (2001). An overall increase in the long timescale correlations during the studied time period is also found. The results of third chapter conclude that this increase of interdependence (Forbes & Rigobon, 2002) plus contagion during the ongoing crisis makes the markets very highly correlated on every scale at the moment.

The findings in the fourth chapter were in line with the previous research. The euro and the Swiss franc had very symmetric cross-correlation functions on all scales with a very strong positive contemporaneous correlation suggesting close connection without any significant lead-lag dynamics. This result is similar to the findings of Nikkinen et al. (2006). Krylova et al. (2009) find an evidence of nonlinear relationship between the Swiss Franc and the euro. There are some very weak findings that support this observation as the lead/lag -relations between the euro and the franc change direction when we move from shorter time scales to longer time scales. The results with the pound were expected. They support the observations of Matsushita et al. (2007) who argue that the pound and euro behave differently and should not be considered the same currency. The asymmetry towards the euro on larger scales suggests the leading role of the euro against the pound which is similar to the results of Krylova et al. (2009) and Nikkinen et al. (2006).

The only other study which also considers the interrelations of exchange rates on different time scales is Wu (2007) and this study only examines the USD/DEM and USD/JPY exchange rates. However there is one clear difference between the results of Wu and the results found using the wavelet cross-correlation methods. Wu argues that the correlations between exchange rates are stronger on a daily time scale than on longer time scales. Nonetheless, the wavelet cross-correlation diagrams suggest just the opposite. Almost without exceptions the correlations become stronger when the time scale increases.

The empirical findings of the fifth chapter demonstrate that market expectations are closely linked among the three major exchange rates. Regardless of time-scales, there are significant lead-lag relationships between the expected probability densities of exchange rates. The linkages in market expectations appear particularly strong between the EUR/USD and GBP/USD exchange rates. On a shorter time-scale, the implied volatility of the JPY/USD exchange rate is found to affect the volatilities of the EUR/USD and GBP/USD rates. Thus, the Japanese yen seems to have a leading role among the exchange rate triplet in terms of short-run dynamics of volatility expectations. On a longer scale, however, there are also significant feedback effects from the GBP/USD volatility expectations to the JPY/USD volatility.

The wavelet cross-correlations of the higher-order moments of option-implied exchange rate distributions indicate that the market expectations about of the JPY/USD exchange rate are virtually unrelated to the developments of the European currencies. The higher-order moments of the expected EUR/USD and GBP/USD densities are strongly linked, especially on a longer time-scale. The results indicate that movements in the skewness and kurtosis of the expected EUR/USD distributions may lead to movements in the GBP/USD distributions. In general, empirical findings suggest that the dynamic structure of exchange rate expectations may vary considerably over different time-scales.

The results of the sixth chapter show that using a non-linear forecasting method does not improve forecasting performance. The wavelet network merely fits to the noise of the training data and forecasts from the network are worse than forecasts from the linear model. These results support the findings of Faust et. al (2002). They question the improvements of the previous contributions and note that reported improvements are seen only with the data used in the original paper. The empirical findings are also quite close to the conclusions of Meese and Rogoff (1983). The predictability does not improve on longer forecasting horizons. On the contrary there is a larger difference between the studied models and the random walk model when working with shorter forecast horizons. This is in contrast to recent research which argues that forecasting performance improves when time horizon increases (Chinn & Meese 1995). The results somewhat support the findings of Carriaro et al. (2009).

The contribution of this thesis is to extend the applications of wavelet methods in finance. Overall, these chapters show that time series analysis in economic and financial research can gain new insight with wavelet analysis by separating processes on different time scales and repeating the traditional analysis on these different scales. The characteristics of wavelet methods fit inherently to the features of financial time series. Economic and financial processes build up naturally from multiple processes on separate time scales. When we are decomposing economic and financial time series to their wavelet components, simultaneously we are decomposing them to their natural building blocks.

The previous chapters introduce many new results and open up new frontiers. Wavelet methods play a vital part in many of these new results. There are two main aspects are behind the success of wavelets in finance. One is the intelligent compromise between the time dimension and the frequency dimension which help wavelets to avoid the obstacles that have plagued time or frequency analysis. Another is the multiscale structure that is a natural part of financial processes. Investors naturally work on many different timescales. And with wavelets we can separate these different timescales. The results also show that the boundaries of the possible applications of wavelets are not yet found and that there are many other uninvestigated frontiers of wavelet applications in finance.

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