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ARBITRAGE PRICING THEORY AND ITS EMPIRICAL APPLICABILITY FOR THE HELSINKI STOCK EXCHANGE*

by

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ABSTRACT

The purpose of this paper is to test the Arbitrage Pricing Theory (APT) using monthly data for Finnish stock returns during the 1970-1986 period. The first stage involves estimating the systematic risks for each asset using factor analysis. The second stage involves testing by transformation analysis if the number and structure of factors which influence the security returns remain unchanged across various time periods. The third stage involves testing the implications of the APT using cross-sectional regression analysis.

1. INTRODUCTION

1.1. Backround

The Capital Asset Pricing Model (CAPM), developed by Treynor (1961), Sharpe (1963 and 1964), Lintner (1965), Mossin (1966) and Black (1972), is a simple and elegant model for pricing risky assets. CAPM is an equilibrium model, and in the CAPM, the systematic risk of an asset is defined to be the covariability of the asset with the market portfolio. CAPM has been the central topic in the empirical work in finance over the past twenty years. Empirical tests of CAPM have produced mixed results. The most powerful evidence in support of CAPM are the findings of Black, Jensen and Scholes (1972), Fama and McBeth (1973) and Foster (1978). They found that portfolios with higher estimated betas also have higher realized returns. The critical point in the estimation of CAPM is the difficulty of measuring the true market portfolio (more about the importance of a relevant data base in empirical research see Ball and Foster 1982). Stock market indices are usually used as a proxy of true market portfolio. Miller and Scholes (1972) found that the results are not very sensitive to the choice of stock market index. The result seems to be directly contrary to the result presented by Roll (1977). Roll presented that CAPM is extremely sensitive to the choice of a market proxy (see also Ball 1978: 110-126). Roll's critique goes further. He casts serious doubts on the testability of the CAPM itself. The CAPM is not testable unless the exact composition of the true market portfolio is used in the tests.

The Arbitrage Pricing Theory (APT) formulated by Ross (1976) provides another model for explaining the relationship between return and risk. The APT is based on similar intuition as CAPM, but it is more general. The CAPM assumes that a return of any security will be linearly related to a single common factor, to a return of the market portfolio, whereas APT assumes that a security return is a linear function, not only of one, but of a set of common factors. The normal empirical procedure to test APT is the

following: First, a factor analysis procedure is used to identify the number of factors and the factor loadings from daily, weekly or monthly time series. Second, the estimated factor loadings are used to explain the cross-sectional variation of estimated expected returns.

Unfortunately, there are many problems in testing of APT. An intensively discussed problem is how to decide the correct number of priced factors. It has been found that the number of significant factors is an increasing function of the size of the groups analyzed (Dhrymes, Friend and Gultekin 1984, and Dhrymes, Friend, Gultekin and Gultekin 1985). There are also some additional methodological problems with the use of factor analysis (Elton and Gruber 1987: 343 - 354). First, the decision as how many factors to extract has been made subjectively. Second, there is no guarantee that factors are produced in a particular order. Third, there is no meaning to the signs of the parameters.

In our opinion, the most relevant questions in testing the APT is neither the question how to determine the "correct number" of the priced factors in different samples nor the question in what kind of order factors are produced in those samples. We may get the same number of factors in different samples or in the same sample in different time periods but the content or empirical interpretation of the factors do not necessarily remain as the same in those groups. Therefore, it is very important to find such common factors which are the same across different samples during the same time period (cross-sectional studies) or across different time periods in the same sample irrespective of what is the number of those factors or in what order the factors are produced. Transformation analysis offers us a versatile methodology with which it is possible to study the stability and invariance existing among different factor structures (for business applications of transformation analysis see Yli-Olli and Virtanen 1990).

1.2. The purposes of the study

The purposes of this study are:

1. to describe briefly the APT,
2. to test the APT using monthly time series data of the Finnish firms quoted on the Helsinki Stock Exchange,
3. in testing the APT, the main effort is made to test, using transformation analysis, the stability of the factor structure over time. That means: transformation analysis tells us if the content of the factors remains the same in different time periods (The empirical content or interpretation of the factors

could be for example market portfolio, unexpected inflation etc. Quite clearly, however, the transformation analysis can tell us if the content of the factors remains the same but it does not tell what the content explicitely is). Analogously, we can find, using transformation analysis, if there are the same common factors (the empirical interpretation of the factors being the same) in different samples. In our analysis, therefore, it is not a problem if the factors in different samples are not produced in a particular order.

2. THE APT-MODEL

The Arbitrage Pricing Theory, originally formulated by Ross (1976) predicts that on the perfectly competitive and frictionless stock markets the stock return is a linear function of a certain number, say k, economic factors. So, the APT starts with the assumption that returns on any stock, R_{it}, are generated by a k-factor model of the form (see e.g. Roll and Ross 1980, 1076-1082):

$$(2.1) \quad R_{it} = E(R_i) + b_{i1}\delta_{1t} + b_{i2}\delta_{2t} + ... + b_{ik}\delta_{kt} + \varepsilon_{it} ,$$

where $E(R_i)$, $i = 1,2,...n$, is the expected return of the stock i, δ_j, $j=1,2,...,k$, are unobserved economic factors, b_{ij} is the sensitivity of the security i to the economic factor j and ε_i are the idiosyncratic risks of the stocks. In addition, we assume that $E(\delta_j) = 0$ for $j = 1,2,...,k$, $E(\varepsilon_i) = 0$ for $i = 1,2,...,n$, $E(\varepsilon_i\varepsilon_h) = 0$ for $i \neq h$, and $E(\varepsilon_i^2) = \sigma_i^2 < \infty$.

Ross (1976) has shown that if the number of stocks is sufficiently large the following linear risk-return relationship can be written:

$$(2.2) \quad E(R_i) = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + ... + \lambda_k b_{ik}$$

where λ_0 is a constant riskless rate of return (the common return on all zero-beta stocks) λ_j, $j = 1,2,...,k$, represents, in equilibrium, the risk premium for the jth factor.

In equation (2.1) each stock i has a unique sensitivity b_{ij} to each factor δ_j but any factor δ_j has a value that is the same for all stocks. These common factors capture the systematic components of risk in equation (2.1). Therefore, any δ_j affects necessarily more than one security return. In the other case it would have been compounded in the

unsystematic component of the risk, i.e. in the residual term ε_i.

In order to test the Arbitrage Pricing Theory we have in principle two alternative aproaches to test the model (2.1):

First, we could try to specify a priori, on the basis of the theory, the general factors that explain pricing in the stock market. Such macroeconomic variables could be e.g. the spread between long-term and short-term interest rates, expected and unexpected inflation, industrial production and spread between high- and low-grade bonds (see Chen, Roll and Ross (1986). In the thin Finnish stock market such variables could be e.g. aggregate future cash-flow of the firms, interest rate of bank deposits or return of the state bonds, the supply of money, and inflation (see Virtanen and Yli-Olli 1987). In the case we have factors based on economic theory the estimation procedure should be as follows. In the first stage, time series regressions are run for each series of stocks (portfolios) to estimate each stock's (portfolio's) sensitivities b_{ij} to macroeconomic variables. Then the risk premia λ_j are estimated by running a cross-sectional regression for each time period examined. In every cross-sectional regression the average return of stocks is used as the dependent variable and the sensitivities of the securities as independent variables.

The more general and also much more problematic approach is to estimate the b_{ij} and unknown factors δ_j simultaneously by factor analysis. In that case a theory does not tell, a priori, what is the exact content or even the number of relevant factors. Without any theory a decision how many factors to extract from the data has to be made subjectively or by statistical criteria. When we have obtained systematic components of the risk, b_{ij}, the risk premia λ_j are estimated again using cross-sectional regressions.

In a factor analysis approach we have many methodological problems. First, there are no meaning to the signs of the factors produced by factor analysis. Second, the scaling of b_{ij}'s and λ_j's is arbitrary. Third, there is no guarantee that factors are produced in a particular order when analysis is performed on separate samples (see Elton and Gruber 1987:336-352). In addition, we have serious difficulties when we try to decide what is the correct number of priced factors. Dhrymes, Friend, Gultekin and Gultekin (1985) used samples of different sizes (30, 60 and 90 stocks) and they found that the number of significant factors is an increasing function of the size of the group analyzed.

In our opinion, a very important but non-discussed and non-analyzed problem is the question if the contents of the factor structures in different samples during the same time period or the contents of the factors in the same sample in different time periods are the same. In this paper we use a method which makes it possible to analyze the stability of factor structures across different samples in the same time period or across different time periods in the same sample. It is not important in this method whether the factors in different samples are produced in a particular order or not. The only limitation is that we have the same number of factors in different samples. After that we can find such common factors which have the same contents, i.e. the same empirical interpretation for different samples.

3. DATA AND STATISTICAL METHODS

The purpose of this study was to test, using Finnish data, the Arbitrage Pricing Theory, especially the stability of factor patterns between different time periods. In Finland, on the Helsinki Stock Exchange, we have three commonly used indices, Unitas and KOP stock market indices published by two Finnish commercial banks and a return index developed by Berglund, Wahlroos and Grandell (1983). From the theoretical point of view, the return index developed by Berglund, Wahlroos and Grandell is the best measure and also selected for our research. This index also includes the dividend component whereas Unitas and KOP indices are pure price indices.

The empirical verification of the APT and the stability analysis require both a large sample in terms of number of securities and also a long time period. We have in use monthly values of selected indices from February 1970 to December 1986. The first sample consists of the shares of 30 firms (Appendix 1). The sample includes all shares which have been quoted on the Helsinki Stock Exchange during the entire sample period. For the stability analysis the whole period is divided into three subperiods: subperiod 1 includes years 1970-1975, subperiod 2 years 1976-1980 and subperiod 3 years 1981-1986. Stability analysis requires the same firms throughout the whole period to be examined. Therefore the firms in the first sample are the same for each subperiods. At the end of the period the total number of quoted firms was 59 (at the moment the number of different securities is much larger). The selection method for the first sample introduces a survival bias because the sample includes only the firms having continuous monthly data during the entire sample period. To escape this bias towards long-lasting firms also another sample was selected. This second sample includes the returns of 30 most traded stocks for each subperiod (the firms for the second sample are not given in Appendix 1, because the sample is a bit different for each subperiod). However, these two samples include in the main the same firms. Comparing the results from different samples it is possible to conclude how sensitive the results are to the choice of the firms in the sample.

The main statistical methods used in the study are factor analysis, regression analysis and transformation analysis. Factor analysis and regression analysis are usual tecniques in business applications. Transformation analysis, on the contrary, has been mainly applied only in Finnish sosiological research. Therefore, this paper contains a short description of this multivariate method.

The degree of stability in factor patterns has been traditionally measured with correlation or congruence coefficients (the same coefficients are used in measuring the stability of estimated betas; see. e.g. Blume (1971)). Both of these measures give an index for the similarity of two different factor solutions in terms of the pattern of correlations among factor loadings across all variables in the reduced factor space. For the dissimilar part of these factor solutions these indices are, however, unable to describe and explain the reason for the non-invariant part prevailing in these factor solutions (see Yli-Olli and Virtanen 1985: 25).

Yli-Olli (1983) introduced the use of transformation analysis for determining the degree and nature of medium-term stability exhibited by the factor patterns of the financial ratios. This approach was further applied and deepened by Yli-Olli and Virtanen (1985).

Originally transformation analysis (initiated by Ahmavaara (1963) and further developed by Ahmavaara (1966) and Mustonen (1966); most applications exist in the area of Finnish sosiological research) was developed to compare factor solutions between two different groups of objects. Yli-Olli (1983) and Yli-Olli and Virtanen (1985 and 1990) have used the technique to compare two different factor solutions among the same group of objects, the two factor solutions being based on measurements made during two different time periods. In the following we sketch out the general idea behind transformation analysis (according to the papers of Yli-Olli and Virtanen 1985 and 1990).

Let us assume that we have two groups of observations G_1 and G_2 with the same variables, both by number and content. Let L_1 and L_2 be the factor matrices for G_1 and G_2, respectively. Let us further assume that the factor models used in deriving L_1 and L_2 are both orthogonal and have the same dimension, pxr, say.

If there exists invariance between the two factor structures, there exists a nonsingular rxr-matrix T such that equation

$$(3.1) \qquad L_2 = L_1 T_{12}$$

holds. Matrix T_{12} is called the transformation matrix (between L_1 and L_2, or in direction $G_1 \rightarrow G_2$). If equation (3.1) holds exactly, it means that the factor structures in groups G_1 and G_2 are, up to a linear transformation, invariant, all the variables have the same empirical meaning in different groups. Depending on the type of the transformation matrix T_{12}, the formation of the factors from the variables and thereby the interpretation of the factors either is preserved (T_{12} is the identity matrix I) or it changes (T_{12} has also non-zero off-diagonal elements).

In practice, situation (3.1) will not be reached, but, after matrix T_{12} has been estimated, we have $L_2 \neq L_1 T_{12}$. The goodness of fit criterion for the model (3.1) may be based on the residual matrix

(3.2) $$E_{12} = L_1 T_{12} - L_2 .$$

Non-zero elements in E_{12} mean that the empirical meaning of the variables in question has changed. This is called abnormal transformation.

The main problem in transformation analysis is the estimation of the matrix T_{12}. The estimation methods are in general based on the minimization of the sum of squares of the residuals e_{ij} (the elements of the residual matrix E_{12}). This is the common method of least squares. The problem is to minimize

(3.3) $$\| E_{12} \| = \| L_1 T_{12} - L_2 \| = \text{trace}\,((L_1 T_{12} - L_2)(L_1 T_{12} - L_2)').$$

Depending on additional constrains set for the matrix T_{12}, we have three different estimation methods, i.e. three transformation analysis models (see e.g. Yli-Olli and Virtanen 1985). Of these three techniques, the symmetric transformation analysis is the most popular one. It is also applied in this study.

With correlation and congruence coefficients one can only measure the degree of similarity of two factor solutions (correlations or congruences among factor loadings). This is also possible via transformation analysis (coefficients of coincidence on the main diagonal of the transformation matrix). In addition to this we obtain a regression type model for shifting of variables from one factor to another (normal or explained transformation). This is revealed by non-zero off-diagonal elements in the transformation matrix and indicates interpretatively changes for the factors in question. And finally, large elements in the residual matrix indicate abnormal or unexplained transformation between the two factor solutions. This means that the empirical content of the corresponding variables has changed. Further, this abnormal transformation can be appointed to separate variables or to separate factors.

4. EMPIRICAL RESULTS

At the beginning we examine some statistical properties of stock returns between different subperiods. First, Appendix 2 (Tables 2.1 - 2.2) shows that the mean and

variance of the variables are not time invariant. Especially the null hypotheses of equal variances between subperiods could be rejected in a large number of firms. However, the unambiguous tests of the Arbitrage Pricing Theory require, in principle, that the first two moments of return series are time invariant (see e.g. Diacogiannis (1986); the security return distributions at the London Stock Exchange were not intertemporally stationary). Second, equilibrium models of security markets are usually based on the assumption of normally distributed security returns. However, the empirical researches have shown that the distributions of individual daily returns are skewed. Intertemporal aggregation to monthly returns has reduced the skewness only slightly (see e.g. Roll and Ross 1980:1095-1096). Table 2.3 (Appendix 2) shows that the null hypotheses of normally distributed returns could be rejected among most firms examined. The non-stationarity and non-normality in return series indicate that the empirical results must be interpreted with relatively high caution.

The following step in our empirical analysis is to use factor analysis procedure to identify the number of factors affecting equilibrium returns. This procedure has been very problematic because it has been shown that the number of factors discovered depends e.g. on the size of the groups of securities one deals with (see Dhrymes, Friend and Gultekin 1984: 345-346). The estimation of factors can be carried out by different factor analytic methods. In this study we use the principal component method and varimax rotation thereafter.

The following step is to divide the whole sample period into three subperiods and to identify the number of factors affecting equilibrium returns during these subperiods. In this research we try to find such common factors which are stable for different subperiods (and among different groups). For this purpose we first extracted three, four, five, six and seven factor solutions for each subperiod. Cumulative proportions of total variance explained (of the unrotated factor patterns) are presented in Table 1.

Table 1. Cumulative proportions of total variance explained.

	Sample 1			Sample 2		
	1970-75	1976-80	1981-86	1970-75	1976-80	1981-86
FACTOR1	0.382	0.335	0.302	0.402	0.306	0.304
FACTOR2	0.473	0.451	0.405	0.475	0.441	0.393
FACTOR3	0.533	0.543	0.482	0.532	0.537	0.468
FACTOR4	0.589	0.608	0.551	0.586	0.600	0.531
FACTOR5	0.638	0.663	0.611	0.633	0.651	0.588
FACTOR6	0.680	0.709	0.656	0.678	0.694	0.637
FACTOR7	0.717	0.744	0.698	0.718	0.732	0.677

Cattell's scree tests (Appendix 3, Figures 1-3) show that we can find 2-5 different factors for each subperiod. The t-tests presented in Table 2 seem to confirm the common factor interpretation. However, we have no absolute guarantee that the factors extracted have the same interpretation when the analysis is performed on separate subperiods, i.e. we can not be sure that the presented factors are those common factors we try to identify.

Table 2. Number of stocks (N=30) associated with each factor at the 5 per cent level of significance (two-tailed t-test).

Subperiod	Number of factors	FACT1	FACT2	FACT3	FACT4	FACT5	FACT6	FACT7
1	4	24	18	15	7	-	-	-
	5	22	18	13	10	2	-	-
	6	19	18	13	10	5	1	-
	7	19	17	16	9	3	5	1
2	4	21	18	12	6	-	-	-
	5	19	15	14	7	5	-	-
	6	20	15	14	11	4	5	-
	7	16	12	12	8	9	5	5
3	4	15	20	16	11	-	-	-
	5	13	18	16	7	3	-	-
	6	15	17	13	8	8	4	-
	7	13	15	12	9	7	4	1

Table 3. Transformation matrix between the factor patterns of returns (subperiod 1 vs. subperiod 2; three-factor solution).

		Subperiod 2		
	Factor	1	2	3
Sub-period 1	1	0.985	-0.145	0.097
	2	0.166	0.947	-0.273
	3	-0.052	0.285	0.957

Table 4. Transformation matrix between the factor patterns of returns (subperiod 2 vs. subperiod 3; three-factor solution).

		Subperiod 3		
	Factor	1	2	3
Sub-period 2	1	0.220	0.975	0.009
	2	0.975	-0.220	0.001
	3	-0.003	-0.009	1.000

In the following, only results obtained from sample 1 (shares quoted at the Helsinki Stock Exchange during the whole period examined) are presented. Sample 2 (containing 30 most traded stocks during each subperiod) was used to control a possible survival bias in the results of sample 1. The results were, however, very similar in both samples.

Next we measure, using transformation analysis, the stability of factor patterns over time. The conclusion about stability is based on the coefficients on the main diagonal of the transformation matrix provided that factors in different samples are produced in the same order. The numerical values of those coefficients are very close to one when the factor structure over time is stable. Tables 3 and 4 present the transformation matrices between

Table 5. Residual matrix E_{12} and abnormal transformation for subperiod 2 (three-factor solution).

Firm	Factor 1	Factor 2	Factor 3	Abnormal transformation t_i^2
KOP	-0.291	0.084	-0.110	0.104
SYP	0.089	-0.010	-0.374	0.148
POHJOLA	0.521	-0.116	-0.464	0.500
EFFOA	-0.324	0.108	-0.212	0.161
KESKO	0.065	-0.066	0.539	0.299
STOCK.	0.264	-0.396	0.728	0.757
TAMRO	0.245	-0.014	0.099	0.070
ENSO	0.171	0.159	-0.179	0.086
FISK.	-0.080	-0.213	0.028	0.053
HUHTAM.	-0.056	0.060	0.071	0.012
KAJAANI	-0.089	-0.015	0.202	0.049
KEMI	0.391	0.242	-0.104	0.222
KONE	-0.065	0.114	-0.256	0.083
KYMMENE	0.088	-0.281	-0.020	0.087
LASSILA	-0.569	-0.141	0.510	0.603
LOHJA	0.274	-0.227	-0.068	0.131
METSÄL.	0.352	0.297	0.027	0.212
NOKIA	0.050	-0.025	-0.028	0.004
OTAVA	0.099	-0.103	-0.243	0.079
PARTEK	0.084	-0.192	-0.121	0.058
RAUMA-R.	0.084	-0.110	0.091	0.027
ROSENLEW	-0.010	-0.374	0.362	0.271
SCHAUMAN	-0.116	-0.464	0.604	0.593
SERLACHIUS	0.108	-0.212	0.093	0.065
SUOMEN S.	-0.066	0.539	0.180	0.327
SUOMEN TR.	-0.396	0.728	-0.124	0.702
TAMFELT	-0.014	0.099	0.084	0.017
TAMPELLA	0.159	-0.179	-0.061	0.061
WÄRTSILÄ	-0.213	0.028	0.268	0.118
YHTYNEET	0.060	0.071	-0.272	0.083
Abnormal transformation s_j^2	1.623	1.906	2.454	5.984

three-factor solutions of subperiods 1 and 2, and of subperiods 2 and 3, respectively. The results show that the stability of factors is very high during different subperiods. This means we have found at least three very stable factors. Table 4 also shows that subperiods 2 and 3 have produced the first and the second factor in different order. That means the first and second factor have changed their places in the third subperiod as compared to the second subperiod. Tables 5 and 6 present the residual matrices between subperiods 1 and 2 and between subperiods 2 and 3, respectively. The residual matrices show that any remarkable abnormal transformation does not exist (there are no large non-zero elements in the matrices).

Table 6. Residual matrix E_{23} and abnormal transformation for subperiod 3 (three-factor solution).

Firm	Factor	Factor	Factor	Abnormal transformation t_i^2
KOP	0.157	0.146	0.025	0.047
SYP	0.010	0.227	-0.128	0.068
POHJOLA	0.102	0.230	0.185	0.097
EFFOA	-0.510	0.052	0.017	0.263
KESKO	0.129	0.174	-0.234	0.102
STOCK.	0.187	0.244	0.464	0.310
TAMRO	-0.045	-0.057	-0.271	0.079
ENSO	-0.471	-0.106	0.359	0.362
FISKARS	-0.336	0.254	0.178	0.209
HUHTAM.	0.125	-0.066	-0.186	0.054
KAJAANI	-0.526	0.649	-0.103	0.708
KEMI	-0.365	0.192	-0.272	0.244
KONE	0.431	-0.537	0.353	0.598
KYMMENE	-0.064	-0.103	0.196	0.053
LASSILA	0.338	-0.043	0.070	0.121
LOHJA	0.066	-0.142	-0.187	0.059
METSÄL.	0.383	-0.247	0.076	0.213
NOKIA	-0.065	-0.217	-0.029	0.052
OTAVA	0.521	-0.304	0.184	0.397
PARTEK	0.050	0.044	-0.050	0.007
RAUMA-R.	0.146	0.025	0.122	0.037
ROSENLEW	0.227	-0.128	0.039	0.070
SCHAUMAN	0.230	0.185	-0.025	0.088
SERLACHIUS	0.052	0.017	0.203	0.044
SUOMEN S.	0.174	-0.234	-0.063	0.089
SUOMEN TR.	0.244	0.464	-0.560	0.589
TAMFELT	-0.057	-0.271	0.229	0.129
TAMPELLA	-0.106	0.359	-0.150	0.163
WÄRTSILÄ	0.254	0.178	-0.366	0.230
YHTYNEET	-0.066	-0.186	0.345	0.158
Abnormal transformation s_j^2	2.143	1.873	1.626	5.642

Table 7. Transformation matrix between the factor patterns of returns (subperiod 1 vs. subperiod 2; four-factor solution).

	Factor	Subperiod 2: 1	2	3	4
Sub-period 1	1	0.813	0.533	-0.206	-0.111
	2	-0.368	0.779	0.383	0.333
	3	0.368	-0.298	0.220	0.853
	4	0.261	-0.141	0.873	-0.387

Table 8. Transformation matrix between the factor patterns of returns (subperiod 2 vs. subperiod 3; four-factor solution).

	Factor	Subperiod 3: 1	2	3	4
Sub-period 2	1	0.944	-0.143	0.292	0.052
	2	0.212	0.942	-0.243	0.097
	3	-0.194	0.302	0.844	-0.399
	4	-0.161	0.041	0.379	0.910

Tables 7 and 8 present, for four-factor solutions, the transformation matrices between subperiods 1 and 2 and between subperiods 2 and 3, respectively. The results show that the factor structure remains very stable also in the case of four-factor solutions. However, compared to the three-factor solutions the factor structure between the second and third subperiod now seems to be slightly more stable than between the first and second subperiod. In the case of three-factor solutions the result was opposite. In addition, the four-factor solution produces the factors in the same order during the second and third subperiod. Respectively, the third and fourth factor have changed their positions between the first and second subperiod. Residual matrices for four-factor solutions are given in Tables 9 and 10. The matrices show also now only minor abnormal transformation.

Appendix 4 shows the results for five- and six-factor solutions. Tables 4.1-4.4 present the tranformation matrices and Tables 4.5 - 4.8 the corresponding residual matrices. The coefficient of coincidence on the main diagonal (if factors are produced in the same order) show considerable instability for five- and six-factor solutions.

Table 9. Residual matrix E_{12} and abnormal transformation for subperiod 2 (four-factor solution).

Firm	Factor 1	2	3	4	Abnormal transformation t_i^2
KOP	-0.056	-0.034	0.129	0.023	0.022
SYP	-0.033	0.253	-0.279	-0.249	0.205
POHJOLA	0.393	-0.070	-0.114	0.146	0.194
EFFOA	-0.132	0.246	-0.128	-0.024	0.095
KESKO	-0.038	0.277	-0.264	0.447	0.348
STOCK.	-0.005	-0.502	0.084	0.429	0.443
TAMRO	0.051	0.334	-0.355	0.276	0.317
ENSO	0.071	0.161	0.020	-0.285	0.113
FISKARS	-0.213	-0.009	-0.243	-0.047	0.106
HUHTAM.	-0.093	-0.085	0.163	-0.043	0.044
KAJAANI	-0.241	0.167	0.147	0.079	0.114
KEMI	0.214	0.358	-0.011	0.090	0.182
KONE	0.204	-0.356	0.431	-0.745	0.909
KYMMENE	-0.037	0.122	-0.206	-0.093	0.067
LASSILA	-0.532	-0.187	-0.017	0.589	0.665
LOHJA	0.309	0.006	-0.269	-0.110	0.180
METSÄL.	-0.013	0.570	-0.064	0.402	0.491
NOKIA	-0.061	0.119	0.141	-0.247	0.099
OTAVA	-0.450	0.521	-0.302	0.004	0.564
PARTEK	0.210	-0.139	-0.366	0.072	0.203
RAUMA-R.	-0.034	0.129	0.023	-0.099	0.028
ROSENLEW	0.253	-0.279	-0.249	0.232	0.258
SCHAUMAN	-0.070	-0.114	0.146	-0.097	0.048
SERLACHIUS	0.246	-0.128	-0.024	-0.181	0.110
SUOMEN S.	0.277	-0.264	0.447	0.231	0.400
SUOMEN TR.	-0.502	0.084	0.429	0.345	0.562
TAMFELT	0.334	-0.355	0.276	-0.210	0.358
TAMPELLA	0.161	0.020	-0.285	0.032	0.109
WÄRTSILÄ	-0.009	-0.243	-0.047	0.421	0.238
YHTYNEET	-0.085	0.163	-0.043	-0.130	0.053
Abnormal transformation s_j^2	1.620	1.979	1.614	2.311	7.523

Table 10. Residual matrix E_{12} and abnormal transformation for subperiod 3 (four-factor solution).

Firm	Factor 1	2	3	4	Abnormal transformation t_i^2
KOP	0.309	0.180	0.036	-0.015	0.130
SYP	0.281	0.124	0.054	-0.096	0.106
POHJOLA	0.015	0.556	-0.175	0.002	0.340
EFFOA	-0.360	0.089	-0.136	0.258	0.222
KESKO	0.087	0.044	-0.004	-0.122	0.024
STOCK.	0.264	0.451	0.091	0.117	0.295
TAMRO	0.023	-0.653	0.216	0.312	0.571
ENSO	-0.260	-0.414	0.530	0.244	0.579
FISKARS	0.403	0.354	0.110	-0.134	0.318
HUHTAM.	0.195	-0.312	-0.072	0.313	0.238
KAJAANI	0.005	0.045	0.010	-0.020	0.003
KEMI	-0.317	0.471	-0.583	-0.208	0.705
KONE	0.078	-0.138	0.108	0.299	0.126
KYMMENE	-0.021	-0.275	0.210	0.171	0.150
LASSILA	0.487	-0.406	0.303	0.012	0.495
LOHJA	-0.135	0.082	-0.076	-0.201	0.071
METSÄL.	-0.036	0.007	0.322	-0.073	0.110
NOKIA	0.063	-0.417	0.020	0.016	0.179
OTAVA	-0.347	0.198	0.709	-0.147	0.683
PARTEK	-0.210	0.172	0.245	-0.103	0.144
RAUMA-R.	0.180	0.036	-0.015	0.093	0.043
ROSENLEW	0.124	0.054	-0.096	-0.023	0.028
SCHAUMAN	0.556	-0.175	0.002	-0.040	0.342
SERLACHIUS	0.089	-0.136	0.258	0.203	0.134
SUOMEN S.	0.044	-0.004	-0.122	-0.004	0.017
SUOMEN TR.	0.451	0.091	0.117	-0.517	0.492
TAMFELT	-0.653	0.216	0.312	0.118	0.584
TAMPELLA	-0.414	0.530	0.244	-0.258	0.578
WÄRTSILÄ	0.354	0.110	-0.134	-0.338	0.270
YHTYNEET	-0.312	-0.072	0.313	0.257	0.266
Abnormal transformation s_j^2	2.573	2.530	1.943	1.197	8.243

So, the analysis presented shows that the stability of factor structure over time is best for three-factor solution (and also quite good for four-factor solutions). First, the factor structures are very stable according to the transformation matrices. Second, the Cattel's scree-tests and eigenvalues also support the results obtained by the transformation analysis (Appendix 3).

The following step involved examining the effect of factors on equilibrium returns. In cross-sections the dependent variable is the monthly mean return and the independent variables are factor loadings from factor analysis. The OLS regression coefficients would be the estimated risk premia. There is no meaning to the signs of the parameters in factor analysis and the scaling of the factors and then also of regression coefficients is arbitrary. Therefore, only the statistical significance of regression coefficients is relevant instead of their numerical values.

The results of the cross-sectional regressions are presented in Tables 11 and 12. They show that at least two different factors are priced, and the third and fourth factor have only a bit more explanatory power compared to the two-factor solution. On the other hand, the transformation analysis showed that we can extract three or four factors with the same content in different subperiods. The seeming inconsistency of the results rises from the fact that transformation analysis gives the number of the factors which have the same content in different subperiods. Regression analysis on the other hand gives the number of priced common factors. So, the transformation analysis gives the maximum number of priced common factors, i.e. the content of factors is the same in different time periods. It is possible that some very stable factors extracted by factor analysis are so firm- or industry-specific that their t-statistic is so low in cross-sectional regression that they are not common. However, it is very important to remember that transformation analysis is necessary in testing if the contents of factors in different subperiods are the same.

Table 11. Regression analysis estimates.

dependent variable: average monthly return for security
independent variables: factor loadings (k=3)

(t-values in parantheses)

Sub-period	Constant	Coefficients of Fact1	Fact2	Fact3	R-square	F
1	0.0257 (5.354)	-0.0067 (-1.122)	-0.0164 (-2.766)	-0.0078 (-1.347)	0.235	2.664
2	0.0196 (4.536)	-0.0237 (-4.160)	-0.0170 (-2.875)	-0.0038 (-0.574)	0.489	8.293
3	0.0185 (4.115)	0.0111 (1.712)	0.0097 (1.388)	0.0039 (0.540)	0.128	1.276

Table 12. Regression analysis estimates.

dependent variable: average monthly return for security
independent variables: factor loadings (k=4)

(t-values in parantheses)

Sub-period	Constant	Coefficients of Fact1	Fact2	Fact3	Fact4	R-square	F
1	0.0172 (2.486)	0.0017 (0.145)	-0.0107 (-1.574)	-0.0002 (-0.025)	0.0091 (1.285)	0.269	2.302
2	0.0199 (3.584)	-0.024 (-3.912)	-0.0150 (-2.241)	-0.0091 (-1.391)	0.0051 (0.740)	0.537	7.240
3	0.0165 (2.609)	0.0125 (1.649)	0.0116 (1.483)	0.0052 (0.580)	0.0036 (0.445)	0.133	0.959

Finally, we perform the test of APT against the "own variance" effect and the "firm size" effect (see Nai-Fu Chen 1983:1405-1409). The apparently significant explanatory power of the own variance suggests that APT may be "false", because investors should be able to diversify away the nonsystematic part of variance, that is, it should not be priced. The small firm effect has attracted attention in research (see Reinganum 1981 and Nai-Fu Chen 1983). Small firms seem to have higher risk-adjusted average returns than large firms. The dependent variable in Table 13 is residual term of equation (2.2) (three-factor case). Respective dependent variable in Table 14 is the residual term of equation (2.2) in four-factor case.

Table 13. Regression analysis estimates.

dependent variable: error term of the equation (2.2); three factors
independent variables: variance of the returns, size of the firm

(t-values in parantheses)

Sub-period	Constant	Coefficients of variance	size	R-square	F
1	-0.0069 (-2.501)	1.7606 (2.861)	0.0000 (0.242)	0.233	4.098
2	-0.0018 (0.718)	0.8774 (1.338)	-0.0000 (-0.377)	0.0845	1.245
3	0.0027 (0.638)	0.5873 (0.073)	-0.0001 (-2.102)	0.1513	2.406

Table 14. Regression analysis estimates.

dependent variable: error term of the equation (2.2); four factors
independent variables: variance of the returns, size of the firm

(t-values in parantheses)

Sub-period	Constant	Coefficients of variance	Coefficients of size	R-square	F
1	-0.0036 (-1.273)	1.0235 (1.609)	-0.0000 (-0.286)	0.0950	1.416
2	-0.0021 (-0.863)	0.9874 (1.550)	-0.0000 (0.365)	0.1059	1.600
3	0.00295 (0.701)	-0.0237 (-0.029)	-0.0001 (-2.046)	0.1412	2.220

The results show that during the first subperiod the own variance of the stock returns seems to have a slight explanatory power. The explanatory power becomes weaker when we go from the three-factor solution to the four-factor solution. This is parallel to our earlier interpretation that there seem to be three or even four priced factors although they are not all common factors. Finally, during the third subperiod we can find a slight size effect.

The results presented above have been extracted from the sample which includes all shares quoted at the Helsinki Stock Exchange during the whole period examined. At the end of the period the number of the firms was much larger. That is why we selected the second sample containing the returns of 30 most traded stocks for each subperiod. However, these two samples included mostly the same firms. Therefore also the results were very similar and the tables for the second subperiod have not been presented in the paper (during the third subperiod the values of t-statistics were a slight higher in the second sample).

5. SUMMARY

The main purpose of this study was to test the APT using monthly time series data of Finnish firms quoted on the Helsinki Stock Exchange and, as a part of this, especially to test, using transformation analysis, the stability of the factor structure over time. That means: transformation analysis tells us if the content of the factors remains the same in

different time periods (and in principle also in different samples during the same time period).

At the beginning of the empirical part of our research we saw that the mean and variance of the returns were not fully time invariant. In addition, hypotheses of normally distributed returns could be rejected among many firms examined. This indicates that the empirical results must be interpreted with moderate caution.

The empirical verification of the APT involved in the first stage the estimation of the systematic risks for each asset using factor analysis. The second stage involved testing by transformation analysis if the number and structure of factors remained unchanged or stable across different time periods. For stability analysis the whole period (from February 1970 to December 1986) was divided into three subperiods: 1970-1975, 1976-1980 and 1981-1986. The factor and transformation analysis showed that we had at least three very stable factors. Also the four factor solution was quite stable.

The next step involved examining the effects of factors on equilibrium returns. The results of the cross-sectional regression showed that we had at least two different factors which were priced. The third and fourth factor had only a slight additional explanatory power as compared to the two-factor solution. As a summary we can state that transformation analysis gave us the maximum number of stable factors which preserved the same content across different time periods and could thus serve as the common priced factors. Regression analysis then gave us the final number of common priced factors.

Finally, we found that the "own variance" and "the firm size" had only a slight explanatory power on equilibrium returns after the two or three priced factors. The analysis was carried out using the residual term of cross-sectional returns as the dependent variable in regression analysis.

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APPENDIX 1. STOCKS INCLUDED IN THE STUDY

Sample 1. (Same for all periods.)

KOP	=	KANSALLIS-OSAKE-PANKKI
SYP	=	UNION BANK OF FINLAND
POHJOLA	=	POHJOLA
EFFOA	=	EFFOA
KESKO	=	KESKO
STOCK.	=	STOCKMANN
TAMRO	=	TAMRO
ENSO	=	ENSO-GUTZEIT
FISK.	=	FISKARS
HUHTAM.	=	HUHTAMÄKI
KAJAANI	=	KAJAANI
KEMI	=	KEMI
KONE	=	KONE
KYMMENE	=	KYMMENE
LASSILA	=	LASSILA & TIKANOJA
LOHJA	=	LOHJA
METSÄL.	=	METSÄLIITTO
NOKIA	=	NOKIA
OTAVA	=	OTAVA
PARTEK	=	PARTEK
RAUMA-R.	=	RAUMA-REPOLA
ROSENLEW	=	W. ROSENLEW
SCHAUMAN	=	SCHAUMAN
SERLACHIUS	=	G.A. SERLACHIUS
SUOMEN S.	=	FINNISH SUGAR
SUOMEN TR.	=	SUOMEN TRIKOO
TAMFELT	=	TAMFELT
TAMPELLA	=	TAMPELLA
WÄRTSILÄ	=	WÄRTSILÄ
YHTYNEET	=	UNITED PAPER MILLS

Subperiods used in the study and numbers of monthly returns (sample 2.)

Subperiod 1:	1.2.1970 - 31.12.1975	N = 70
Subperiod 2:	1.1.1976 - 31.12.1980	N = 59
Subperiod 3:	1.1.1981 - 31.12.1986	N = 71

APPENDIX 2. STATIONARITY IN RETURN SERIES

Table 2.1. Numbers of stocks (N=30) in which the null hypothesis, H_0: means from subperiods are equal, could be rejected (two-tail t-test).

Significance level	1. vs. 2. subperiod	1. vs. 3. subperiod	2. vs. 3 subperiod
.10	10	7	23
.05	7	7	17
.01	2	3	13

Table 2.2. Numbers of stocks (N=30) in which the null hypothesis, H_0: variances from subperiods are equal, could be rejected (F-test).

Significance level	1. vs. 2. subperiod	1. vs. 3. subperiod	2. vs. 3 subperiod
.10	16	16	20
.05	16	14	17
.01	14	8	14

Table 2.3. Test of normality in stock returns (Shapiro - Wilk's W-test statistics).

	1970-75	1976-80	1981-86
KOP	0.960	0.882**	0.981
SYP	0.943**	0.977	0.897**
POHJOLA	0.928**	0.955	0.946**
EFFOA	0.972	0.904**	0.829**
KESKO	0.939**	0.972	0.981
STOCK.	0.942**	0.899**	0.873**
TAMRO	0.953*	0.785**	0.783**
ENSO	0.978	0.979	0.829**
FISK.	0.975	0.968	0.933**
HUHTAM.	0.894**	0.717**	0.913**
KAJAANI	0.970	0.915**	0.713**
KEMI	0.955*	0.968	0.905**
KONE	0.894**	0.512**	0.936**
KYMMENE	0.939**	0.982	0.973
LASSILA	0.920**	0.963	0.957*
LOHJA	0.952*	0.979	0.972
METSÄL.	0.816**	0.738**	0.763**
NOKIA	0.979	0.961	0.956*
OTAVA	0.669**	0.804**	0.878**
PARTEK	0.954*	0.885**	0.913**
RAUMA-R.	0.953*	0.975	0.925**
ROSENLEW	0.933**	0.987	0.953*
SCHAUMAN	0.969	0.963	0.981
SERLACHIUS	0.888**	0.928**	0.854**
SUOMEN S.	0.912**	0.901**	0.942**
SUOMEN TR.	0.906	0.917**	0.975
TAMFELT	0.941**	0.860**	0.928**
TAMPELLA	0.966	0.980	0.890**
WÄRTSILÄ	0.922**	0.893**	0.956*
YHTYNEET	0.918**	0.956*	0.975

H_0: returns are normally distributed
** H_0 will be rejected at 0.01 risk level
* H_0 will be rejected at 0.05 risk level

APPENDIX 3. SCREE TESTS FOR THE NUMBER OF FACTORS TO BE EXTRACTED

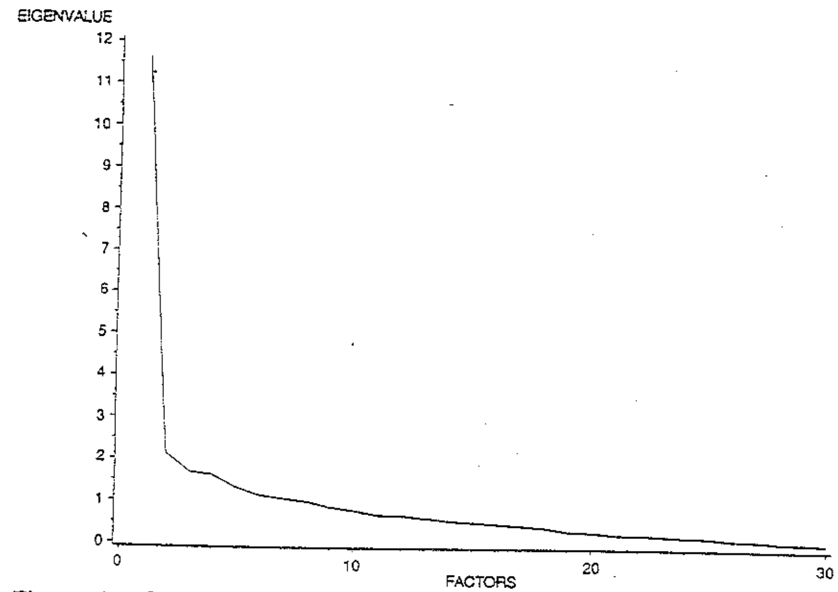

Figure 1. Subperiod 1.

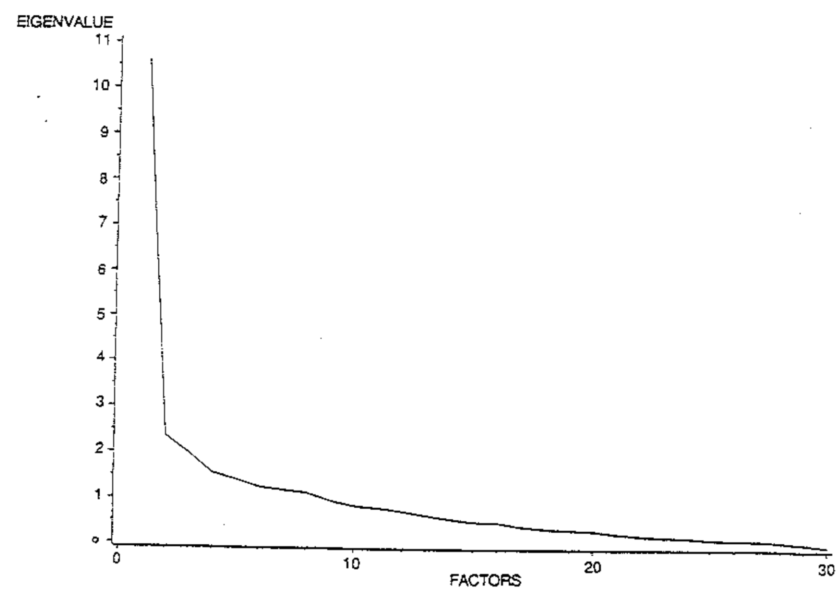

Figure 2. Subperiod 2.

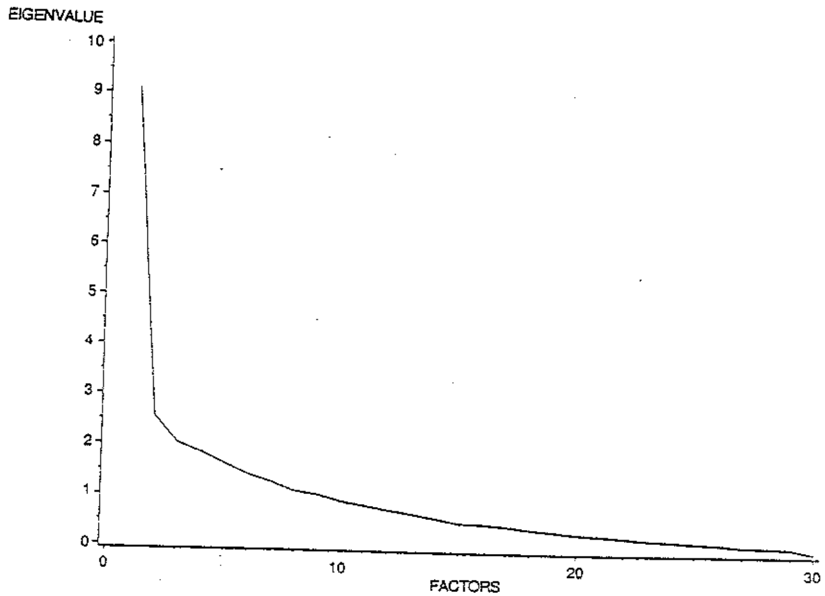

Figure 3. Subperiod 3.

APPENDIX 4. TRANSFORMATION AND RESIDUAL MATRICES FOR FIVE- AND SIX-FACTOR SOLUTIONS

Table 4.1. Transformation matrix between the factor patterns of returns (subperiod 1 vs. subperiod 2, five-factor solution).

		Subperiod 2				
	Factor	1	2	3	4	5
Sub-period 1	1	0.643	0.651	0.135	-0.377	-0.046
	2	-0.287	0.647	0.175	0.683	0.052
	3	0.340	-0.159	0.047	0.213	0.901
	4	0.494	-0.365	0.525	0.448	-0.384
	5	-0.380	-0.003	0.821	-0.382	0.190

Table 4.2. Transformation matrix between the factor patterns of returns (subperiod 2 vs. subperiod 3, five-factor solution).

		Subperiod 3				
	Factor	1	2	3	4	5
Sub-period 2	1	0.612	-0.309	0.647	-0.002	0.333
	2	0.275	0.847	-0.070	-0.173	0.415
	3	0.558	0.228	-0.073	0.428	-0.669
	4	-0.430	0.361	0.754	-0.034	-0.339
	5	-0.231	0.068	0.051	0.887	0.391

Table 4.3. Transformation matrix between the factor patterns of returns (subperiod 1 vs. subperiod 2, six-factor solution).

		Subperiod 2					
	Factor	1	2	3	4	5	6
Sub-period 1	1	0.506	0.119	0.226	0.716	-0.405	0.047
	2	0.775	0.344	-0.138	-0.351	0.366	-0.070
	3	0.048	-0.405	0.427	0.319	0.741	-0.027
	4	-0.122	0.341	0.019	0.069	0.187	0.910
	5	-0.325	0.751	0.404	0.086	0.148	-0.371
	6	0.138	-0.152	0.764	-0.501	-0.312	0.161

Table 4.4. Transformation matrix between the factor patterns of returns (subperiod 2 vs. subperiod 3, six-factor solution).

	Factor	Subperiod 3 1	2	3	4	5	6
	1	0.233	0.620	0.011	-0.226	0.538	0.470
Sub-	2	0.835	-0.184	0.401	-0.210	-0.254	0.008
period	3	0.279	0.559	-0.145	0.334	-0.019	-0.690
2	4	-0.184	0.387	0.462	0.453	-0.513	0.375
	5	-0.188	-0.137	0.745	0.130	0.536	-0.295
	6	0.321	-0.321	-0.225	0.756	0.313	0.275

Table 4.5. Residual matrix E_{12} and abnormal transformation for subperiod 2 (five-factor solution).

Firm	Factor 1	2	3	4	5	Abnormal transformation t_i^2
KOP	0.095	-0.145	0.170	0.074	-0.066	0.069
SYP	0.009	0.251	-0.223	-0.039	-0.302	0.205
POHJOLA	0.430	-0.117	-0.230	0.425	-0.127	0.449
EFFOA	-0.025	0.190	-0.076	0.011	-0.148	0.065
KESKO	0.006	0.454	-0.377	0.207	0.474	0.616
STOCK.	-0.099	-0.333	-0.148	0.400	0.393	0.457
TAMRO	0.148	0.286	-0.202	-0.284	0.408	0.392
ENSO	0.015	0.045	0.230	-0.410	-0.108	0.234
FISKARS	-0.233	0.000	-0.203	0.007	-0.045	0.098
HUHTAM.	-0.032	-0.141	0.270	-0.319	0.160	0.221
KAJAANI	-0.296	0.117	0.252	0.028	0.070	0.170
KEMI	0.084	0.521	-0.162	0.101	0.213	0.360
KONE	-0.138	-0.136	0.247	-0.266	-0.063	0.174
KYMMENE	-0.026	0.049	0.129	-0.331	-0.095	0.139
LASSILA	-0.543	-0.212	0.103	0.015	0.602	0.712
LOHJA	0.271	0.078	-0.117	-0.300	-0.008	0.183
METSÄL.	0.010	0.512	0.150	0.046	0.264	0.357
NOKIA	-0.055	-0.013	0.360	-0.120	-0.292	0.232
OTAVA	-0.368	0.307	0.200	-0.281	-0.232	0.403
PARTEK	0.271	-0.034	-0.288	-0.225	0.105	0.226
RAUMA-R.	-0.146	0.170	0.074	-0.066	-0.030	0.061
ROSENLEW	0.251	-0.223	-0.039	-0.302	0.340	0.321
SCHAUMAN	-0.117	-0.230	0.425	-0.127	-0.195	0.302
SERLACHIUS	0.190	-0.076	0.011	-0.148	-0.082	0.071
SUOMEN S.	0.454	-0.377	0.207	0.474	0.008	0.616
SUOMEN TR.	-0.333	-0.148	0.400	0.393	0.133	0.465
TAMFELT	0.286	-0.202	-0.284	0.408	-0.178	0.401
TAMPELLA	0.045	0.230	-0.410	-0.108	0.215	0.281
WÄRTSILÄ	0.000	-0.203	0.007	-0.045	0.509	0.302
YHTYNEET	-0.141	0.270	-0.319	0.160	-0.092	0.229
Abnormal transformation s_j^2	1.531	1.785	1.717	1.888	1.889	8.810

Table 4.6. Residual matrix E_{23} and abnormal transformation for subperiod 3 (five-factor solution).

Firm	Factor 1	2	3	4	5	Abnormal transformation t_i^2
KOP	0.419	0.268	-0.020	-0.039	0.120	0.264
SYP	0.258	0.121	0.148	-0.188	0.046	0.141
POHJOLA	0.119	0.389	-0.166	0.112	0.028	0.206
EFFOA	-0.429	0.028	-0.227	0.365	0.101	0.380
KESKO	0.080	0.037	0.044	-0.147	-0.124	0.047
STOCK.	0.343	0.250	0.223	0.115	-0.230	0.295
TAMRO	-0.174	-0.208	0.037	0.086	0.225	0.133
ENSO	-0.316	-0.071	0.406	0.090	0.038	0.279
FISKARS	0.137	0.340	0.096	-0.050	-0.316	0.246
HUHTAM.	0.170	-0.087	-0.128	0.066	0.468	0.277
KAJAANI	-0.240	0.001	0.318	-0.136	0.336	0.290
KEMI	0.057	0.436	-0.730	-0.166	-0.425	0.933
KONE	0.464	-0.219	-0.238	0.405	-0.284	0.565
KYMMENE	-0.249	-0.148	0.170	0.274	0.127	0.204
LASSILA	0.322	-0.404	0.313	0.226	0.010	0.416
LOHJA	-0.053	0.085	-0.114	-0.169	-0.191	0.088
METSÄL.	-0.164	0.037	0.381	-0.063	0.033	0.178
NOKIA	-0.196	-0.336	0.122	-0.052	0.252	0.232
OTAVA	-0.574	0.371	0.638	0.114	-0.220	0.935
PARTEK	-0.236	0.164	0.281	-0.094	-0.048	0.173
RAUMA-R.	0.268	-0.020	-0.039	0.120	-0.018	0.088
ROSENLEW	0.121	0.148	-0.188	0.046	-0.100	0.084
SCHAUMAN	0.389	-0.166	0.112	0.028	0.122	0.207
SERLACHIUS	0.028	-0.227	0.365	0.101	0.258	0.262
SUOMEN S.	0.037	0.044	-0.147	-0.124	0.093	0.049
SUOMEN TR.	0.250	0.223	0.115	-0.230	-0.270	0.251
TAMFELT	-0.208	0.037	0.086	0.225	-0.426	0.284
TAMPELLA	-0.071	0.406	0.090	0.038	-0.625	0.570
WÄRTSILÄ	0.340	0.096	-0.050	-0.316	-0.097	0.236
YHTYNEET	-0.087	-0.128	0.066	0.468	-0.281	0.326
Abnormal transformation s_j^2	2.111	1.526	2.060	1.100	1.843	8.640

Table 4.7. Residual matrix E_{23} and abnormal transformation for subperiod 2 (six-factor solution).

Firm	Factor 1	2	3	4	5	6	Abnormal transformation t_i^2
KOP	-0.359	0.155	-0.142	0.106	-0.089	-0.117	0.206
SYP	0.046	-0.158	0.217	-0.064	0.074	-0.325	0.190
POHJOLA	0.261	-0.266	-0.132	0.416	0.088	-0.041	0.338
EFFOA	-0.347	-0.029	0.231	-0.050	0.034	-0.061	0.182
KESKO	-0.047	-0.370	0.472	0.252	0.154	0.336	0.563
STOCK.	0.506	-0.170	-0.197	0.354	-0.191	0.510	0.745
TAMRO	0.331	-0.216	0.240	-0.242	0.169	0.329	0.409
ENSO	-0.015	0.279	0.091	-0.514	-0.052	0.075	0.359
FISKARS	0.147	-0.107	0.229	-0.109	-0.301	0.096	0.197
HUHTAM.	0.010	0.300	0.000	-0.387	-0.235	0.284	0.376
KAJAANI	0.091	-0.316	0.306	-0.117	-0.057	0.197	0.257
KEMI	0.581	0.199	-0.244	-0.171	0.211	0.050	0.513
KONE	-0.050	-0.058	0.180	-0.190	-0.205	-0.089	0.125
KYMMENE	0.113	0.138	0.085	-0.256	-0.307	-0.149	0.221
LASSILA	-0.255	-0.607	0.109	-0.086	0.065	0.596	0.812
LOHJA	0.034	-0.047	-0.090	0.508	-0.314	0.061	0.372
METSÄL.	0.539	0.093	0.090	-0.125	0.140	0.119	0.356
NOKIA	-0.017	-0.019	0.423	-0.100	-0.262	-0.125	0.274
OTAVA	0.196	-0.501	0.264	0.087	-0.295	-0.211	0.498
PARTEK	-0.079	0.130	-0.294	0.258	-0.233	0.136	0.249
RAUMA-R.	0.155	-0.142	0.106	-0.089	-0.117	0.032	0.078
ROSENLEW	-0.158	0.217	-0.064	0.074	-0.325	0.391	0.340
SCHAUMAN	-0.266	-0.132	0.416	0.088	-0.041	-0.298	0.359
SERLACHIUS	-0.029	0.231	-0.050	0.034	-0.061	-0.194	0.099
SUOMEN S.	-0.370	0.472	0.252	0.154	0.336	0.147	0.582
SUOMEN TR.	-0.170	-0.197	0.354	-0.191	0.510	-0.023	0.490
TAMFELT	-0.216	0.240	-0.242	0.169	0.329	-0.090	0.308
TAMPELLA	0.279	0.091	-0.514	-0.052	0.075	-0.014	0.359
WÄRTSILÄ	-0.107	0.229	-0.109	-0.301	0.096	0.312	0.273
YHTYNEET	0.300	0.000	-0.387	-0.235	0.284	-0.266	0.447
Abnormal transformation s_j^2	1.985	1.840	1.939	1.634	1.477	1.703	10.578

Table 4.8. Residual matrix E_{23} and abnormal transformation for subperiod 3 (six-factor solution).

Firm	Factor 1	2	3	4	5	6	Abnormal transformation t_i^2
KOP	0.142	0.069	-0.069	0.207	0.236	0.002	0.128
SYP	0.158	0.266	-0.203	-0.220	0.003	0.451	0.389
POHJOLA	0.394	0.586	-0.396	0.178	-0.036	0.114	0.702
EFFOA	-0.363	0.129	0.200	0.129	0.020	-0.496	0.451
KESKO	0.225	-0.179	0.230	-0.128	-0.028	-0.101	0.163
STOCK.	-0.157	0.250	0.143	0.083	-0.225	0.274	0.240
TAMRO	0.233	-0.278	0.410	-0.116	0.146	-0.263	0.403
ENSO	-0.338	-0.092	0.452	-0.047	-0.086	0.225	0.387
FISKARS	-0.033	0.112	-0.195	0.433	-0.165	0.215	0.313
HUHTAM.	0.116	-0.033	-0.117	0.055	0.449	-0.092	0.241
KAJAANI	-0.375	0.210	0.398	-0.030	-0.313	0.364	0.575
KEMI	-0.317	-0.101	-0.187	-0.063	0.641	-0.207	0.603
KONE	0.567	-0.426	-0.081	0.310	0.090	-0.317	0.714
KYMMENE	-0.173	-0.146	0.101	0.357	-0.105	0.117	0.214
LASSILA	0.682	-0.092	-0.182	0.017	-0.288	-0.143	0.611
LOHJA	0.083	-0.089	-0.142	-0.288	0.214	-0.213	0.209
METSÄL.	-0.122	0.022	0.371	0.031	-0.022	0.028	0.155
NOKIA	-0.257	-0.244	0.223	0.113	-0.312	0.260	0.353
OTAVA	-0.159	0.166	0.205	0.070	0.383	-0.297	0.334
PARTEK	-0.176	0.274	0.153	-0.127	-0.047	-0.100	0.158
RAUMA-R.	0.069	-0.069	0.207	0.236	0.002	0.061	0.112
ROSENLEW	0.266	-0.203	-0.220	0.003	0.451	-0.082	0.371
SCHAUMAN	0.586	-0.396	0.178	-0.036	0.114	0.124	0.562
SERLACHIUS	0.129	0.200	0.129	0.020	-0.496	0.138	0.339
SUOMEN S.	-0.179	0.230	-0.128	-0.028	-0.101	0.102	0.123
SUOMEN TR.	0.250	0.143	0.083	-0.225	0.274	-0.265	0.286
TAMFELT	-0.278	0.410	-0.116	0.146	-0.263	-0.499	0.598
TAMPELLA	-0.092	0.452	-0.047	-0.086	0.225	-0.634	0.675
WÄRTSILÄ	0.112	-0.195	0.433	-0.165	0.215	0.024	0.312
YHTYNEET	-0.033	-0.117	0.055	0.449	-0.092	-0.306	0.321
Abnormal transformation s_j^2	2.422	1.806	1.626	1.082	1.992	2.115	11.043